Cost of Business Cycle with Heterogeneous Trading Technology

(Preliminary  Do Not Circulate)

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Abstract

Our paper investigates the welfare cost of business cycle in an economy where households have heterogeneous trading technologies. In the economy with aggregate risk, the difference portfolio choices induced by heterogeneous trading technologies lead to a larger consumption inequality in equilibrium, while this source of inequality vanish in a economy without business cycle. Put simply, the consumption inequality due to the aggregate output fluctuation is amplified by heterogeneous trading technologies. The welfare cost of business is therefore larger in such an economy. In our benchmark economy with a reasonable low risk aversion rate, the business cycle costs 6.49% per period consumption to an average household, when we calibrate our economy to match the high risk premia.

Keywords: Cost of Business Cycle, Asset Pricing, Household Finance, Risk Sharing, Limited Participation (JEL code E32, E25, G12)
1 Introduction

In a calibrated representative agent model, Lucas (1987) shows a very insignificant welfare gain from the elimination of business cycle. His work suggests that the benefit of stabilizing the cyclical fluctuations in an economy is very limited. Hence, the study of business cycle might not be the top priority of macroeconomics. More recently, Lucas (2003) argues that most macroeconomics models still fail to generate a sizeable welfare cost of business cycle.

Our paper investigates the welfare cost of business cycle in an economy where households have heterogeneous trading technologies. In contrast to most papers in the incomplete market literature, the menu of assets available in our economy is quite rich. However, households in our model have heterogeneous abilities to access the menu of assets from the market. We distinguish between passive traders, who hold fixed portfolios of stock and bond, and active traders, who frequently adjust their portfolios in response to the change of investment opportunity set.

In equilibrium of our calibrated economy, heterogeneous trading technologies result in a clear difference between active traders and passive traders with respect to their portfolio choices. In response to the high risk premia, households with more sophisticated trading technologies take more aggregate risks by holding a large fraction of equity in their portfolios. They also optimally adjust their portfolios in response to the frequent change of investment opportunity set. Nevertheless, households with less sophisticated trading technology take a more cautious approach. On average, they bear less aggregate risk by holding less equity share in their portfolios and do not actively respond to the change of investment opportunity set. The active traders ultimately earn a high rate of return in their portfolios, accumulating more wealth and enjoying a high level of consumption, while the passive investors earn a low return in their portfolios, acquiring relatively low levels of wealth and consuming less. Hence, heterogeneous trading technologies induce more consumption inequality in this economy.

Clearly, this source of consumption inequality heavily depends on the level of risk premium as well as the variation of market price of risk. Both of them link tightly to the business cycle fluctuation. A reduction of aggregate output volatility helps to reduce not only the size but also
the time variation of risk premium. This reduction downplays the role of portfolio choice, and hence, improves the consumption inequality and welfare in the economy. Without aggregate risk, the inequality of consumption caused by heterogeneous trading technologies disappears since the composition and timing of portfolio choice no longer matter to the rate of return. In short, all assets are risk free and give exactly the same rate of return. A sophisticated trading technology does not have any advantage in an environment without aggregate risk.

We conjecture that heterogeneous trading technologies may contribute to the welfare cost of business cycle. In the economy with aggregate risk, the difference portfolio choices across households lead to a larger consumption inequality in equilibrium, while this source of inequality vanish in a economy without business cycle. Put simply, the consumption inequality due to the aggregate output fluctuation is amplified by heterogeneous trading technologies. The welfare cost of business is therefore larger in such an economy.

We use a modified macroeconomics model developed by Chien, Cole, and Lustig (2010b) to evaluate our conjecture quantitatively. Their model incorporates heterogeneous trading technologies into an otherwise standard macroeconomics model. In our use of the model, the heterogeneity in trading technology is calibrated to match the high risk premia shown in historical US data. The welfare cost is measured by the percentage of consumption compensation to households in an economy without business cycle so that they are indifference to an environment with aggregate fluctuations. We find that the welfare gain from the elimination of business cycle is large with a reasonably low risk aversion coefficient. In our benchmark case in which risk aversion coefficient is 4, the business cycle costs 6.49% per period consumption to an average household in our economy. The welfare cost is even larger, 9.37% of per period consumption, when the risk aversion rate is lowered to 3 and our economy is recalibrated to match the risk premia. These welfare cost numbers are definitely much larger than that of Lucas’s calculation.

Also, we compute the case in which the entire households are active traders and endowed with the same sophisticated trading technologies. Given the parameter values in our benchmark calibration, the results show a low risk premium and a much smaller cost of business cycle. The
importance of this computational exercise is twofold. First, it shows how an inferior investment technology among part of the investors influences the pattern of returns in asset markets. If all households make no investment mistake, the nice asset pricing result is dampened comparing to our benchmark economy. In addition, it demonstrates a large welfare loss in regards to unskillful investment strategies. The exercise shows that the welfare cost of business cycle is much smaller if no household makes investment error.

The assumption of heterogeneous trading technologies is critical to our results. The question thus arises: how realistic is the assumption of heterogeneous trading technologies. The answer can be found in empirical studies and data that have shown the high amount of heterogeneity in household portfolio choices. Different households behave as if they had access to different menu of tradable assets. In the United States, a majority of households do not invest directly in equity in spite of the sizeable historical equity premium. Even for those who participate in equity market, most do not frequently adjust the composition of their portfolios, regardless of the large countercyclical variation of Sharpe ratios in equity market. Put simply, they miss the market timing. However, there is a small fraction of households who hold a large share of stock and constantly change their equity position in response to the high, variable risk premia. Therefore, these households end up richer and have more exposure to aggregate risk. Parker and Vissing-Jorgensen (2009) show that the consumption of the richest 10% of US households is five times more exposed to aggregate risk than that of average households.

Our paper is closely related to a body of literature in which the distribution effects on consumption inequality might justify a large welfare cost of business cycle. Krusell and Smith (1999) propose an idea that business cycle might worsen the consumption inequality across a population while the impact to an average household is insignificant. The higher cost of business cycle is due to the distributional impact of consumption among the rich and poor. Evidently, the distributional impact is missing in a representative agent economy. Krusell, Mukoyama, Sahin, and Anthony A. Smith (2009) employ an incomplete market model calibrated to the wealth distribution in the United States, in order to evaluate the welfare effect from the elimination of business
cycle. Using the same parameter for risk aversion as in Lucas (1987), they find the welfare cost is approximately 0.1% of households’ consumption. Although the welfare cost number is already one magnitude larger than that from Lucas’s calculation, it is still neglectable in an economic sense. Storesletten, Telmer, and Yaron (2001) consider the welfare cost of business cycle in an environment with the countercyclical variation of idiosyncratic shock. A more volatile idiosyncratic income risk during the recession can amplify the cost of aggregate risk on individual consumption and leads to a higher distributional impact. Although the welfare cost of business cycle is still insignificant, the cost increases fast as the risk aversion coefficient increases. Krebs (2007) extends the concept of idiosyncratic labor income shock by adding a permanent job displacement risk. The risk of job displacement is assumed to be closely associated with the business cycle. His paper finds a sizeable cost of business cycle due to the important role played by the displacement risks.

The central idea of this corpus of literature is to translate a small scale of aggregate risk into a large consumption inequality. We follow this concept, but the large consumption inequality in our model is caused by a novel feature: heterogeneity in trading technologies. Most articles in this literature operate within incomplete market models in which all households can only trade a very limited menu of tradable assets. However, the actual menu of assets that households can trade is quite rich. Instead of assuming a limited set of tradable assets, we introduce the heterogeneous ability in accessing the menu of assets. This idea is motivated by the empirical evidence of heterogeneity in portfolio choices. With heterogeneous trading technologies, households’ total incomes not only differ because of their idiosyncratic risk in labor income but also because of the variation of their investment returns resulting from the heterogeneity in trading technologies. In addition, heterogeneous trading technologies affect to the return of portfolio choices only in an economy with aggregate risk. Without business cycle, the cost of consumption inequality from different trading technologies vanishes. Therefore, the heterogeneity in trading technologies only enlarges the consumption inequality in an economy with aggregate risk and hence amplifies the cost of business cycle.

Alvarez and Jermann (2004) demonstrate a close link between cost of business cycle and risk
premium. They offer an alternative way to measure the cost of business cycle by asset pricing data. Their work illustrates that, under a representative agent economy, the welfare cost of business cycle is limited by an upper bound, which can be approximated by the risk premium between an aggregate consumption claim and a risk free asset, regardless of the assumption of utility function. One of the contributions in Alvarez and Jermann (2004) is the following observation: if one model can generate a reasonable asset pricing result, then it might produce a large welfare cost of business cycle as well. Our calibrated model does produce a reasonable asset pricing result; however, the large welfare cost of business cycle results mainly from the consumption inequality induced by heterogeneous trading technologies, not directly from the variation of aggregate consumption.

Our paper also relates to a fast growing body of literature on household finance. Campbell (2006) points out that some households might make various mistakes when facing complicated financial decisions. Our paper evaluates the welfare cost of some of these mistakes. In our model economy, passive traders make two types of mistakes. Households that do not participate in equity market forgo the large equity premium. Those equity investors who do not frequently change their portfolio choice miss the market timing. By comparing the results of two model economies, with and without heterogeneous trading technologies, we demonstrate that these investment mistakes not only affect the patterns of risk premium but also cause a large welfare cost. If all households comprise active traders who do not make any investment mistake, then the risk premium is low and stable in our calibrated economy. Moreover, the welfare cost of business cycle is almost neglectable and similar to the result found by Lucas (1987). This finding emphasizes the importance of studying household finance because preventing investment mistakes can considerably improve welfare.

The outline of the paper is as follows: Section 2 describes the environment and the trading technologies. Section 3 discusses the calibration of the model. Section 4 displays the results for the benchmark model. Section 6 demonstrates that our results are robust to Epstein-Zin preference. Finally, section 7 offers our conclusion.
2 Model

The model setup follows closely to the one in Chien, Cloe and Lustig (2010). The novel feature of their model is imposing the restrictions on the menu of assets that households are able to trade, which defines the trading technology a household owns. These restrictions are imposed exogenously. The goal of these restrictions is to capture the observed portfolio behavior of most households.

We will refer to households as being passive traders if they take their portfolio composition as given and simply choose how much to save or dissave in each period. Other households constantly manage their portfolio in response to changes in the investment opportunity set. We refer to these traders as active traders since they optimally adjust the composition of their portfolio every period. Note that those passive traders are completely rational expect their portfolio choice decision. They fully acknowledge the rate of return of their portfolios and adjust their consumption and saving decisions optimally. Hence, our results are clearly driven by the only additional, novel assumption, heterogeneous trading technologies, in contrast to most papers in the incomplete market literature.

2.1 Environment

This endowment economy consists of a continuum of heterogeneous households who are subject to idiosyncratic income shocks as well as aggregate output shocks. The total measure of households is normalized to be one. The heterogeneity across households arises from two assumptions. In the planning period 0, households are received a onetime permanent shock on their trading technologies, while all other characteristics of households are identical. Starting at period 1, these households also differ in terms of the realization of idiosyncratic income shock at all subsequent periods. Initially, all households start with the same initial wealth and face identical stochastic process of idiosyncratic income shocks.

In the model time is discrete, infinite, and indexed by $t = 0, 1, 2, ...$ The first period, $t = 0$, is a planning period in which financial contracting takes place. We use $z_t \in Z$ to denote the aggregate shock in period $t$ and $\eta_t \in N$ to denote the idiosyncratic shock in period $t$. $z^t$ denotes the history of aggregate shocks, and similarly, $\eta^t$ denotes the history of idiosyncratic shocks for a household.
The idiosyncratic events $\eta$ are i.i.d. across households with mean normalized to be one. We use $\pi(z^t, \eta^t)$ to denote the unconditional probability of state $(z^t, \eta^t)$ being realized. The events are first-order Markov, and we assume that

$$\pi(z_{t+1}, \eta_{t+1}|z^t, \eta^t) = \pi(z_{t+1}|z_t)\pi(\eta_{t+1}|z_{t+1}, \eta_t).$$

Since we can appeal to a law of large number, $\pi(z^t, \eta^t)/\pi(z^t)$ also denotes the fraction of agents in state $z^t$ that have drawn a history $\eta^t$. We use $\pi(\eta^t|z^t)$ to denote that fraction. We introduce some additional notation: $z^{t+1} \succ z^t$ or $y^{t+1} \succ y^t$ means that the left hand side node is a successor node to the right hand side node. We denote by $\{z^t \succ z^t\}$ the set of successor aggregate histories for $z^t$ including those many periods in the future.

There is a single non-durable goods available for consumption in each period, and its aggregate supply is given by $Y_t(z^t)$, which evolves according to

$$Y_t(z^t) = \exp\{z_t\}Y(z^{t-1}),$$

with $Y(z^0) = 1$. This endowment goods comes in two forms. The first part is non-diversifiable income that is subject to idiosyncratic risk and it is given by $\gamma Y(z^t)\eta_t$; hence $\gamma$ is the share of income that is non-diversifiable. The second part is diversifiable income, which is not subject to the idiosyncratic shock, and is given by $(1-\gamma)Y_t(z^t)$.

All households are infinitely lived and rank stochastic consumption streams according to the following criterion

$$U(\{c_t\}) = \sum_{t \geq 1}^{\infty} \beta^t \pi(z^t, \eta^t) c_t(z^t, \eta^t)^{1-\alpha} 1 - \frac{1}{\alpha},$$

where $\alpha > 0$ denotes the coefficient of relative risk aversion, and $c_t(z^t, \eta^t)$ denotes the household’s consumption in state $(z^t, \eta^t)$. 
2.2 Assets Traded

There are three assets available in this economy: equity, bond and contingent claims on aggregate shocks. All of these assets are claims to diversifiable income. The actual menu of assets that a household can trade depends on the trading technology. However, this is still an incomplete market economy since there is no state contingent claim to the non-diversifiable income (labor income).

Following Abel (1999), we simply consider equity as a leveraged claim to aggregate diversifiable income \( ((1 - \gamma)Y_t(z^t)) \). The leverage ratio is assumed to be constant over time and denoted by \( \psi \). Let \( B_t(z^t) \) denotes for the supply of one period risk free bond in period \( t \) and \( R_{t,t-1}^f(z^{t-1}) \) denotes the risk free rate between period \( t - 1 \) and \( t \) given the aggregate history \( z^{t-1} \). With a constant leverage ratio, the total supply of \( B_t(z^t) \) has to adjusted such that

\[
B_t(z^t) = \psi \left[ \omega_t(z^t) - B_t(z^t) \right],
\]

where \( \omega_t(z^t) \) is denoted for the price of a claim to aggregate diversifiable income. Because the aggregate diversified income can be decomposed into the interest payment to bondholders and dividend payment to shareholders, the dividend payment, \( d_t(z^t) \), is given by

\[
D_t(z^t) = (1 - \gamma)Y_t(z^t) - R_{t,t-1}^f(z^{t-1})B_{t-1}(z^{t-1}) + B_t(z^t)
\]

A trader who invests a fraction \( \psi/(1 + \psi) \) of their wealth in bonds and the rest in equity is holding the market portfolio. We denote the price of the equity (a claim to dividends payment \( D_t(z^t) \)) by \( V_t(z^t) \).

The third available asset is the aggregate state contingent claims. We denote the price of a unit claim to the final good in aggregate state \( z^{t+1} \) acquired in aggregate state \( z^t \) by \( Q_t(z_{t+1}, z^t) \).

We consider a household entering the period with net financial wealth \( \hat{a}_t(z^t, \eta^t) \). This household buys securities in financial markets (state contingent claims \( a_t(z^{t+1}, \eta^{t+1}) \), risk free bonds \( b_t(z^t, \eta^t) \), and equity shares \( s_t^D(z^t, \eta^t) \)) and consumption \( c_t(z^t, \eta^t) \) in the good markets subject to this one-
period budget constraint:

\[
\sum_{z^{t+1} > z^t, \eta^{t+1} > \eta^t} Q_t(z_{t+1}, z^t) a_t(z^{t+1}, \eta^{t+1}) \pi(\eta_{t+1} | z_{t+1}, \eta_t) + s_t^D(z^t, \eta^t) V_t(z^t) \\
+ b_t(z^t, \eta^t) + c_t(z^t, \eta^t) \leq \hat{a}_t(z^t, \eta^t) + \gamma Y_t(z^t) \eta_t, \text{ for all } z^t, \eta^t,
\]  

(3)

where \( \hat{a}_t(z^t, \eta^t) \), the agent’s net financial wealth in state \((z^t, \eta^t)\), is given by the payoffs of his state-contingent claim acquired last period, the payoffs from his equity position and the risk free bond payoffs:

\[
\hat{a}_t(z^t, \eta^t) = a_{t-1}(z^t, \eta^t) + s_t^D(z^{t-1}, \eta^{t-1}) \left[ D_t(z^t) + V_t(z^t) \right] + R_{t,t-1}(z^{t-1}) b_{t-1}(z^{t-1}).
\]  

(4)

2.3 Trading Technology

There are two main classes of traders: active traders and passive traders. Active traders are able to trade state contingent claims on aggregate shock. They change their portfolio composition of equity of bond optimally every period in response to the variation of state contingent prices. These active traders make no mistake on their investment choices. Passive traders cannot trade state contingent claims and their portfolio choice is limited by an exogenously assigned and fixed target \( \varpi \) for the equity share. We refer to these traders as passive precisely because of their inelastic respond to the changes of investment opportunity. These passive traders potentially make two kinds of investment mistakes. First, they miss the market timing if the volatility of the market price of risk is not constant in equilibrium. Second, for those passive traders who hold small or zero fraction of equity in their portfolios, they relinquish the risk premia. The welfare cost of their mistakes may be large in the equilibrium exhibiting a large risk premia and a volatile Shape Ratio in equity.

In addition, households face exogenous limits on their net asset positions, or solvency constraints,

\[
\hat{a}_t(z^t, \eta^t) \geq 0.
\]  

(5)
Traders cannot borrow against their future labor income.

### 2.4 Measurability Restrictions

To capture these portfolio restrictions implied by the different trading technologies, we use measurability constraints (see Chien, Cole, and Lustig (2010b) for a detailed discussion) on net wealth. These restrictions allow us to solve for equilibrium allocations and prices without searching for all the equilibrium prices that clear each security market.

**Active Trader** Since idiosyncratic shocks are not spanned for the active trader, his net wealth needs to satisfy:

\[
\hat{a}_t (z^t, [\eta_t, \eta^{t-1}]) = \hat{a}_t (\tilde{z}^t, [\tilde{\eta}_t, \eta^{t-1}]),
\]

for all \( t \) and \( \eta_t, \tilde{\eta}_t \in N \).

**Passive Trader** Passive traders who hold a fixed fraction \( \varpi \) in levered equity and \( 1 - \varpi \) in non-contingent bonds in their portfolio earn a portfolio return:

\[
R^p_t (\varpi, z^t) = \varpi R^d_{t,t-1} (z^t) + (1 - \varpi) R^f_{t,t-1} (z^{t-1})
\]

where \( R^d_{t,t-1} (z^t) \) denotes for the equity return between period \( t \) and \( t - 1 \) given the realization of history state \( z^t \). Hence, their net financial wealth satisfies this measurability restriction:

\[
\frac{\hat{a}_t ([z_t, z^{t-1}], [\eta_t, \eta^{t-1}])}{R^p_t (\varpi^*, [z_t, z^{t-1}])} = \frac{\hat{a}_t ([\tilde{z}_t, z^{t-1}], [\tilde{\eta}_t, \eta^{t-1}])}{R^p_t (\varpi^*, [\tilde{z}_t, z^{t-1}])},
\]

for all \( t, z_t, \tilde{z}_t \in Z \), and \( \eta_t, \tilde{\eta}_t \in N \). If \( \varpi = 1/(1 + \psi) \), then this trader holds the market in each period and earns the return on a claim to aggregate tradable income. There is a special type of passive traders who do not participate the equity market and only hold risk free asset. We call them non-participants, which can be think as those passive traders with zero equity target share, \( \varpi = 0 \).
2.5 Equilibrium

An equilibrium for this economy is defined in a standard way. It consists of a list of bond, equity and state contingent claims holdings, a consumption allocation and a list of bond, equity and state contingent prices such that: (i) given these prices, a trader’s asset and consumption choices maximize her expected utility subject to the budget constraints, the solvency constraints and the measurability constraints, and (ii) all asset markets clear.

3 Calibration

This section discusses the calibration of the parameters and the endowment processes, and the composition of trader pools. Section 4 use a calibrated version of the model to evaluate the welfare effect of eliminating business cycle.

To compute the equilibrium of this economy, we follow the algorithm described by Chien, Cole, and Lustig (2010b), who use truncated aggregate histories as state variables. We keep track of lagged aggregate histories up to 7 periods.

3.1 Preferences and Endowments

Lucas (2003) suggested that a reasonable risk aversion coefficient should lie between 1 and 4. Our benchmark calibration set the coefficient of relative risk aversion $\alpha$ to four. In order to check the robustness of our results with respect to the choice of risk aversion rate, we conduct a sensitivity analysis in subsection 4.3. The model is calibrated to annual data. The time discount factor $\beta$ is set to .95. To match the collateralizable wealth to income ratio in the data, the tradable or collateralizable income share $1 - \gamma$ is set to 10%, as discussed below. The average ratio of household wealth to aggregate income in the US is 4.30 between 1950 and 2005. The wealth measure is total net wealth of households and non-profit organizations (Flow of Funds Tables). With a 10% collateralizable income share, the implied ratio of wealth to consumption is 5.40 in
The process of aggregate output is calibrated to match the aggregate consumption growth moments from Alvarez and Jermann (2001) and Mehra and Prescott (1985). The average consumption growth rate is 1.83% and the standard deviation is 3.15%. Expansions are more frequent than recessions: 73% of realizations are high aggregate consumption growth states. We calibrate the labor income process as in Storesletten, Telmer, and Yaron (2004) and Storesletten, Telmer, and Yaron (2007), except that we eliminate the counter-cyclical variation of labor income risk. The variance of labor income risk is constant in our model. A constant labor income risk setup highlights the role of the new feature (heterogeneous trading technologies) considered in this paper. The main driving force of our result comes from the heterogeneity in trading technology, not from the counter-cyclical variation of labor income risk. The Markov process for $\log \eta(y, z)$ has a standard deviation of 0.71, and the autocorrelation is 0.89. We use a 4-state discretization for both aggregate and idiosyncratic risk. The elements of the process for $\log \eta$ are 0.38 and 1.61 for low and high shock respectively.

Equity in our model is simply a leveraged claim to diversifiable income. In the Flow of Funds, the ratio of corporate debt-to-net worth is around 0.65, suggesting a leverage parameter $\psi$ of 2. However, Cecchetti, Lam, and Mark (1990) report that standard deviation of the growth rate of dividends is at least 3.6 times that of aggregate consumption, suggesting that the appropriate leverage level is over 3. Following Abel (1999) and Bansal and Yaron (2004), we choose to set the leverage parameter $\psi$ to 3.

3.2 Composition of Trader Pools and Equity Target Share

We set the fraction of non-participant to 50% according to the fact that 51.1% of households reported owning stocks directly or indirectly form the most recent Survey of Consumer Finances. In order to match the large equity premium (7.53%) measured in post-war US data, a relatively

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1 As is standard in this literature, we compare the ratio of total outside wealth to aggregate non-durable consumption in our endowment economy to the ratio of total tradeable wealth to aggregate income in the data. Aggregate income exceeds aggregate non-durable consumption because of durable consumption and investment.
small fraction of active traders need to bear the residual aggregate risks created by non-participant. Hence, we set the share of active traders equal to 10%, and passive equity traders to 40%.

Among those households who hold equity, we are not able to distinguish in the data between active traders and passive equity traders. It is hard to calibrate the equality target share of passive equity traders, since we do not know who they are. However, empirical studies have shown that rich households tend to be more sophisticated traders. We consider the richest 10% households are active traders and the poorest 50% households are non-participants. The equity target share of passive equity traders is therefore calibrated to match the average fraction of equality share among those households who possess a percentile of wealth between 50% and 90%. According to the data from Survey of Consumer Finances, the average equity share among these middle wealthy households is 24.2%. We therefore set the equity target share of passive equity traders to 24%. This calibration also reflects the observation that the rich tends to hold a higher fraction of equity than the poor.

[Table 1 about here.]

4 Quantitative Results

We consider two cases in our quantitative exercise. The first is our benchmark economy where the parameters are calibrated as described above. We refer this case as BM (benchmark) economy. We also consider another economy in which there is no heterogeneity in trading technologies. All households are able to access all assets available in the market without any restriction. We label this economy as NHT (no heterogeneous trading) economy. Table 1 reports moments of asset prices in both economies we consider. These results are generated by simulating data of a model with 12,000 agents for 10,000 periods. Panel I and II report results for the case of BM economy and NHT economy respectively.
4.1 Asset Prices

In the upper part of table I, we report the maximum unconditional Sharpe ratio (SR) or market price of risk (\( \frac{\sigma(m)}{E(m)} \)), the standard deviation of the maximum SR (\( Std(\frac{\sigma(m)}{E(m)}) \)), the equity risk premium \( E(R_{t+1,t}^D - R_{t+1,t}^f) \), the standard deviation of excess returns \( \sigma(R_{t+1,t}^D - R_{t+1,t}^f) \), the Sharpe ratio on equity, the mean risk-free rate \( E(R_{t+1,t}^f) \) and the standard deviation of the risk-free rate \( \sigma(R_{t+1,t}^f) \).

**Benchmark Economy** In our benchmark economy, the maximum SR is 0.37 and the standard deviation of the maximum SR is 4.04%. The equity premium is 7.53% and the Sharpe ratio on equity is .37. The average risk-free rate is 1.91% and its volatility is 2.27%. Clearly, our benchmark economy generates several key features of asset pricing observed in the data, such as high equity premia; low and stable risk free interest rate and a relatively volatile Shape Ratio.

The large fraction of non-participant traders is critical to our result of high risk premia. Those households who only hold risk free asset do not take any aggregate risk since their portfolio return is independent of the realization of aggregate shocks. Additionally, passive equity traders only take a limited amount of aggregate risk because of their constant target equity share. Therefore, a large amount of aggregate risk has to be absorbed by a small fraction of active traders. In equilibrium, a high risk premium is necessary so that active traders are willing to bear these extra aggregate risks. The key mechanism is to concentrate the aggregate risk on a small fraction of population.

**No Heterogeneous Trading Economy** In an economy where the entire households are active traders, the asset pricing results are dampened. Comparing to our benchmark case, the maximum SR is only 0.15 and the standard deviation of the maximum SR is down to 1.01%. The equity premium reduces to 3.01% and the Sharpe ratio on equity is only 0.15. The average risk-free rate increases to 3.09% and its volatility remains roughly the same, 2.27%. The heterogeneity in

\(^2\)The Shape ratio estimated from the data is enormous and highly counter-cyclical. Our model still falls short to match data quantitatively. However, Chien, Cole, and Lustig (2010a) extend a similar version of our model by introducing inertia investment behavior into the model. Their work shows that the inertia investment behavior does help to explain the large counter-cyclical variation of SR.
trading technologies considerably affects the patterns of asset pricing results. The reason for the low equity premium is clear: the aggregate risk is equally borne by all households and there is no concentration of risk.

**Approximation** In general, the prices of state contingent claims depend on the entire aggregate history. However, we are unable to keep track of the entire aggregate history of shocks in the state space. Following Chien, Cole, and Lustig (2010b), we use truncated aggregate histories as state variables to forecast state contingent prices. In order to show the accuracy of our approximation, we report the implied $R^2$ in a linear regression of the actual realization of state contingent prices on the predicted state contingent prices which based on the truncated aggregate histories. This measure of precision is closer to the one by Krusell and A. Smith (1998). As shown in table I, the $R^2$ for this regression is higher than 0.9995 in our benchmark case and higher than 0.9999 in the case without heterogeneous trading technologies. This result shows that our approximation is accurate and comparable to others reported in the literature for models with heterogeneous agents and incomplete markets.

### 4.2 Welfare Costs of Business Cycle

The welfare cost of eliminating business cycle is defined as the average welfare difference between two economies: one with aggregate shocks and the other without. Given the fact that households are heterogeneous in terms of their wealth, income shocks and trading technologies in the long run equilibrium, the average welfare of one economy is computed by taking the expectation across all idiosyncratic features over the population. In addition, we measure the average welfare gap between two economies by the percentage of per period consumption. Therefore, the welfare cost is defined as the expected percentage of consumption compensation to a household assigned to an economy without business cycle so that this household is indifferent to join our benchmark economy. The welfare cost is reported in the bottom of table I.
**Benchmark Economy**  In our benchmark economy, the welfare cost of business cycle is 6.49%. This number means that an average household in our benchmark economy is willing to relinquish up to 6.49% of his per period consumption in order to be in another economy without aggregate uncertainty given all others equal. The welfare cost is much large than those findings in the body of literature. This result demonstrates that heterogeneous trading technologies play an important role not only on the patterns of asset pricing but also on the distributional effects of consumption. In our benchmark economy, those households with better trading technologies earn a higher return on their wealth, while those households with less sophisticated trading technologies earn a lower return. This phenomenon generates a distributional impact on consumption and eventually widens the welfare gap across households. However, the welfare inequality caused by heterogeneous trading technologies vanishes in an economy without business cycle – the reason for which is quite simple: since all assets are risky free, the portfolio choice between equity and risk free bond does not affect the return of their portfolios. There is no investment advantage of a household who owns an advanced trading technology. The return of wealth between active trader and passive trader are identical in an environment without aggregate risk.

**No Heterogeneous Trading Economy**  In our second exercise in which the entire households are active traders, the welfare cost is only 1.46%. The low welfare cost is consistent with those findings in the cost of business cycle literature. This result suggests that the welfare cost of business cycle is less significant in an environment where all agents have sophisticated trading technology and make no investment mistakes. This outcome can be easily understood. Because all households have equal trading technology, there is no heterogeneity in portfolio choice. The income and consumption inequality are reduced in this case. The aggregate risk does not amplify the distributional impact on consumption anymore, so the welfare cost of business decreases considerably.

The amount of reduction in the welfare cost of business cycle can be thought as the average welfare gain form preventing the investment mistakes made by passive traders in our model. Clearly, our results show that the average welfare loss due to these investment errors is large, 5.03% of per period consumption (the welfare cost difference between BM economy and NHT economy). This
number implies that the welfare cost of being stuck with inferior trading technologies is sizeable. Also, our findings also shed a light on the importance of understanding the investment mistakes made by passive traders, since avoiding them can improve the average welfare of the society.

4.3 Sensitivity Analysis

Risk Aversion Coefficient Our benchmark calibration set the risk aversion coefficient to 4. Although our choice of risk aversion is in the range considered in many macroeconomics models, it is different from the choice by Lucas (1987), who uses a log utility. More importantly, the welfare cost of business cycle might be sensitive to the risk aversion rate. Here, we investigate the sensitive of our results to the change of risk aversion coefficient. We conduct two sensitivity analyses with respect to the change in risk aversion rate. In each analysis, we vary the risk aversion coefficient from 3 to 1.

The first analysis only considers the change of risk aversion rate and keeps all other parameters unchanged. Table II reports the results of our first analysis. The decrease of risk aversion rate lowers the risk premia as well as welfare cost. The risk premium drops substantially, from 5.18% with risk aversion coefficient 3, to 1.28% in the log utility case. In addition, the welfare cost of eliminating business cycle decreases in a non-linear pattern. With risk aversion rate 3 or 2, the welfare costs are still very significant, 5.27% and 4.22% respectively. However, it reduces sharply to 0.6% when we consider the case of log utility. This analysis demonstrates a close relationship between risk premia and welfare cost of business cycle. This is not surprising, because the welfare cost of business cycle in our paper depends critically on the magnitude of consumption dispersion, which is based upon the return difference between equity and risk free bond. As the risk premium decreases, the heterogeneity in wealth returns reduces as does the welfare cost.

The first analysis indicates that when households become less and less risk averse, our model misses the calibration target, equity premia, more and more. Therefore, we conduct a second sensitivity analysis. For each risk aversion rate considered above, we adjust the composition between
active traders and passive equity traders in order to match the historical risk premia as much as possible, while keeping all other parameters fixed. The results of the second analysis are shown in table 3.

The first panel reports the results of the case in which risk aversion coefficient is 3. In order to match the high historical risk premia, the fraction of active traders and passive equity traders are adjusted to be 3% and 47% respectively. The asset pricing results are similar to the one in our benchmark economy. The risk premia is high, 7.38%, and volatile, 19.20% in standard deviation, while the risk free rate is low, 2.25% and stable, 1.66% in standard deviation. Most importantly, the welfare cost of inflation increases to 9.37%. The higher welfare cost result can be understood in the followings. First, active traders are those who respond to the change in state contingent prices and are those who bear extra aggregate risk. Put simply, they are marginal traders who price the risk premia. Second, if these active traders still bear the same amount of aggregate risk as in our benchmark case, the risk premium in this economy will drop since their risk aversion rate is lower now. In order to maintain the same, high level of risk premia while giving a lower risk aversion rate, a larger amount of aggregate risk has to be concentrated and borne by a smaller fraction of active traders. As the fraction of active trader adjusts from 10% to 3%, each active trader bears more aggregate risk, but able to enjoy a higher level of consumption in terms of compensation. The reduction in the fraction of active traders worsens the consumption inequality and consequently increases welfare cost of business cycle.

Panel 2 and 3 report the results of the case with $\alpha = 2$ and 1 respectively. In both cases, we are unable to match the high risk premia shown in the data even when the fraction of active traders is set to be only 1% of total population. The risk premia of both cases is significantly smaller: 5.78% for the case with $\alpha = 2$ and only 3.04% for the case with log utility. Nevertheless, the welfare costs of business is even higher, 9.56%, in the case where risk aversion coefficient is 2. The reason for which is simply because of a higher inequality in consumption. Although the lower risk premia reduce the inequality of consumption by shirking the heterogeneity of wealth return across population, the smaller fraction of active traders amplifies the consumption inequality even more.
The second effect on consumption inequality due to the shirking size of active traders dominates the first effect resulting from lower risk premia. Consequently, the welfare cost increases slightly. The last panel reports the result in a log utility case. The welfare cost drops substantially from 9.56% to 3.81% when the risk aversion coefficient changes from 2 to 1. The result is not surprising given the same composition of traders in both panel II and III.

The second sensitivity analysis demonstrates that the welfare cost of business cycle is even larger with a lower risk aversion coefficient whenever the historical, high risk premia can be matched in our calibration economy. Additionally, in the log utility case, the welfare cost of business cycle is still significant even if our calibration did not match the risk premia. The welfare cost number is 3.81% in the log utility when active traders comprise 1% of total population.

[Table 3 about here.]

5 Conclusion

Our paper demonstrates that heterogeneous trading technologies can play an important role not only to the patterns of asset pricing but also to the welfare cost of business cycle. In our calibrated model, a large amount of aggregate risk is borne by a small fraction of households while a large fraction of households bear little or even no aggregate risk. The concentration of risk on a limited set of households drives a large risk premia in our model. As a result, sophisticated investors who hold large fraction of equity compensated with much higher return in wealth while less sophisticated investors earn a low return on their wealth. The wealth return difference worsens the income and consumption inequality. In addition, the new feature of our model, heterogeneous trading technologies, has no distributional effect on consumption in an economy without aggregate shocks, because the return difference between stock and bond vanishes. A cease of aggregate shocks can greatly improve the consumption inequality caused by the heterogeneity in investment behavior. Therefore, the welfare cost of business cycle is much more significant in an economy with heterogeneous trading technologies.
In the case with homogeneous trading technologies, our result shows an insignificant welfare cost of business cycle. This result implies a large welfare difference between economies with or without heterogeneous trading technologies, which can be thought as the cost of investment mistakes made by passive traders. These mistakes include relinquishing high risk premium and missing the market timing. The significant welfare cost of investment errors highlights the importance of studying in household finance. If we can find a way to avoid these investment mistakes, the average welfare of the society can be improved considerably. Additionally, our results indicate that the welfare improvement by avoiding these investment errors is comparable to that by eliminating business cycle. Therefore, if the elimination of aggregate output volatility is infeasible or extremely expensive, then spending a lot of resources on preventing investment mistakes made by households may be reasonable.
References


Table I: Results of BM and NHT economy

<table>
<thead>
<tr>
<th>Active Traders</th>
<th>Panel I: BM Economy</th>
<th>Panel II: NHT Economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passive Equality Traders</td>
<td>10%</td>
<td>100%</td>
</tr>
<tr>
<td>Non-participant Traders</td>
<td>40%</td>
<td>0%</td>
</tr>
</tbody>
</table>

### Asset Pricing

<table>
<thead>
<tr>
<th></th>
<th>Panel I: BM Economy</th>
<th>Panel II: NHT Economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(M)$</td>
<td>0.3739</td>
<td>0.1528</td>
</tr>
<tr>
<td>$\text{Std}\left[\sigma_t(M)\right]$</td>
<td>4.0440</td>
<td>1.0106</td>
</tr>
<tr>
<td>$E\left(R_{t+1,t}^D - R_{t+1,t}^f\right)$</td>
<td>7.5368</td>
<td>3.0077</td>
</tr>
<tr>
<td>$\sigma\left(R_{t+1,t}^D - R_{t+1,t}^f\right)$</td>
<td>20.3867</td>
<td>19.7216</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.3697</td>
<td>0.1525</td>
</tr>
<tr>
<td>$E\left(R_{t+1,t}^f\right)$</td>
<td>1.9141</td>
<td>3.0900</td>
</tr>
<tr>
<td>$\sigma\left(R_{t+1,t}^f\right)$</td>
<td>2.2729</td>
<td>2.2539</td>
</tr>
</tbody>
</table>

### Approximation

<table>
<thead>
<tr>
<th></th>
<th>Panel I: BM Economy</th>
<th>Panel II: NHT Economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^2$</td>
<td>0.9995</td>
<td>0.9999</td>
</tr>
</tbody>
</table>

### Welfare Cost

<table>
<thead>
<tr>
<th>Welfare Cost of Business Cycle(%)</th>
<th>Panel I: BM Economy</th>
<th>Panel II: NHT Economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.49</td>
<td>1.45</td>
<td></td>
</tr>
</tbody>
</table>

BM refers to our benchmark economy and NHT refers to no heterogeneous trading economy. Storesletten, Telmer, and Yaron (2007) calibration of idiosyncratic shocks without counter cyclical variation risk; Alvarez and Jermann (2001) calibration of aggregate consumption growth shocks. Parameters: $\alpha = 4$, $\beta = 0.95$, collateralized share of income is 10%. The results are generated by simulating an economy with 12,000 agents and 10,000 periods.
Table II: Results of Sensitivity Analysis I

<table>
<thead>
<tr>
<th></th>
<th>Panel I</th>
<th>Panel II</th>
<th>Panel III</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>risk aversion rate ((\alpha))</strong></td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td><strong>Active Traders</strong></td>
<td>10%</td>
<td>10%</td>
<td>10%</td>
</tr>
<tr>
<td><strong>Passive Equality Traders</strong></td>
<td>40%</td>
<td>40%</td>
<td>40%</td>
</tr>
<tr>
<td><strong>Non-participant Traders</strong></td>
<td>50%</td>
<td>50%</td>
<td>50%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Asset Pricing</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\frac{\sigma(M)}{E(M)})</td>
<td>0.2856</td>
<td>0.1868</td>
<td>0.0872</td>
</tr>
<tr>
<td>(Std\left[\frac{\sigma_t(M)}{E_t(M)}\right])</td>
<td>3.2615</td>
<td>2.1882</td>
<td>1.0307</td>
</tr>
<tr>
<td>(E\left(R_{t+1,t}^D - R_{t+1,t}^f\right))</td>
<td>5.1849</td>
<td>3.0434</td>
<td>1.2799</td>
</tr>
<tr>
<td>(\sigma\left(R_{t+1,t}^D - R_{t+1,t}^f\right))</td>
<td>18.3498</td>
<td>16.4103</td>
<td>14.6313</td>
</tr>
<tr>
<td><strong>Sharpe Ratio</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.2826</td>
<td>0.1855</td>
<td>0.0875</td>
</tr>
<tr>
<td>(E\left(R_{t+1,t}^f\right))</td>
<td>2.8186</td>
<td>3.8275</td>
<td>4.8322</td>
</tr>
<tr>
<td>(\sigma\left(R_{t+1,t}^f\right))</td>
<td>1.6744</td>
<td>1.0946</td>
<td>0.5360</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Approximation</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.9997</td>
<td>0.9997</td>
<td>0.9998</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Welfare Cost</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Welfare Cost of Business Cycle(%)</strong></td>
<td>5.27</td>
<td>4.22</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Storesletten, Telmer, and Yaron (2007) calibration of idiosyncratic shocks without counter cyclical variation risk; Alvarez and Jermann (2001) calibration of aggregate consumption growth shocks. Parameters: \(\beta = 0.95\), collateralized share of income is 10%. The results are generated by simulating an economy with 12,000 agents and 10,000 periods.
Table III: Results of sensitivity Analysis II

<table>
<thead>
<tr>
<th></th>
<th>Panel I</th>
<th>Panel II</th>
<th>Panel III</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>risk aversion rate (α)</strong></td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td><strong>Active Traders</strong></td>
<td>3%</td>
<td>1%</td>
<td>1%</td>
</tr>
<tr>
<td><strong>Passive Equality Traders</strong></td>
<td>47%</td>
<td>49%</td>
<td>49%</td>
</tr>
<tr>
<td><strong>Non-participant Traders</strong></td>
<td>50%</td>
<td>50%</td>
<td>50%</td>
</tr>
<tr>
<td><strong>Asset Pricing</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{\sigma(M)}{E(M)}$</td>
<td>0.3957</td>
<td>0.3461</td>
<td>0.2238</td>
</tr>
<tr>
<td>Std $\left[ \frac{\sigma_t(M)}{E(M)} \right]$</td>
<td>7.5114</td>
<td>9.5149</td>
<td></td>
</tr>
<tr>
<td>$E\left( R^D_{t+1,t} - R^f_{t+1,t} \right)$</td>
<td>7.3828</td>
<td>5.7938</td>
<td>3.0422</td>
</tr>
<tr>
<td>$\sigma\left( R^D_{t+1,t} - R^f_{t+1,t} \right)$</td>
<td>19.2098</td>
<td>17.7495</td>
<td>15.6695</td>
</tr>
<tr>
<td><strong>Sharpe Ratio</strong></td>
<td>0.3843</td>
<td>0.3264</td>
<td>0.1941</td>
</tr>
<tr>
<td>$E\left( R^f_{t+1,t} \right)$</td>
<td>2.2503</td>
<td>3.0917</td>
<td>4.3834</td>
</tr>
<tr>
<td>$\sigma\left( R^f_{t+1,t} \right)$</td>
<td>1.6619</td>
<td>1.0823</td>
<td>0.5280</td>
</tr>
<tr>
<td><strong>Approximation</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.9995</td>
<td>0.9995</td>
<td>0.9997</td>
</tr>
<tr>
<td><strong>Welfare Cost</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Welfare Cost of Business Cycle(%)</td>
<td>9.37</td>
<td>9.56</td>
<td>3.81</td>
</tr>
</tbody>
</table>

Storesletten, Telmer, and Yaron (2007) calibration of idiosyncratic shocks without counter cyclical variation risk; Alvarez and Jermann (2001) calibration of aggregate consumption growth shocks. Parameters: $\beta = 0.95$, collateralized share of income is 10%. The results are generated by simulating an economy with 12,000 agents and 10,000 periods.