Growth, Unemployment and Wage Inertia*

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Abstract

We introduce a non-competitive labor market in the neoclassical one-sector growth model to study the growth effects of wage inertia. We compare the dynamic equilibrium of an economy with wage inertia with the equilibrium of an economy with flexible wages. Both economies converge to the same long run equilibrium, but the transitional dynamics will be different because wages are a state variable when wage inertia is introduced. We study the growth effects of technological and fiscal policy shocks in these two economies. During the transition, the growth effects of technological shocks obtained when wages exhibit inertia will be the opposite from the ones obtained when wages are flexible. We also show that the growth effects of fiscal policies will be delayed when there is wage inertia.

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1 Introduction

Wage inertia implies that current wages depend on past wages. This is a well-known fact in labor economics that has been justified in different wage settings (Bell, 1996; Blanchard and Katz, 1997 and Montuenga-Gómez and Ramos-Parreño, 2005). In wage bargaining models, the wage is set over a reference wage that is interpreted as a fall back position, which typically is related to the unemployment benefit (Burkhard and Morgenstern, 2000 and Beissinger and Egger, 2004). In most OECD countries, this unemployment benefit depends on past wages. This introduces a relationship between current and past wages. Wage inertia has also been justified in efficiency wages models, where the wage is set by the firm as a markup over a reference wage in order to make workers exert the right amount of effort (de la Croix and Collard, 2000; Danthine and Donaldson, 1990, and Danthine and Kurmann, 2004). In this wage setting, the reference wage is interpreted as a social norm that affects the disutility of effort in the workers utility function. This social norm is typically related to past labor income. In this way, wage inertia is introduced in the efficiency wage model.

Wage inertia has been introduced in New Keynesian models to explain facts of the business cycle (Danthine and Kurmann, 2004; and Blanchard and Gali, 2007). However, in growth models wages are assumed to be flexible even when unemployment is introduced in the model (Aricó, 2000; Brättinger, 2000; Daveri and Mafezzoli, 2000; Domenech and García, 2008; Eriksson, 1997). Two exceptions that introduce wage inertia in a growth model are the papers by Raurich, Sala and Sorolla (2006) and Greiner and Flaschel (2010). However, these papers only study the long run growth effects of the interaction between wage inertia and some particular fiscal policies. Therefore, to study how wage inertia modifies the time path of the GDP growth rate during the transition is an open question. To address this question is the aim of this paper.

We study a version of the neoclassical one-sector exogenous growth model with a non-competitive labor market. We follow the wage bargaining approach and we assume that real wages are set by a monopolistic union as a markup over an external reference wage. We assume that this reference wage depends on the weighted average of past average labor income. In this way, real wage inertia is introduced in the model, which implies that wages are a state variable of the model. The weights determine the intensity of wage inertia and, therefore, the speed of wage adjustment. Wage inertia will then modify the transitional dynamics of the neoclassical growth model as two forces will drive the transition: first, as in the neoclassical growth model, the diminishing returns to capital that drive capital accumulation; and second, wage inertia drives the dynamics of wages.

We distinguish two effects of wage inertia. On the one hand, the time path of the employment rate depends on the time path of both capital and wages. First, capital accumulation increases labor demand and, thus, increases employment. Second, wage
growth reduces employment. Thus, a fast (slow) accumulation of capital in comparison to the speed of wage adjustment will imply an increase (decrease) in the employment rate. Therefore, the interaction between capital accumulation and wage adjustment explains periods of fast employment creation and also non-monotonic transitions of the employment rate.

On the other hand, the returns on capital and wages are related by the zero profit condition. Then, wage inertia modifies the time path of capital accumulation because it modifies the returns on capital. If wages are initially high, the interest rate will be initially low, implying low capital accumulation. The opposite holds when wages are initially low.

During the transition, the GDP growth rate depends on the exogenous growth rate of technology, capital accumulation and employment growth. As wage inertia modifies both capital accumulation and employment growth, the introduction of wage inertia modifies the GDP growth rate in a non-obvious way and causes non-monotonic transitions in this variable, which do not arise in the neoclassical growth model with flexible wages. In contrast, in the long run, the GDP growth rate coincides with the exogenous growth rate of technology and, thus, wage inertia will not modify the long run growth rate.

We use numerical simulations to study the effects of wage inertia during the transition. We show that in economies with initially high wage, a low wage inertia implies an initially low but increasing employment rate. As a consequence, in these economies, the growth rate will be initially large and decreasing during the transition. In contrast, initially low wages imply that the employment rate will be initially high and decreasing during the transition. This implies that the growth rate of GDP will be initially low but it will increase during the transition. We conclude that economies with the same initial stock of capital exhibit different time paths of the growth rate of GDP if initial wages are different. Therefore, in this model, transitional dynamics will crucially depend on the initial conditions on both state variables: capital and wages.

We also show that initially high wages may cause a process of fast employment creation that may imply a non-monotonic transition of the GDP growth rate. Before this process starts, GDP growth will be low. Then, during the process of employment growth, the GDP growth rate will be large because of the employment creation process. Finally, when the creation of employment ends, the GDP growth rate decreases until it converges to its long run value. This non-monotonic transition implies that the log of the GDP exhibits a S-Shaped curve along the transition. This is a well-known fact of the development process that the neoclassical growth model fails to show.

We compare the growth effects of technological shocks in the model with flexible wages and in the model with wage inertia. We consider two different technological shocks: a permanent increase in the level of total factor productivity and a permanent increase in the growth rate of total factor productivity. When wages are flexible and transitional
dynamics are driven only by diminishing returns to capital, both shocks initially increase
the marginal product of capital and therefore the GDP growth rate. When there is
wage inertia, transitional dynamics also depend on the time path of the employment
rate. Both shocks imply an initially decreasing path of the employment rate which causes
a negative effect on GDP growth. This negative growth effect dominates the positive
growth effect associated to capital accumulation during the first periods and, therefore,
both shocks initially reduce the GDP growth rate. Thus, permanent technological shocks
imply opposite transitions of the GDP growth rate when there is wage inertia.

Finally, we study the growth effects of the introduction of an unemployment benefit.
This unemployment benefit rises the reference wage and therefore wages. If wages
are flexible, they immediately adjust implying an immediate jump downwards on both
employment and GDP. When there is wage inertia, wages do not immediately adjust to
this fiscal policy and, therefore, the employment rate and the level of GDP will suffer a
slow decline. Thus, the macroeconomic effects of this fiscal policy suffer a delay when
wages exhibit inertia.

2 The economy

In this section we describe a version of the neoclassical growth model of exogenous growth
with a non-competitive labor market. First, we introduce the technology and the firms’
optimal decisions; second, we describe the consumers’ problem and, finally, we specify a
wage setting rule with wage inertia.

2.1 Firms

We consider the following neoclassical production function:

\[ Y_t = F(K_t, A_t L_t), \]

where \( Y_t \) is gross domestic product (GDP), \( K_t \) is capital, \( A_t L_t \) are efficiency units of labor
employed. We assume that technology, \( A_t \), exogenously increases at a constant growth
rate \( x \geq 0 \), and the production function satisfies the following properties: constant returns
to scale, twice continuously differentiable, \( F_K > 0, F_L > 0, F_{KK} < 0, F_{LL} < 0 \) and the
Inada conditions. The production function can be rewritten in intensive form as follows

\[ \hat{y}_t = f(\hat{k}_t), \]

where \( \hat{y}_t = \frac{Y_t}{A_t L_t} \) is output per efficiency unit of labor employed and \( \hat{k}_t = \frac{K}{A_t L_t} \) is capital
per efficiency unit of labor employed. The production function satisfies the following
properties: $f' > 0$ and $f'' < 0$.

Firms are price takers and maximize profits, $F(K_t, A_t L_t) - w_t L_t - (r_t + \delta)K_t$, where $w_t$ is the real wage, $r_t$ is the interest rate and $\delta$ is the constant depreciation rate, $0 \leq \delta \leq 1$. The first order conditions are

$$\hat{w}_t = f'\left(\hat{k}_t\right) - \hat{k}_t f''\left(\hat{k}_t\right), \quad (1)$$

and

$$r_t + \delta = f'\left(\hat{k}_t\right), \quad (2)$$

where $\hat{w}_t = \frac{w_t}{A_t}$ is the wage per efficiency unit of labor.

Using (1), we get

$$\hat{k}_t = \tilde{k}(\hat{w}_t), \quad (3)$$

where $\tilde{k}' > 0$ and from (3) we obtain the labor demand function

$$L_t^d = \frac{K_t}{A_t \tilde{k}(\hat{w}_t)}. \quad (4)$$

Combining (2) and (3), we obtain the zero profit condition as a function relating the interest rate and wages

$$r_t = f'\left[\tilde{k}(\hat{w}_t)\right] - \delta \equiv \tilde{r}(\hat{w}_t), \quad (5)$$

where $\tilde{r}' < 0$. Thus, when the wage increases faster than efficiency units of labor the interest rate must decrease in order to have zero profits.

We assume that the labor supply is exogenous and equal to population, $N_t$, that grows at a constant rate $n$.\(^1\) We can define GDP per efficiency unit of population $y_t = \frac{Y_t}{A_t N_t}$, capital per efficiency unit of population $k_t = \frac{K_t}{A_t N_t}$ and the employment rate $l_t = \frac{L_t}{N_t} \in [0, 1]$. Because of the constant returns to scale assumption, we rewrite the production function as

$$y_t = g(k_t, l_t).$$

In models with wage setting, factor markets do not clear if the wage is different from the competitive wage. When the wage is smaller than the competitive wage, there is excess supply in the capital market and excess demand in the labor market; and if the wage is larger than the competitive wage then there is excess demand in the capital market and excess supply in the labor market. In this paper, we will consider the latter case, which implies that the equilibrium stock of capital is determined by the supply and

\(^1\)Conclusions regarding the growth effects of wage inertia would not be modified if an endogenous labor supply had been assumed and leisure had been introduced additively in the utility function.
the equilibrium level of employment is determined by the labor demand, that is

\[ L_t = \frac{K_t}{A_t k(\widehat{w}_t)}. \] (6)

Using (6), we obtain the employment rate function

\[ l_t = \bar{l}(\widehat{w}_t, k_t) = \frac{k_t}{\bar{k}(\widehat{w}_t)}. \] (7)

Finally, we use (7) to rewrite the production function in per capita terms as follows

\[ \frac{y_t}{k_t} = g(1, -\frac{1}{\bar{k}(\widehat{w}_t)}) = h' \left[ \bar{k}(\widehat{w}_t) \right], \] (8)

where \( h' < 0 \).

2.2 Consumers

Consider a representative family composed of the \( N_t \) agents in the economy that chooses the consumption of each agent, \( C_t \), in order to maximize the discounted sum of the utilities of the \( N_t \) agents

\[ N_0 \int_{t=0}^{\infty} e^{-(\rho-n)t} u(C_t) \, dt \]

subject to the following budget constraint

\[ \dot{S}_t = r_t S_t + (1 - \tau) w_t l_t + \xi_t (1 - l_t) + T_t - C_t - nS_t, \]

where \( \rho > n \) is the subjective discount rate, \( S_t \) is the stock of assets of each member of the family, \( \tau \) is the tax rate on employed workers, \( \xi_t \) the unemployment benefit and \( T_t \) is a lump-sum subsidy to an individual. Note that the revenues of this family accrue from total labor income because we assume that all workers, employed and unemployed, belong to the same family. This big family assumption implies that there is no heterogeneity associated to the risk of becoming unemployed. Daveri and Maffezzoli (2000), Domenech and Garcia (2008), Eriksson (1997) also follow the big family assumption. If instead of a big family we have heterogeneous agents, the solution would not change as long as we assume complete competitive insurance markets for unemployment (de la Croix and Collard, 2000) or that the union pursues a redistributive goal, acting as a substitute for the insurance markets (Maffezzoli, 2001 and Benassy, 1997).

The solution to this maximization problem is characterized by the Euler condition
\[ \frac{\dot{C}_t}{C_t} = \frac{r_t - \rho}{\sigma(C_t)} \]

where \( \sigma(C_t) = \frac{u''(C_t)C_t}{u'(C_t)} \), the budget constraint and the following transversality condition:

\[ \lim_{t \to \infty} e^{-(\rho-n)t} u'(C_t) S_t = 0. \tag{9} \]

Let us define \( c_t = \frac{C_t}{A_t} \) as consumption per efficiency unit of labor and note that in equilibrium \( S_t = A_t k_t \). Using these relationships, conditions (1), (5), and (8) and assuming the following government balanced budget constrain:

\[ \tau l_t \omega_t - \xi (1 - l_t) = T_t, \]

the Euler condition and the budget constraint can be rewritten as follows

\[ \frac{\dot{c}_t}{c_t} = \frac{f' \left( \tilde{k} \left( \tilde{\omega}_t \right) \right) - \delta - \rho - \sigma(c_t A_t) x}{\sigma(c_t A_t)}, \tag{10} \]

and

\[ \frac{\dot{k}_t}{k_t} = h \left[ \tilde{k} \left( \tilde{\omega}_t \right) \right] - \frac{c_t}{k_t} - (n + x + \delta). \tag{11} \]

Equations (10) and (11) characterize the growth rates of consumption and capital per efficiency unit of labor. These equations depend on the time path of the wage per efficiency unit of labor and, therefore, they will depend on the particular assumptions regarding wage setting.

When wages are flexible and they are set in order to clear the markets, then there is full-employment implying that \( l_t = 1 \) and that \( \tilde{k} \left( \tilde{\omega}_t \right) = k_t \). In this case, equations (10) and (11) characterize the dynamic equilibrium of the standard neoclassical growth model with full-employment.

When wages are flexible but they are set above the competitive wage, so that markets do not clear, then \( l_t < 1 \) and \( \tilde{k} \left( \tilde{\omega}_t \right) = k_t / l_t \), where \( l_t = \tilde{l}(\tilde{\omega}_t, k_t) \) is defined in (7). In this case, the system of differential equations (10) and (11) alone do not characterize the dynamic equilibrium. The equilibrium will also depend on the wage equation obtained from the wage setting and that determines the wage. Under appropriate assumptions regarding the wage setting, the employment rate is constant along the transition. In this case, the transitional dynamics are identical to the ones obtained in the version of the neoclassical growth model with full-employment.

Finally, when there is wage inertia, the equilibrium dynamics will also depend on the wage setting process. On the one hand, an increase in the wage decreases the interest rate and reduces the rate of growth of the optimal consumption per capita. On the other hand,
an increase in the wage reduces employment and the output per unit of capital decreases. This reduces the rate of growth of capital per capita. Therefore, in order to characterize the dynamic equilibrium, we need an additional dynamic equation that governs the wage dynamics. This equation is obtained from the wage setting process.

2.3 Wage setting rule with inertia

We assume a simple model of firm-level wage setting, where unions set the wage in order to maximize:

$$Max V = [(1 - \tau) w_t - w_t^s]^\gamma \tilde{l}(\tilde{w}_t, k_t),$$

where $w_t^s$ is the outside alternative reference wage, $\tau$ is the direct tax rate and $\gamma > 0$ provides a measure of the wage gap weight in the unions’ objective function. When setting the wage, unions take into account the firm’s labor demand. The solution of the unions’ maximization problem is the following wage equation:

$$w_t = \frac{w_t^s}{(1 - \tau) \left( 1 - \frac{\gamma}{\varepsilon(w_t, k_t)} \right)}, \tag{12}$$

where $\varepsilon(\tilde{w}_t, k_t) = - \left( \frac{\partial l(\tilde{w}_t, k_t)}{\partial w_t} \right)$ is the elasticity of the labor demand.\(^2\)

The literature has introduced several assumptions regarding the reference wage. Layard et al. (1991, chapter 2) identifies the reference wage with the average labor income. Blanchard and Katz (1999) and Blanchard and Wolfers (2000) assume that the reference wage also depends on past wages. In this case, wages depend on past wages and, thus, there is wage inertia. In this paper we follow de la Croix and Collard (2000) and Raurich, Sala and Sorolla (2006) and assume that the reference wage is a social norm that depends on the following weighted average of past average labor income:

$$w_t^s = w_0^s e^{-\theta t} + \theta \int_0^t e^{-\theta(t-i)} b(i) di, \tag{13}$$

where $w_0^s$ is the initial value of the reference wage, $\theta > 0$ provides a measure of the wage

\(^2\)Note that if $\gamma = 1$ and $w^s$ is the unemployment benefit then maximize the unions’ objective function is equivalent to maximize the average labor income obtained by both employed and unemployed workers.

\(^3\)Alternatively, we may follow de la Croix and Collard (2000) and consider an efficiency wage model of the gift exchange type where the utility of consumption and the disutility of effort are additively separable and that the disutility of effort is $G(e_t) = \left[ e_t - \beta \left( \ln [(1 - \tau) w_t] - \ln \left( \frac{w_t^s}{1 - \tau} \right) \right) \right]^2$. The first order conditions imply that $G'(e_t) = 0$ and the effort function is: $e_t = \ln [(1 - \tau) w_t] - \ln \left( \frac{w_t^s}{1 - \tau} \right)$.

Assume that the firm sets the wage considering the effort function. From the first order conditions, we obtain $\frac{\partial e_t}{\partial w_t} w_t = 1$. This implies that $e_t = 1$ and that the wage equation is $w_t = \frac{w_t^s}{(1 - \tau)(1 - \gamma)}$. Note that this wage equation coincides with the one obtained in the wage bargaining model when $\varepsilon(\tilde{w}_t, k_t)$ is constant. This happens when the technology is characterized by a Cobb-Douglas production function.
adjustment rate and \( b(t) \) is the workers’ average labor income

\[
b_t = (1 - \tau) l_tw_t + [1 - l_t] \xi_t,
\]

with the unemployment benefits being \( \xi_t = \lambda (1 - \tau) w_t \) and \( \lambda \in (0, 1) \). The government satisfies the following budget constraint:

\[
\tau l_tw_t - \lambda (1 - \tau) [1 - l_t] w_t = T_t,
\]

where, as we said, \( T_t \) is a lump-sum subsidy. In this way, labor income taxes do not cause wealth effects.

Note that a larger value of \( \theta \) implies lower wage inertia. Actually, if \( \theta \) diverges to infinite, the reference wage coincides with the current average labor income and, in this limiting case, there is no wage inertia. In order to see this, we solve the system of equations (12) and (14) when \( w^*_t = b_t \) and we obtain that the employment rate when wages are flexible is

\[
l_t = 1 - \frac{\gamma}{(1 - \lambda) \varepsilon(\hat{w}_t, k_t)}.
\]

If there is wage inertia, the law of motion of the reference wage is obtained by differentiating (13) with respect to time

\[
\hat{w}_t^* = \theta [b_t - w^*_t].
\]

Combining (12), (16) and (14), we obtain

\[
\frac{\hat{w}_t^*}{w_t^*} = \theta \left[ l_t + \lambda [1 - l_t] \frac{1}{1 - \varepsilon(\hat{w}_t, k_t)} - 1 \right].
\]

We log-differentiate (12) and \( \hat{w}_t = \frac{w_t}{\dot{w}_t} \) and substitute the expression of the growth rate of the reservation wage to obtain the growth rate of wages

\[
\frac{\dot{\hat{w}}_t}{\hat{w}_t} = \theta \left( \frac{1 - \lambda \bar{I}(\hat{w}_t, k_t) + \lambda}{1 - \bar{\varepsilon}(\hat{w}_t, k_t)} - 1 \right) - \left( \frac{1}{\bar{\varepsilon}(\hat{w}_t, k_t)} - 1 \right) \left( \frac{\bar{\varepsilon}(\hat{w}_t, k_t)}{\hat{\varepsilon}(\hat{w}_t, k_t)} - x. \right)
\]

3 Equilibrium

We must distinguish between the equilibrium when wages are flexible because \( \theta \) diverges to infinite and the equilibrium when wages exhibit inertia.

**Definition 1** An equilibrium with flexible wages is a path \( \{c_t, k_t, \hat{w}_t, l_t\} \) that solves the
system of differential equations (10) and (11), satisfies (15), (7), the transversality condition (9) and the initial condition $k_0$.

**Definition 2** An equilibrium with wage inertia is a path $\{c_t, k_t, \hat{w}_t, l_t\}$ that solves the system of differential equations (10), (11), and (17), satisfies (7), the transversality condition (9) and the initial conditions $k_0$ and $w_0$.

Note that the main difference between these two equilibrium definitions is the number of state variables. If wages are flexible, capital is the unique state variable. If wages exhibit inertia, capital and wages are the two state variables.

In order to simplify the analysis and obtain results, we will assume the following isoelastic utility function:

$$u(C_t) = \frac{C_t^{1-\sigma} - 1}{1 - \sigma}$$

and the following Cobb-Douglas production function:

$$Y_t = K_t^\alpha (A_t L_t)^{1-\alpha}, \alpha \in (0, 1).$$

In this case, $\sigma(c_t A_t) = \sigma, \hspace{0.5cm} f'\left(\hat{w}_t\right) = \alpha \left(\frac{\hat{w}_t}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}}$, $\hspace{0.5cm} f'\left(\hat{k}_t\right) = \alpha \left(\frac{\hat{k}_t}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}}$ and $h\left(\hat{w}_t\right) = \frac{\hat{w}_t}{\alpha}$ and $\hspace{0.5cm} \varepsilon(\hat{w}_t, k_t) = \frac{1}{\alpha}$. Therefore, the system of differential equations characterizing the equilibrium with wage inertia, (10), (11), and (17), simplifies as follows:

$$\frac{\dot{c}_t}{c_t} = \frac{\alpha \left(\frac{\hat{w}_t}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}} - \delta - \rho - \sigma x}{\sigma}, \hspace{0.5cm} (18)$$

$$\frac{\dot{k}_t}{k_t} = \left(\frac{\hat{w}_t}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}} - c_t \frac{k_t}{k_t} - (n + x + \delta), \hspace{0.5cm} (19)$$

and

$$\frac{\hat{w}_t}{\hat{w}_t} = \theta \left(\frac{(1-\lambda) k_t \left(\frac{\hat{w}_t}{1-\alpha}\right)^{-\frac{1}{\alpha}} + \lambda}{1 - \gamma \alpha} - 1\right) - x. \hspace{0.5cm} (20)$$

If there is no wage inertia, $\theta \rightarrow \infty$, then (15) implies the following constant employment rate:

$$\bar{l} = 1 - \frac{\gamma \alpha}{1 - \lambda}. \hspace{0.5cm} (21)$$

In this case, from (7), we obtain that the wage is the following function of capital:

$$\hat{w}_t = (1 - \alpha) \left(\frac{k_t}{\bar{l}}\right)^{\alpha}. \hspace{0.5cm} (22)$$

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4This last property implies that the wage set by the union is a constant mark up over their reservation wage, i.e, $\omega_t = \frac{\hat{w}_t}{(1-\sigma)(1-\gamma \alpha)}$. 

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Note that this implies that capital is the only state variable. In this case, the system of differential equations characterizing the equilibrium, (10), (11), simplify as follows:

\[
\frac{\dot{c}_t}{c_t} = \frac{\alpha \left( \frac{k_t}{\bar{L}} \right)^{\alpha-1} - \delta - \rho - \sigma x}{\sigma},
\]

and

\[
\frac{\dot{k}_t}{k_t} = \left( \frac{k_t}{\bar{L}} \right)^{\alpha-1} - \frac{c_t}{k_t} - (n + x + \delta).
\]

Note that if there is no wage inertia then the equilibrium is characterized by one control variable, \(c_t\), and one state variable, \(k_t\). This equilibrium has a transition which is qualitatively equivalent to the transition in the neoclassical growth model with full-employment because the employment rate remains constant along the transition. In fact, the model coincides with the neoclassical growth model with full-employment when \(\gamma = 0\).

In this paper, we show the effects of wage inertia by comparing the dynamic equilibrium with flexible wages with the dynamic equilibrium with wage inertia. First, we analyze the effects of wage inertia on the steady state equilibrium and then we study the effects of inertia on the transition. We define a steady state equilibrium as an equilibrium path along which \(\dot{c}_t, c_t, k_t\) remain constant.

**Proposition 3** There is a unique steady state equilibrium of the economy with wage inertia. The steady state values of the employment rate and of per efficiency units of wages, capital, consumption and GDP satisfy

\[
I^* = 1 - \frac{\gamma \alpha}{1 - \lambda} + \left( \frac{1 - \gamma \alpha}{1 - \lambda} \right) \frac{\omega^*}{\bar{\theta}},
\]

\[
\dot{\omega}^* = (1 - \alpha) \left( \frac{\delta + \rho + \sigma x}{\alpha} \right)^{\frac{1}{\alpha-1}},
\]

\[
k^* = \left( \frac{\delta + \rho + \sigma x}{\alpha} \right)^{\frac{1}{\alpha-1}} I^*,
\]

\[
c^* = \frac{\delta + \rho + \sigma x}{\alpha} - (n + x + \delta),
\]

\[
y^* = \frac{\delta + \rho + \sigma x}{\alpha}.
\]

Note that the long run values of the ratios \(\frac{k}{k^*}\) and \(\frac{c}{c^*}\) and the wage \(\dot{\omega}^*\) do not depend on the parameters characterizing the labor market. Thus, the equilibrium with wage inertia converges to a steady state where the values of these variables coincide with the long run value of these variables in an economy with full employment. However, the long run values of the employment rate and the levels of capital, GDP and consumption will depend on both \(\gamma\) and \(\theta\). On the one hand, a larger value of \(\gamma\) causes a reduction in the employment rate. This reduction implies a decrease in the levels of capital, consumption and GDP.
On the other hand, the employment rate decreases with $\theta$ if $x > 0$. Thus, a stronger wage inertia has a positive effect on the long run employment rate if there is sustained growth in the economy. Sustained growth implies that the labor demand increases even in the long run. Wage inertia prevents wages to increase with the labor demand, which explains the positive effect on the employment rate. The same argument explains that the employment rate increases with $x$ only if there is wage inertia because if there is no wage inertia any increase in the marginal product of labor completely translates into a larger wage. Therefore, wage inertia positively affects the employment rate in the long run when there is sustained growth, because unions do not take into account the growth of labor income implied by technological progress when bargaining next period wages.

**Proposition 4** The steady state equilibrium of the economy with wage inertia is locally saddle path stable.

This results shows that there is a unique dynamic equilibrium. The main departure from the competitive equilibrium paradigm is in the introduction of wage inertia that introduces a new state variable and modifies the transitional dynamics. The aim of the following section is to show how wage inertia affects the transitional dynamics. This is done by means of a numerical analysis.

**Remark 5** Given that we have a two dimensional manifold, there are two roots with a negative real part. In the numerical example of the following section, the roots are complex numbers when $\theta$ is sufficiently low. This implies that for a sufficiently high intensity of wage inertia the equilibrium exhibits endogenous cycles. In the examples of the following section, we assume a value of $\theta$ such that the roots are real numbers. Therefore, the non-monotonic behavior of the variables will be just a consequence of the existence of two different forces driving the transition.

## 4 Transitional dynamics

When wages are flexible, transitional dynamics are governed only by the diminishing returns to capital and, thus, they only depend on the initial capital stock. In contrast, when there is wage inertia, transitional dynamics are governed by both the diminishing returns to capital and wage dynamics. In this case, transitional dynamics will depend on the initial conditions on capital and wages. The existence of two different forces driving the transition implies that the dynamic equilibrium will exhibit relevant differences along the transition with respect to the equilibrium path of a model with flexible wages. In this section, we compare these different transitional dynamics by means of a numerical analysis. Therefore, the value of the parameters must be fixed.
We assume that \( \sigma = 2 \) which implies a value of the \( IES = 0.5 \) and \( \alpha = 0.35 \) which implies a constant value of the labor income share equal to 0.65. We fix \( x = 2\% \) and \( n = 1\% \), which are within the range of empirically plausible values of these two parameters. The depreciation rate \( \delta = 6.46\% \) implies a long run interest rate equal to 5.2\% and the subjective discount rate \( \rho = 0.012 \) implies that the long run ratio of capital to GDP equals 3. We assume that there is no unemployment benefit in the benchmark economy, \( \lambda = 0 \), and the value of \( \gamma \) is set so that the long run employment rate equals \( l^* = 0.9 \). As \( \gamma \) is used to calibrate the long run value of the employment rate, the value of this parameter will be different in the economy with flexible wages than in the economy with wage inertia. In particular, \( \gamma = 0.52 \) in the economy with wage inertia and \( \gamma = 0.28 \) in the economy with flexible wages. Finally, the parameter \( \theta \) determines the speed of wage adjustment. Obviously, a larger value of \( \theta \) implies a larger speed of wage adjustment. We set the value of \( \theta \) equal to 0.2, which implies an intensity of wage inertia that makes half distance in wages be satisfied in only three periods.\(^5\)

In what follows, we show the results of four different numerical exercises aimed to illustrate the transitional dynamics when there is wage inertia and compare the growth effects of shocks in an economy with flexible wages and in an economy with wage inertia.

### 4.1 Convergence

In a first numerical exercise we compare two economies with the same initial stock of capital but different initial wages. Therefore, this numerical exercise allows us to show how the initial conditions on wages modify the transitional dynamics. These transitional dynamics are illustrated in Figures 1 and 2.

Figure 1 compares the transitional dynamics of two economies with the same initial capital stock, which is 5\% larger than the long run capital stock and a different initial wage. In particular, the economy described by the continuous line has an initial wage that is 1\% smaller than the long run wage and the economy illustrated by the dashed line has an initial wage that is 5\% larger than its long run value. The first two panels show the time path of the two state variables: wages and capital. The third panel shows the time path of the interest rate that follows from the time path of wages and equation (5). This equation implies a negative relationship between wages and the interest along the growth process. Panel (iv) displays the time path of the employment rate. The path of this variable is obtained from equation (7). As follows from these equations, the path of the employment rate is completely described by the path of capital and wages. As capital accumulates, the labor demand increases and so does the employment rate. However, an increases in wages reduces the employment rate. Thus, the two forces driving the transition have opposite

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\(^5\)Half distance is defined as \( \frac{\bar{w}_t - \bar{w}_0}{\bar{w}_0 - \bar{w}_0} = \frac{\tau}{2} \), where \( \tau \) is the number of periods needed to satisfy half distance.
effects on employment, which may cause a non-monotonic behavior in the time path of the employment rate. To see this, consider the economy with initially low wages and high capital stock described by the continuous line. On the one hand, the low wages imply that initially the long run value of the interest rate is large, which explains the initial increase in the capital stock. On the other hand, the large labor demand due to the initially large capital stock and the low wages imply that the employment rate is initially high. Obviously, this high employment rate implies that the current average income is high, which causes an increase in the reference wage and in actual wages. The increase in wages explains both the reduction in the employment rate and also the reduction in the interest rate. The later causes the reduction in the capital stock. Therefore, the interaction between the labor market and the interest rate explains the non-monotonic time path of the capital stock. Moreover, the low wage inertia assumed implies that wages decrease faster than capital. This implies that eventually the employment rate decreases below its long run value and the time path of this variable exhibits a non-monotonic transition. Panels v) and vi) display, respectively, the time path of the growth rate of per capita GDP and the logarithm of per capita GDP. We use the production function to obtain the logarithm of per capita GDP as the following function

$$\ln GDP = a + \alpha \ln k + (1 - \alpha) \ln l + xt,$$

where $a$ is a constant. This equation shows that in the long run the logarithm of GDP is a linear function implying a constant long run growth rate equal to $x$. However, during the transition, both the accumulation of capital and the evolution of the employment rate determine the time path of this variable. In the economy illustrated by the continuous line, the employment rate is initially high and suffers a process of fast reduction. The high employment rate explains the initially large per capita GDP and the fast reduction in employment explains the low growth rate.

The dashed line in Figure 1 illustrates the transitional dynamics of an economy that has the same initial capital stock but an initially larger wage. The transitional dynamics of this economy are the opposite from the transitional dynamics of the economy with initially low wages. In particular, the growth rate of GDP decreases in the economy with initially high wages, whereas increases in the other economy. We then conclude that the growth process during the transition crucially depends on the initial value of wages.

Figure 2 also compares the transitional dynamics of two economies with the same initial capital stock, which is 5% smaller than the long run capital stock. Again, the only difference is the initial value of the wage. The economy described by the continuous line has an initial wage that is 5% smaller than the long run wage and the economy illustrated by the dashed line has an initial wage that is 1% larger than its long run value. The transitional dynamics displayed in this figure also show that the time path of the GDP
growth rate crucially depends on the initial conditions in the labor market.

We conclude from these two figures that economies with the same initial capital stock but different initial wages exhibit different time paths of the GDP growth rate. Therefore, the initial cost of labor should be taken into account as a relevant variable explaining convergence.

In Figure 3, we show that differences in wage inertia also cause large differences in transitional dynamics. We compare two economies with the same initial conditions, but different assumptions on wage inertia. In particular, in the economy characterized by the continuous line we assume that $\theta = 0.2$, whereas the dashed line shows the time path of the variables when there is no wage inertia, i.e. $\theta \to \infty$. In both economies the initial stock of capital is 50% smaller than its long run value and the initial wage is set so that wages in both economies coincide in the initial period. It follows that the differences along the transition are only due to a different intensity of wage inertia. The main differences are in the time paths of the employment rate (panel iv) and of the growth rate of GDP (panel v). In particular, in the economy with flexible wages the employment rate is constant and the transition of the growth rate is driven only by the diminishing returns to capital. In the economy with wage inertia, capital increases faster than wages in the initial periods implying an increase in the employment rate. As a consequence, the reference wage increases, which accelerates the process of wage growth. This implies that eventually the employment rate decreases until it converges to its long run value. While the employment rate increases, the growth rate of GDP is larger than that of the economy with flexible wage, whereas it is smaller when the employment rate decreases. This implies a fast and sharp transition of the GDP growth rate in the economy with wage inertia.

Figure 4 illustrates the transitional dynamics of two economies with different initial conditions and different wage inertia. The continuous line shows the time path of the variables of an economy with wage inertia ($\theta = 0.2$), an initial capital stock that is 50% smaller than its long run value and an initial wage that is 10% larger than its long run value. The dashed line shows the time path of the variables of an economy with flexible wages and an initial capital stock that is 50% smaller than its long run value. In the economy with flexible wages, wages are set to have an employment rate constant and equal to its long run value. Given that the stock of capital is initially low, wages will be initially small and the interest rate will be large. This large interest rate causes a fast capital accumulation, which explains the increasing time path of wages and the decreasing time path of the interest rate. The behavior of the interest rate also causes the growth rate to be initially large and then, during the transition, it decreases until it converges to its long run value. Thus, along this growth process, the growth rate exhibits a monotonic transition.

The economy with wage inertia exhibits a completely different transition. In this case,
wages are a state variable and their initial large value imply an initially low employment rate and interest rate. On the one hand, the low interest rate implies that the stock of capital in the initial periods decreases, which reduces the growth rate. On the other hand, the low employment rate implies that wages will decrease and the employment rate will increase fast during the first periods of the transition. As a consequence, the interest rate increases and the stock of capital, after the initial reduction, will increase. Both the accumulation of capital and the growth of the employment rate imply that the growth rate of GDP will be large. Eventually, the process of employment and capital growth is reduced, which implies a decrease in the growth rate of GDP. Note that the time path of the growth rate of GDP exhibits a non-monotonic behavior along the growth process. This non-monotonic behavior implies that the time path of the log of GDP exhibits a S-shaped curve. Note that this behavior cannot be explained by the neoclassical growth model when wages are flexible.6

4.2 Technological and fiscal policy shocks

In this subsection we compare the transitional dynamics implied by different shocks. In the figures, the continuous line illustrates the transitional dynamics in our benchmark economy with wage inertia and the dashed line shows the transitional dynamics of an economy with flexible wages. In the examples of this subsection we assume that the economies before the shock are initially in the steady state.

Figure 5 shows the transitional dynamics implied by a positive and permanent technological shock that increases the level of total factor productivity by a 5%. In the economy with flexible wages, this shock implies an initial increase both in the interest rate and in wages. The increase in the interest rate implies that capital accumulates during the transition, which explains both the increasing time path of wages and the decreasing time path of the interest rate. The accumulation of capital explains the decreasing time path of the growth rate of GDP. As it is well-known from business cycle literature, models with flexible wages do not explain the facts of the labour market after a shock, as they imply a large increase in wages and almost no effect on the employment rate. In contrast, the model with wage inertia implies that the wage initially does not jump due to the shock. This has two direct effects. First, the increase in the interest rate will be larger than

6S-shaped curves are a well-known fact of the development process, implying that initially the GDP growth rate is low and when the development process starts, it exhibits a period of fast growth that eventually stops. The neoclassical growth model fails to show this kind of transition and thus fails to explain the development process. Also models with unemployment but no-wage inertia do not explain this process. Recent growth literature has developed models aimed to explain this behavior. Examples are the endogenous preferences literature (Alonso-Carrera, et al., 2004 and Steger, 2006) and models of technological change (Parente and Prescott, 1999). In this paper, we follow a different approach and we consider the effect of employment on the development process. In particular, we show that a version of the neoclassical growth model that introduces wage inertia can explain this non-monotonic transition.
without wage inertia. As a consequence, the accumulation of capital will be faster when there is wage inertia. Second, as a consequence of the shock, the labor demand increases and, as wages do not increase, the employment rate will initially increase. This causes the increase in GDP. However, during the transition, wages will increase and the employment rate will then decrease. This implies a reduction in the growth rate of GDP. Thus, the growth rate of GDP will be driven by two different and opposite effects: the reduction in the employment rate and the increase in the capital stock. Given our assumption of low wage rigidity, the employment rate will experience a process of rapid reduction, implying that this effect initially dominates the transition. This causes the reduction in the growth rate of GDP after the positive technological shock. We then conclude that the growth effects of a technological shock will be the opposite from the ones obtained when wages do not exhibit inertia.

Figure 6 shows the transitional dynamics implied by a positive technological shock that increases the exogenous growth rate from 2% to 3%. In the economy with flexible wages, the employment rate is constant and the transitional dynamics will be governed only by the diminishing returns to capital. The growth rate increases along the transition until it converges to its long run value. In the economy with wage inertia, capital per efficiency unit of labor initially decreases faster than wages per efficiency unit, implying and initial reduction in the employment rate. This reduction decreases the reference wage and accelerates the reduction in wages per efficiency unit. This makes the employment rate increase after some periods. The initial reduction in the employment rate implies that the growth rate initially decreases. Therefore, this technological shock also has opposite growth effects when there is wage inertia.

Figure 7 shows the transitional dynamics implied by the introduction of an unemployment benefit, $\lambda = 1/3$. This unemployment benefit rises the reference wage and thus increases wages. In the economy with flexible wages, this causes an initial jump upwards in wages that has two different effects. On the one hand, the employment rate is reduced, which implies a reduction in GDP. On the other hand, the interest rate decreases which causes the reduction of capital during the transition. The reduction in the capital stock explains the lower growth rate during the transition. In the economy with wage inertia, wages do not jump after the introduction of the unemployment benefit. As a consequence, the interest rate initially does not decrease, nor does the employment rate. This implies that the stock of capital and the level of GDP remain constant after the introduction of the unemployment benefit. However, during the transition, wages will experience a process of rapid growth that reduces the interest rate and the employment rate. As a consequence, the stock of capital decreases during the transition. The combined effect of the reduction in the capital stock and in the employment rate explains the larger

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7Galí (1999) shows evidence that employment decreases after positive technological shocks.
reduction in the growth rate and the time path of the log of per capita GDP. Note that
this example shows that wage inertia delays the effects of fiscal policy on GDP.

5 Conclusions

In this paper we study a version of the neoclassical growth model with wage inertia. We
compare the equilibrium with wage inertia with the equilibrium of another economy
where wages are flexible. We show that both economies converge to the same long run
equilibrium, but the transitional dynamics will be different. Thus, we show that wage
inertia modifies the time path of the dynamic equilibrium because it introduces a new
state variable: wages.

We study the transitional dynamics using several numerical exercises. In a first
exercise, we compare the transitional dynamics of economies with the same initial capital
stock but different initial wages. We show that economies with the same initial capital
stock may exhibit opposite transitions of the GDP growth rates because of different
assumptions on the initial wage.

In a second exercise, we compare the transitional dynamics in an economy with wage
inertia and in an economy with flexible wages. In the economy with flexible wages, the
time path of the variables exhibits a monotonic behavior, while we show that the time
path is non-monotonic when wage inertia is introduced. Obviously, this non-monotonic
transition is a consequence of the interaction between two forces: diminishing returns to
capital and wage inertia. We show that this non-monotonic transition may imply that the
time path of the logarithm of per capita GDP exhibits a S-shaped curve. This analysis
suggests that a closer analysis of labor market performance in growth models may be an
important element in explaining the development process.

In a third numerical exercise, we study the transitional dynamics implied by the
following two technological shocks: a permanent increase in the level of total factor
productivity and a permanent increase in the TFP growth rate. We show that the effects
on these two shocks on the growth rate of GDP will be the opposite if wage inertia is
introduced.

Finally, in a last numerical exercise, we study the effects of a fiscal policy that consists
of introducing an unemployment benefit. We show that wage inertia causes a delay on
the effects of this fiscal policy on GDP.
References


A Appendix

Proof of Proposition 4

The Jacobian matrix associated to (18), (19) and (??) is

\[
J = \begin{pmatrix}
\frac{\partial \dot{w}_t}{\partial w_t} & \frac{\partial \dot{w}_t}{\partial k_t} & \frac{\partial \dot{w}_t}{\partial c_t} \\
\frac{\partial \dot{k}_t}{\partial w_t} & \frac{\partial \dot{k}_t}{\partial k_t} & \frac{\partial \dot{k}_t}{\partial c_t} \\
\frac{\partial \dot{c}_t}{\partial w_t} & \frac{\partial \dot{c}_t}{\partial k_t} & \frac{\partial \dot{c}_t}{\partial c_t}
\end{pmatrix}
\]

\[
= \begin{pmatrix}
-\left(\frac{\theta (1-\lambda)}{1-\gamma \alpha}\right) \left(\frac{k^*}{\alpha}\right) \left(\frac{w^*}{1-\alpha}\right)^{-\frac{1}{\alpha}} & \left(\frac{\theta (1-\lambda)}{1-\gamma \alpha}\right) \left(\frac{w^*}{1-\alpha}\right)^{-\frac{1}{\alpha}} w^* & 0 \\
-\left(\frac{1}{\alpha}\right) \left(\frac{w^*}{1-\alpha}\right)^{-\frac{1}{\alpha}} k^* & \frac{c^*}{k^*} & -1 \\
-\left(\frac{w^*}{1-\alpha}\right)^{-\frac{1}{\alpha}} c^* & 0 & 0 
\end{pmatrix}
\]

The characteristic polynomial is

\[
P(J) = -\mu^3 + \mu^2 Tr(J) + \mu H + Det(J),
\]

where the determinant of the Jacobian Matrix is

\[
Det(J) = \left(\frac{\theta (1-\lambda)}{1-\gamma \alpha}\right) \left(\frac{w^*}{1-\alpha}\right)^{-\frac{2}{\alpha}} w^* c^* > 0,
\]

the trace of this matrix is

\[
Tr(J) = -\left(\frac{\theta (1-\lambda)}{1-\gamma \alpha}\right) \left(\frac{k^*}{\alpha}\right) \left(\frac{w^*}{1-\alpha}\right)^{-\frac{1}{\alpha}} + \frac{c^*}{k^*} = \\
= -\left(\frac{1}{\alpha}\right) \left(\frac{w^*}{1-\alpha}\right)^{-\frac{1}{\alpha}} k^* \left(\frac{\theta (1-\lambda)}{1-\gamma \alpha}\right) \left(\frac{w^*}{1-\alpha}\right)^{-\frac{1}{\alpha}} w^* + \left(\frac{\theta (1-\lambda)}{1-\gamma \alpha}\right) \left(\frac{k^*}{\alpha}\right) \left(\frac{w^*}{1-\alpha}\right)^{-\frac{1}{\alpha}} \frac{c^*}{k^*}
\]

and

\[
H = \frac{\partial k_t}{\partial w_t} \frac{\partial \dot{w}_t}{\partial k_t} - \frac{\partial \dot{w}_t}{\partial w_t} \frac{\partial k_t}{\partial k_t} = \\
= -\left(\frac{1}{\alpha}\right) \left(\frac{w^*}{1-\alpha}\right)^{-\frac{1}{\alpha}} k^* \left(\frac{\theta (1-\lambda)}{1-\gamma \alpha}\right) \left(\frac{w^*}{1-\alpha}\right)^{-\frac{1}{\alpha}} w^* + \left(\frac{\theta (1-\lambda)}{1-\gamma \alpha}\right) \left(\frac{k^*}{\alpha}\right) \left(\frac{w^*}{1-\alpha}\right)^{-\frac{1}{\alpha}} \frac{c^*}{k^*}
\]

where the inequality follows from the bounded utility condition. Note that the equilibrium is saddle path stable regardless of the sign of the trace.
Figures

Figure 1. Transitional Dynamics: different initial conditions
Figure 2. Transitional Dynamics: different initial conditions

(i) WAGES

(ii) CAPITAL

(iii) INTEREST RATE

(iv) EMPLOYMENT RATE

(v) GROWTH RATE

(vi) LOG GDP
Figure 3. Transitional Dynamics: different wage inertia

(i) wages

(ii) capital

(iii) interest rate

(iv) employment rate

(v) growth rate

(vi) log GDP
Figure 4. Transitional Dynamics: different wage inertia and initial conditions
Figure 5. Permanent increase in the level of TFP
Figure 6. Permanent increase in the long run growth rate
Figure 7. Unemployment benefit

(i) WAGES

(ii) CAPITAL

(iii) INTEREST RATE

(iv) EMPLOYMENT RATE

(v) GROWTH RATE

(vi) LOG GDP