The Pre-Borrowing Motive:
A Model of Coexistent Debt and Cash Holdings.

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Abstract
This paper demonstrates how costly default gives rise to the risk-averse type of behavior by firms. Firms are exposed to the risk of change in the terms of borrowing. With costly default, firms are better off hedging this risk. Hedging motivates firms to borrow earlier with long term debt and keep proceeds in cash until the funds are needed. The finding is novel in the light that the result does not rely on collateral constraints. We examine full implications of the pre-borrowing motive in the dynamic neo-classical model of the firm and characterize optimal borrowing and cash holding policies. In a calibrated version of the model, levels of cash holdings are consistent with that observed in data.
1 Introduction

Bates, Kahle, and Stulz [2009] document the fact in 2006 an average U.S. (Compustat) firm was holding more than 20% of its assets in cash. Authors argue that precautionary motive for holding cash has the strongest empirical support because cash to asset ratio was found to be positively related to the measure of cash flow volatility. Theoretical models that justify holding cash for hedging purposes rely heavily on the assumption that financial constraints are binding. Yet empirically both financially constrained and financially unconstrained firms exhibit positive relationship between cash holdings and volatility of cash flows.

This paper builds a model of precautionary demand for cash holdings that is relevant for both financially constrained and unconstrained firms. News about future prospects of the firm influence the interest rate at which the firm can issue debt. We argue that firms are hedging the volatility in the terms of borrowing because default costs introduce concavity in the value function of the firm. Firms act as if they are risk averse with respect to the news signal\(^1\). By borrowing before the arrival of the news signal and keeping proceeds in cash until the investment date firms avoid exposure to the risk in the credit spread.

The interesting detail about observed cash holdings of firms is that they coexist with debt. As can be seen from figure 1, there is a considerable mass of firms that has positive both Cash/Assets and Debt/Assets. In fact, in 2009 half of firms had Cash/Assets and Debt/Assets above 5%. Moreover, the proportion of firms that has Cash/Assets and Debt/Assets above 20% was almost 10%\(^2\). Since the most plausible explanation for cash holdings is the precautionary motive, we would expect the models of precautionary demand for cash holdings to predict observed coexistence.

Yet delivering that prediction for the financially unconstrained firm has proven to be challenging. One stream of literature considers cash to be negative of debt (Hennessy and

\(^1\)Under some regularity conditions.

\(^2\)Appendix D demonstrates that observed cash and debt co-existence is mostly driven by long-term debt rather than short-term debt. Figure 8 illustrates that cash and debt coexist in a sample for a particular year as well as for a pool of 1985-2010 data points. From Table 1 we can see that the proportion of firms with high levels of both cash and debt was consistently high in the past quarter-century.
Figure 1: Empirical distribution of Cash and Leverage ratios. Data is taken from Compustat, it is a panel of firms in 1985-2010 excluding financial and utility companies.

Whited [2007], Moyen [2007], Moyen [2004]). These models feature unrestricted access to capital markets but do not model cash and debt policies simultaneously. Other models provided a distinct role for cash by imposing collateral constraints on borrowing firms (Acharya, Almeida, and Campello [2007], Almeida, Campello, and Weishbach [2004], Han and Qiu [2007], Acharya, Davydenko, and Strebulaev [2011]). That is, in their models firms have restricted access to financial markets. Boileau and Moyen [2010] study cash policy in a dynamic setting with cash and non-risky debt, hence also effectively restricting the access of a firm to external financing.

The model presented in this paper delivers coexistence of debt and cash holdings as a result of optimal financing policy by financially unconstrained firm. The only friction that firms face is costly default. The model predicts that firms have an incentive to hedge the risk of the arrival of news and they use cash holdings to do so. Firms anticipate an investment to be made at an intermediate date. They prefer to borrow immediately and avoid exposure to risk in the terms of borrowing, which is derived from the risk of news arrival. In the static version of the model, firms hold cash on their balance sheets from

\[ \text{It is financially unconstrained in the sense that it can always fund a positive NPV investment.} \]
date 0 to date 1 and have debt outstanding.

In the dynamic version of the model, pre-borrowing motive manifests in positive contemporaneous correlation between new debt issuances and cash holdings. Firms exploit benefits of the tax shield of debt by rolling-over two period debt, but not every two periods, as you might expect. Most firms borrow to re-finance the debt before it matures and keep proceeds in cash.

It is worth noting that this model establishes the importance of a liquidity policy for a financially unconstrained firm. This is in contrast to the widely held view that firms use cash holdings to overcome financial constraints. For example, Almeida, Campello, and Weishbach [2004] write:

If a firm has unrestricted access to external capital — that is, if a firm is financially unconstrained — there is no need to safeguard against future investment needs and corporate liquidity becomes irrelevant.

In our model the objective function of the firm is concave with respect to the level of the signal about future prospects, which induces risk-averse type of behavior. The firm has unrestricted access to capital markets both prior to investment and at the date of the investment opportunity. It prefers to borrow earlier rather than later because borrowing earlier helps it to avoid exposure to the news risk.

This paper contributes to the existing literature in three ways. First, it establishes that with costly default firms have an incentive to hedge the risk of information arrival. Second, it shows how firms can use cash holdings (proceeds from early issues of debt) as an internal hedge against that risk. Third, it shows how the resulting pre-borrowing motive breaks the equality between cash and (long term) debt in otherwise standard neo-classical setting.

The paper also extends the existing literature on behavior of risk-neutral agents that act in risk-averse ways due to frictions. For example, in the risk-shifting problem of Meckling and Jensen [1976], option-like structure of equity claim induces risk-seeking behavior on the shareholders’ side and risk-averse preferences for bondholders. The key risk in this paper is news arrival. Firms face an endogenous default boundary that depends

\[ 4 \]

\[ 4 \]In that case interest rate paid on debt is not subject to change due to news arrival.
on a random signal about the future prospects of the company. In a frictionless world, firms are risk neutral with respect to that risk. The presence of default costs induces a value maximizing firm to minimize the probability of default. The firm’s objective function becomes concave in the risk of default boundary. Companies optimally choose to minimize this exposure through borrowing before the arrival of the signal, thereby avoiding randomness in the default boundary. Hence, firms are acting in a risk-averse manner due to the presence of default costs, but are not directly hedging the event of default.

Froot, Scharfstein, and Stein [1993] established a somewhat similar result with respect to the risk of cash flows — they show that expected default costs are convex in cash inflow; hence, a firm would be interested in hedging it. They argue that this generates a demand for external hedging instruments by firms. Unlike cash flow, an easily verifiable variable, news is harder to verify and hence to contract upon. Therefore, it may be harder for firms to hedge news arrival with external instruments. In our paper we show first that news arrival in general is a risk that firms are willing to hedge. Second, since external hedging instruments are not readily available, we show how internal instruments (early issue of debt and cash holdings) can be used to hedge the risk of the news signal. In other words, Froot, Scharfstein, and Stein [1993] showed that firms should be hedging the volatility in the amount of external borrowing. In contrast, in our paper we show that firms should also be hedging the volatility in the terms of borrowing.

We provide empirical evidence that supports our idea. In a regression that uses controls from Bates, Kahle, and Stulz [2009] we show that the estimate of volatility in firm’s market to book ratio has strong positive impact on cash holdings. This results suggest that firms hold more cash because of higher volatility in market to book ratio. It is natural to think of market to book ratio as an indicator of future prospects of the firm. That is, the updated on news expected payoff from the assets in the firm. Hence, the variance of the market to book ratio is a good proxy for the volatility in the news about future prospects of the firm. According to our model (since default costs are convex in the news signal) the more volatile the signal is, the more incentives the firm has to hedge it. Therefore, the more cash holdings it will have. Data supports that claim — volatility of market to book ratio has positive impact on cash holdings.
We use solution of the dynamic model to simulate a panel of firms and show that most of simulated moments resemble that of empirical data. For example, cash holdings are highly persistent — [1]. [Essentially, go over the table of moments].

The paper is organized in the following way: static model establishes the pre-borrowing motive in 3-period setting. Building of that result, dynamic model shows implications for introducing two period debt in the otherwise standard neo-classical model of the firm (without investment).

2 Static Model

We now consider a risk-neutral firm in 3-period setting that is choosing between short and long term financing for an investment project.

2.1 Motive for Hedging

The firm lives for 2 periods. An investment opportunity occurs at date 1. The firm has to pay \( I \) out of either internal or external funds or both. It also has some assets in place. A firm’s payoff in period 2 would be \( G + \nu \), where \( G \) is a constant and \( \nu \in [0, \infty) \) follows distribution with the cumulative distribution function \( K(\nu) \). Default costs are fixed at \( \xi \).\(^5\)

The risk-free interest rate is normalized to be zero.

The investment amount is assumed to be fixed to avoid the discussion of Myers [1977] debt-overhang problem. The hedging motive that arises due to endogenous default boundary in this model is different from the debt-overhang problem. As will soon be clear, the costs of default and the endogenous default boundary make both the total value of the firm and shareholder value as of period 1 concave in the news signal, thereby creating a hedging motive. Hence, the hedging motive is not related to the conflict of interest between shareholders and bondholders, as the debt-overhang problem is.

At date 1, the firm receives a signal \( \lambda \) about its future prospects. The updated distribution function for \( \nu \) becomes \( F(\lambda, \nu) \), such that:

\(^5\)Support for \( \nu \) starts at 0. Hence, if the worst possible scenario happens, bond holders would receive \( G - \xi \), which is assumed to be non-negative WLOG. That is the only purpose of having constant \( G \) in the final payoff.
\[
\int_{\lambda}^{X} F(\lambda, \upsilon)g(\lambda) \, d\lambda = K(\upsilon),
\] (1)

where \(g(\lambda)\) is the marginal distribution of \(\lambda\), the information signal. Also, assume that \(F_\lambda < 0, F_{\lambda\lambda} > 0\). That is, with better news, we expect that the probability that \(\upsilon\) is below some number \(a\) is a decreasing with decreasing marginal effect.

At date 1, the firm has access to an outside capital market that has infinitely many risk neutral price taking lenders. Firm is issuing unsecured bonds that are payable at date 2. The interest rate at which lenders are willing to give money to the firm is determined by setting the expected profit of lenders to zero. Since the firm has no internal funds carried from date 0, it needs to borrow the total amount of investment \(I\) at date 1. Assume that it can do so for any realization of the news signal. That is, there always exist an \(r \geq 0\) such that the expected promised payoff to bond holders is at least \(I\). Then interest rate \(r(\lambda)\), is endogenously determined by:

\[
I(1 + r) \left[ 1 - F(\lambda, I(1 + r) - G) \right] + \int_{0}^{I(1+r)-G} (x + G - \xi) f(\lambda, x) \, dx = I. \tag{2}
\]

The first term in 2 is the bond’s face value times the probability of no default. The second term is the expected payment to bond holders in case of default. Hence, the interest rate sets expected payments on the bond equal to the proceeds from issuing it.

It is intuitively clear that the better the news, the better the prospects of the company, hence, the more likely the firm to repay all of its obligations in period 2. Hence, the risk premium required by the bond market is smaller. The next lemma states this formally.

**Lemma 2.1.** If \(F(\lambda, \upsilon)\) satisfies \(F_\lambda < 0\), then \(r_\lambda < 0\).

Proof can be found in the appendix.

The total expected value of the firm at \(t = 1\), contingent on the realization of the signal is

\[
V_1(\lambda) = E \left( G + \upsilon - \xi 1 \{ G + \upsilon \leq I(1 + r(\lambda)) \} \right) \bigg| \lambda \bigg) \tag{3}
\]
The expectation is taken with respect to $F_\lambda$. We can re-write 3 to be

$$V_1(\lambda) = E \left( G + v | \lambda \right) - \xi F \left( \lambda, I(1 + r(\lambda)) - G \right)$$

If we now look at the value of the firm from the perspective of period 0 we will see that the dependence on news signal is integrated out of the first term in a risk-neutral way, while the second term depends on news signal non-linearly. The next proposition establishes that default costs are a convex function of the news signal.

**Proposition 2.2.** Define $C(\lambda) = \xi F \left( \lambda, I(1 + r(\lambda)) - G \right)$. If $F(\lambda, v)$ satisfies

$$\left( \frac{f(\lambda, v)}{1 - F(\lambda, v)} \right)_\lambda' < 0$$

(i)

and

$$\left( \frac{f(\lambda, v)}{1 - F(\lambda, v)} \right)_v' > 0$$

(ii)

then $C(\lambda)$ is a convex function.

The conditions imposed on the cumulative distribution function are fairly mild. Condition (i) can be interpreted as decreasing marginal probability to be at the default boundary given that there is no default. It is natural to think that the better the news, the less likely the firm is to end up close to the default boundary if it has not defaulted. Condition (ii), as discussed by Froot et al. [1993], is satisfied by a wide class of distributions.

The convexity of the expected default costs $C(\lambda)$ with respect to the news signal makes the total value of the firm $V_1(\lambda)$ a concave function of the signal. Hence, the firm has an incentive to hedge the risk of news arrival.

The convexity of cost function has a simple intuitive explanation — the better the news, the lower the expected default costs, since the company is expected to perform better. However, the marginal decrease in expected default costs is smaller with each additional piece of good news. After all, even if the news is extremely good, there is still

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6Exponential distribution with mean parameter $\lambda$ satisfies it. I was also able to show numerically that normal distribution with mean $\lambda$ also satisfies it.
Figure 2: Value function of the firm. Default costs introduce a ”penalty” $\varepsilon E(\text{default}|\lambda)$ to the firm valuation that is decreasing in the news signal. Hence, if we start with value of the firm that is linear in the news signal, the introduction of default costs brings concavity to the value function. Hence, firms have incentives to behave as if they are risk averse.

a chance that the firm will default on its obligations so that costs never fall down to zero. That implies that the gap between the value function with and without costly default is decreasing in the news signal (see figure 2).

The proposition above establishes that if the firm is borrowing at the moment when it needs funds for investment, the firm is subject to the volatility in terms of borrowing (as a result of news arrival). And since the value of the firm is concave in the news signal, the firm is interested in hedging that risk. In the next section, we will show how pre-borrowing (that is, borrowing before the news arrival), and using hoarded cash to finance the investment in period 1 reduces the exposure to the risk in terms of borrowing.

2.2 Pre-borrowing policy

Now let us allow the firm to choose when to borrow — at date 0 before the realization of the signal (and hence on average terms), or at date 1 after the news are revealed (and hence subject to the volatility in terms of borrowing). The face value of debt issued at date 1 is denoted by $B$ (that is, principal plus the interest payable at date 2 is equal to $B$). Let $D$ stand for the face value of debt issued at period 0. The firm is allowed to chose
any combination of $D$ and $B$ that will deliver it enough liquid assets to pay $I$ at date 1.

Note that B-bonds holders are assumed to have junior priority in asset distribution in case of default. The assumption of seniority of debt claims that were issued earlier is not the one driving our results. It is made for tractability of model results. The same intuition goes through with say reversed priority — debt holders at period zero predict the optimal behavior by the firm for each realization of $\lambda$ and incorporate that into their pricing decision. Hence, they are still making decision based on average terms and the firm is not exposed to the risk in the news signal if it decides to borrow at period 0 the whole amount of investment\textsuperscript{7}.

If the firm decides to borrow funds at date 0 before the news is revealed, it promises to pay an interest rate that does not depend on the news. Denote by $\delta(D)$ the proceeds from the issue of that debt:

$$\delta(D) = E^\lambda \left( \int_0^{D-G+\xi} (v + G - \xi)f(\lambda, v) \, dv + D\left(1 - F(\lambda, D - G + \xi)\right) \right) \quad (4)$$

The firm keeps $\delta(D)$ until date 1 when investment needs arise. Then it will have to borrow the remaining amount at period 1 so that the total sum of proceeds from both issues of debt is equal to investment expenditure. Denote by $P(\lambda, B)$ the proceeds from issuing debt at date 1:

$$P(\lambda, B) = \int_{D-G+\xi}^{D+B-G} (v + G - \xi - D)f(\lambda, v) \, dv + B[1 - F(\lambda, D + B - G)]. \quad (5)$$

Then we know that $D$ and $B$ are chosen so that the firm has enough resources to make the investment:

$$\delta(D) + P(\lambda, B) = I \quad (6)$$

The interest rate promised on the debt issued in period 1 depends on the news signal, and hence, the face value of debt is a function of the news signal: $B(\lambda)$. It is implicitly defined by 6.

\textsuperscript{7}More details can be found in Appendix.
The total value of the firm as of period 0 can be written as

\[ V = E^\lambda \left( E\left( G + v - \xi 1_{\{G + v < B(\lambda) + D\}} \mid \lambda \right) \right) \]  
(7)

The outer expectation is taken with respect to news signal distribution. We can re-write 7 to be:

\[ V = E(G + v) - \xi E^\lambda \left( F(\lambda, B(\lambda) + D - G) \right) \]

The next proposition shows that when the firm decides to borrow marginally more in period 0 and therefore can borrow marginally less after the news arrival, the company is expected as of date 0 to default with a smaller probability. That is, the company is better off borrowing earlier rather than later if it wants to save expected default costs.

**Proposition 2.3.** If \( F(\lambda) \) satisfies

\[ \left( \frac{f(\lambda, v)}{1 - F(\lambda, v)} \right)'_\lambda < 0 \] (i)

and

\[ \left( \frac{f(\lambda, v)}{1 - F(\lambda, v)} \right)'_\upsilon > 0 \] (ii)

then \( V'_D > 0 \).

The firm’s decision to borrow slightly more before the news arrival rather than after reduces the firm’s exposure to risk of the news signal. We have already seen that default costs are convex in the news signal, hence, the firm is better off hedging this risk. The risk hedging is delivered by borrowing earlier, that is in period 0 rather than in period 1.

When the firm borrows early, it holds proceeds in the form of cash from the moment of debt issue until investment needs arise. That is, we observe outstanding debt and cash holdings on the balance sheets of firms at the same time. Both cash and debt balance serve the same goal - finance investment expenditure. While debt was issued to raise sufficient funds, cash is used to transfer resources to the date of investment. This financing structure allows firms to lock in the interest rate.

Proposition 2.3 merely establishes the incentive for firms to issue debt earlier rather
than later. In a more realistic environment, firms would balance this incentive against interest expense paid on cash funds (interest-dominated asset) for a period before investment. Firms might also consider equity issuance as a source of funding the investment needs. This will influence optimal debt issuance policy of the firm. However, as we will see in dynamic version of the model, as long as firms have an incentive to finance expenditures (outflows)\(^8\) with debt (tax shield, etc), firms will also have an incentive to borrow earlier to avoid exposure to the risk of news arrival.

## 3 Dynamic Model

We now consider implications of pre-borrowing motive for standard neo-classical firm that has a choice between short and long term debt financing. In every period the firm can issue both one and two period debt. That is, the environment is set up as if 3-period models are overlapping instead of being stacked next to each other. Every period there are long term and short term debt maturing and long term debt outstanding (that will mature in the next period). Hence, at the same time there are 3 types of debt. The firm in this setting is not restricted to only debt financing and can issue equity.

### 3.1 Environment

Consider a firm that has 1 unit of capital. Its idiosyncratic productivity shock, \(z_t\) follows an log-AR(1) process with correlation parameter \(\rho_z\) and variance of innovations \(\sigma_z^2\).

Denote by \(LB_{t+1}\) proceeds from issue of debt that is to be repaid in the future period. If \(c_{t+1}^B\) is the coupon rate, then in the next period creditors will get \((1 + c_{t+1}^B)LB_{t+1}\).

Denote

\[
B_{t+1} = (1 + c_{t+1}^B)LB_{t+1}.
\]

The amount of debt \(B_t\) can be negative. In that case we can think of \(B_t\) representing cash carried over from the last period.

\(^8\)Dynamic version of the model does not have investment choice for the sake of computational feasibility. The pre-borrowing idea though applies to any sort of expense or outflow that is expected to happen. For example, firms that roll-over debt need funds to refinance the debt and, hence, would have an incentive to borrow one period before the debt maturity and keep proceeds in cash.
In the same way, let $LD_{t+1}$ stand for the proceeds from issuing debt that will be payable two periods from now. The coupon on 2-period debt is due only at the maturity. So, at $t + 2$ creditors will be paid $(1 + c_{t+1}^D)LD_{t+1}$.

Equity issuance triggers both fixed and proportional costs. If $Eq_t$ is the amount of equity to be raised by the firm (net equity distribution), then shareholders pay $Eq_t(1 + \lambda_1) + \lambda_0$. In case shareholders decide to declare default, assets that are in the firm are sold with a discount $\varepsilon$.

### 3.2 Equity Problem

Firm is entering the period with operating cash inflow of $z_t$. It has long term debt outstanding $D_t$ and liquid assets $LA_t$, which are the cash carried from previous period (if any) less maturing long term debt $-B_t - D_{t-1}$. If the firm decides to continue operations, it needs to make an investment to cover for the depreciated capital. Equity distributions can be written as:

$$Eq_t = z_t - F + LB_{t+1} + LD_{t+1} + LA_t$$

where $F$ is fixed per-period cost of operation.

The firm has inflow of operating profit and proceeds from issue of short and long term debt. These funds are allocated to repayment of maturing debt (if any), saving and the rest is distributed to shareholders. Equity issuance is costly. Hence, distributions to shareholders taking into account costs of equity issuance are:

$$DE_t = Eq_t + (Eq_t\lambda_1 - \lambda_0)1_{Eq_t < 0}.$$

Each period shareholders choose the financing structure of firms operation: how much long term debt to issue and either borrow in short term debt or save some cash for the next period. Formally, the problem can be written as:
\[ V(z_t, D_t, LA_t) = \max \left[ 0, \left( z_t - F + 1 + LA_t - \frac{D_t}{1 + r} \right) (1 - \varepsilon), \max_{B_{t+1}, D_{t+1} \geq 0} \left\{ z_t - F + LB_{t+1} + LD_{t+1} + LA_t + \Phi_t + \frac{1}{1 + r} E(V(z_{t+1}, D_{t+1}, LA_{t+1})) \right\} \right]; \]

where liquid assets within the firm next period are \( LA_{t+1} = -B_{t+1} - D_t \) and equity issuance costs in the current period are \( \Phi_t = (Eq_t\lambda_1 - \lambda_0)1_{Eq_t < 0} \).

Shareholders apart from continuing operation (11), can default on their debt obligations and walk away (10). In addition to that, shareholders might decide to liquidate the firm (11). That option allows shareholders sell physical assets within the firm, repay the debt holders and walk away from the firm with what is left. This decision might be optimal even for an all equity financed firm. The firm will find it optimal to liquidate when expected future operating cash flows are less than costs of operation. Since in the model the firm has to restore depreciated capital in order to survive until the next period, the firm behaves as if it has fixed costs of operation.

Gomes [2001] incorporates liquidation decision explicitly in the choice set for all equity financed firms. Many papers that are dealing with defaultable debt in dynamic setting abstract from fixed costs of operation (for example, Hennessy and Whited [2007]).\(^9\) Some papers (Gomes and Schmid [2010], Kuehn and Schmid [2011]) that have both risky debt and fixed costs of operation, do not explicitly account for liquidation option for the firm. If shareholders are not allowed to sell assets at \( t \) but the expected cash inflow from operation is smaller than operating costs, shareholders might decide to sell assets to investors through issuing too much debt. Too much in that case would mean that shareholders guarantee default in \( t + 1 \). That behavior contradicts U.S. bond regulations and hence biases simulated results towards issuing too much debt.

\(^9\)Firms that do not face fixed costs of operation would not find it optimal to liquidate because their operating profit is always positive and the only reason to stop operation would be debt overhang.
3.3 Bond pricing

Recovery in case of default consists of operating cash inflow, depreciation tax shield, proceeds from sale of physical assets and cash if any.

\[ R(z_{t+1}) = (z_{t+1} - f + 1 - B_{t+1}1_{B_{t+1} < 0})(1 - \varepsilon). \]

Those funds are allocated to the three existing groups of bond holders. The earliest issued long term debt is assumed to have the highest priority.

- long term bond holders receive

\[ R_1(z_{t+1}, D_t) = \min(R(z_{t+1}), D_t); \]

- long term recent bond holders receive

\[ R_2(z_{t+1}, D_t, D_{t+1}) = \min\left[\max(0, R(z_{t+1}) - D_t, D_{t+1}/(1 + r))\right]; \]

- short term bond holders essentially receive anything that was left after paying the two groups of long term bond holders -

\[ R_3(z_{t+1}, D_{t+1}, D_t, B_{t+1}) = \min\left[\max(0, R(z_{t+1}) - D_t - D_{t+1}/(1 + r)), B_{t+1}\right]. \]

Risk-neutral competitive lenders decide on price of short and long term debt by guaranteeing zero profit from lending:

\[ LB_{t+1} = \frac{1}{1 + r} \left[ (1 + c_{t+1}^B)LB_{t+1}E_t(1_{V_{t+1} > 0}) + E_t(R_{3,t+1}1_{V_{t+1} = 0}) \right] \]

(12)

\[ LD_{t+1} = \frac{1}{(1 + r)^2} \left[ (1 + c_{t+1}^D)LD_{t+1}E_t(1_{V_{t+1} > 0 \& V_{t+2} > 0}) + E_t(R_{1,t+2}1_{V_{t+1} > 0}1_{V_{t+2} = 0}) + (1 + r)E_t(R_{2,t+1}1_{V_{t+1} = 0}) \right]; \]

(13)
Or, expressing \((1 + c^D_{t+1})LD_{t+1}\) through \(D_{t+1}\) and \((1 + c^B_{t+1})LB_{t+1}\) though \(B_{t+1}\) we have:

\[
LB_{t+1} = \frac{1}{1 + r} \left[ B_{t+1}E_t(1_{V_{t+1}>0}) + E_t(R_{3,t+1}1_{V_{t+1}=0}) \right]
\] (14)

and

\[
LD_{t+1} = \frac{1}{(1 + r)^2} \left[ D_{t+1}E_t(1_{V_{t+1}>0}1_{V_{t+2}>0})
+ E_t(R_{1,t+2}1_{V_{t+1}>0}1_{V_{t+2}=0}) + (1 + r)E_t(R_{2,t+1}1_{V_{t+1}=0}) \right];
\] (15)

As have been pointed out by Gomes and Schmid [2010], working with face value as a choice variable \((B_{t+1} and D_{t+1})\) and evaluating market value of the two debts has computational advantage over specifying coupon schedule \(c^B_{t+1}\) and \(c^D_{t+1}\) explicitly. In the latter case the procedure should have included outer loop for convergence of coupon schedule on top of the inner loop for value function convergence. In the current specification, in order to price the debt we need only to evaluate two functions.

### 3.4 Calibration

The model is solved using value function iterations. Grid for \(z_t\) has 15 points and its dynamics is approximated using Tauchen [1986] method. Grid for \(LA_t\) has 41 points while \(D_t\) has 11 grid points. For each value function iteration, short and long term debt is priced fairly to obtain firm’s optimal choice functions. Pricing of debt is done through evaluation of functions summarized in 14 and 15. Hence, there is only one convergence loop for the value function of shareholders.

The model is calibrated at the annual frequency. I have estimated AR(1) model for log(Sales/Assets) for US COMPUSTAT non-financial, non-utilities firms on 1985-2010 data. The estimates are 0.501 for persistence parameter and 0.16 for standard deviation of innovations. The size of per-period operating costs was chosen to match average value of profitability (3.5% of total assets for US COMPUSTAT non-financial, non-utilities firms on 1985-2010 data). Equity issuance costs are chosen to make borrowing with additional
equity issuances prohibitively costly for the firm. Risk-free interest rate is set at 3% annual, which implies discount factor of approximately 0.97, smaller than 0.98 used by Kuehn and Schmid [2011] but larger than Bhamra, Fisher, and Kuehn [2011]’s 0.96 and Gomes [2001]’s 0.939. The default costs are the same as liquidation costs and are set at 35% of asset value.

3.5 Optimal financing choice

The firm decides on debt and liquidity policy simultaneously. As diagram 3 illustrates, firms save more if it enters the period with more liquid assets, which is consistent with intuition that cash-rich firms carry the cushion from period to period. Firms find it optimal to decrease cash savings if the debt outstanding is too large — foreseeing the likely default next period, shareholders take cash out of the company in the current period (see Figure 4).

A pre-borrowing motive is illustrated in Figure 5. At low levels of debt outstanding, the firm has no need to borrow to cover repayment of debt next period. As the level of long-term debt outstanding increases, firm decides to borrow more in the current period. If the level of debt maturing next period is too high though, the optimal policy is not to issue any debt and wait for the default in the next period.

The steady state distribution of firms over financing choices is summarized in Figure 6. As you can see, there is a mass of firms that chooses to issue long-term debt as well as save some positive amount of cash until the next period. This is consistent with observed empirical evidence — firms have significant cash holdings at the same time as having substantial amount of debt outstanding .

The model matches some of the empirical moments well. For example, cash holdings that are as persistent as those observed in the data (0.8394 and 0.805 — estimated from a sample of US COMPUSTAT non-financial firms).

Simulated data also shows positive relationship between cash and new debt issuances, which is consistent with empirical finding by Bates, Kahle, and Stulz [2009]. The reported coefficient of new (long term) debt issuances in cash regressions is positive and statistically significant. This finding provides support for the idea that firms keep at least a part of
Figure 3: Optimal cash holdings as a function of liquid assets within the firm. The more cash the firm has at the beginning of the period, the more it saves for the next period.
Figure 4: Optimal cash holdings as a function of debt outstanding. The more debt the firm has outstanding, the larger the probability it will default next period. Hence, after a certain level, shareholders find it more optimal to take liquid funds out of the firm in the current period and save no cash.
Figure 5: Optimal (Risky) LT borrowing.
Figure 6: Distribution of Cash and Debt of a panel of simulated firms.
the proceeds from debt issue in cash.

The current parameter calibration results in leverage ratios that are too high. This might be partly due to the fact that in the model the firm has access to debt of maturity up to 2 years. In practice, the array of available maturities is richer. In the current specification, the firm is issuing long term debt every period. If the model did allow for say a 5 period debt then the firm might have found it optimal to borrow say 1 or 2 periods before the maturity, not 4 periods. Therefore, with 5 year debt available there would be firms that have only one type of debt outstanding while in the current model the firm finds it optimal to have two issues of the long term debt.

Model produces very persistent leverage ratios (with correlation coefficient above 90%). That is consistent with the idea of ‘hysteresis’ discussed by Gomes and Schmid [2010].

4 Conclusions

This paper explains why financial policy can be important for a financially unconstrained firm. It also offers a precautionary explanation for coexistence of debt and cash on the balance sheets of U.S. firms.

Presence of default costs provides incentives for the firm to minimize the likelihood of default. The chance that the firm will default is a convex function of the news signal. Therefore, the firm is better off hedging the news risk. The financial policy that delivers hedging can be summarized in the following way: borrow early before the arrival of the news signal and hoard cash. When the investment needs arise finance them with the internal funds (cash). This policy allows firms to lock in the interest rate and avoid the exposure to the volatility in the terms of borrowing.

This pre-borrowing motive helps explain large levels of cash holdings observed empirically. A calibrated version of the model produces mean cash to asset ratios that are comparable with those observed in the data. It also reproduces the positive relationship between debt issuances and cash holdings that was documented by various empirical studies.
APPENDIX

A Lemma 2.1

Proof. For a given \( \lambda \), \( r(\lambda) \) is defined endogenously by the following equation:

\[
I(1 + r)
\left[1 - F\left(\lambda, I(1 + r) - G\right)\right] + \int_0^{I(1+r)-G} (x + G - \xi)f(\lambda, x) \, dx = H(r, \lambda) = I \quad (16)
\]

Since the firm is assumed capable of borrowing for any realization \( \lambda \), equation 16 has at least one solution. Depending on the particular shape of \( F(\lambda, \cdot) \) the equation can be satisfied for more than single \( r \). In that case we define \( r(\lambda) \) to be the minimal \( r \) that satisfies equation 16.

Before using the implicit function theorem to find the sign of \( r_\lambda \), simplify \( H(r, \lambda) \):

\[
H(r, \lambda) = I(1 + r)[1 - F(\lambda, I(1 + r) - G)] + \left(I(1 + r) - \xi\right)F(\lambda, I(1 + r) - G)
- \int_0^{I(1+r)-G} F(\lambda, x) \, dx
= I(1 + r) - \int_0^{I(1+r)-G} F(\lambda, x) \, dx - \xi F(\lambda, I(1 + r) - G) \quad (17)
\]

Now we compute the derivatives of \( H(r, \lambda) \):

\[
\frac{\partial H(r, \lambda)}{\partial \lambda} = -\int_0^{I(1+r)-G} F_\lambda(\lambda, x) \, dx - \xi F_\lambda(\lambda, I(1 + r) - G) > 0 \quad (18)
\]

\[
\frac{\partial H(r, \lambda)}{\partial r} = I - IF(\lambda, I(1 + r) - G) - I\xi f(\lambda, I(1 + r) - G)
= I[1 - F(\lambda, I(1 + r) - G) - \xi f(\lambda, I(1 + r) - G)] \quad (19)
\]

Let’s look closer at expression 20. It is the marginal change in the market value of debt due to infinitesimal increase in the promised interest on the debt, evaluated at the default boundary. If it was negative, then there was a smaller level of \( r \) that would satisfy equation 16. Intuitively, it is irrational for the firm to promise the interest rate that delivers \( 20 < 0 \).

The debt holders would have been better off getting a smaller promised payment since the
firm would default with smaller probability. Hence, the value maximizing firm operates only with $20 > 0$.

Then

$$ r_\lambda = -\frac{\partial H(r,\lambda)}{\partial \lambda} < 0 $$

(21)

B Proposition 2.2

Proof. Costs of external borrowing are

$$ C(\lambda) = \xi F\left(\lambda, I(1 + r(\lambda)) - G\right) $$

Interest rate is defined by the following bond market clearing condition

$$ \int_0^{I(1+r)-G} (x + G - \xi) f(\lambda, x) \, dx + I(1+r)[1 - F(\lambda, I(1+r) - G)] = I $$

(22)

Integrating by parts we obtain an alternative expression for market value of debt:

$$ I(1+r) - \int_0^{I(1+r)-G} F(\lambda, x) \, dx - \xi F(\lambda, I(1+r) - G) = I $$

(23)

$$ [1 - F(\lambda, \cdot) - \xi f(\lambda, \cdot)] I \, dr + \left[ - \int_0^{I(1+r)-G} F(\lambda, x) \, dx - \xi F(\lambda, \cdot) \right] d\lambda = 0 $$

(24)

where $F(\lambda, \cdot)$ stands for $F(\lambda, I(1+r) - G)$.

$$ r_\lambda = \frac{dr}{d\lambda} = \frac{1}{I} \frac{\int_0^{I(1+r(\lambda))} -G} {1 - F(\lambda, \cdot) - \xi f(\lambda, \cdot)} < 0 $$

(25)

Now go back to cost function

$$ C_\lambda = \xi \left[ f\left(\lambda, I(1 + r(\lambda)) - G\right) I r_\lambda + F_\lambda\left(\lambda, I(1 + r(\lambda)) - G\right) \right] $$

(26)

Plug in expression for derivative of interest rate with respect to news:
\[ C_\lambda = \xi \left( \frac{f(\lambda, \cdot)}{1 - F(\lambda, \cdot) - \xi f(\lambda, \cdot)} \int_0^{I(1+r(\lambda))} F_\lambda(\lambda, x) \, dx \right) + \frac{f(\lambda, \cdot)}{1 - F(\lambda, \cdot) - \xi f(\lambda, \cdot)} \xi F_\lambda(\lambda, \cdot) + F_\lambda(\lambda, \cdot) \]

Now denote

\[ \tilde{HR} = \frac{f(\lambda, I(1+r(\lambda)) - G)}{1 - F(\lambda, I(1+r(\lambda)) - G) - \xi f(\lambda, I(1+r(\lambda)) - G)} \]

So

\[ C_\lambda = \xi \left( \tilde{HR} \left[ \int_0^{I(1+r(\lambda))} F_\lambda(\lambda, x) \, dx \right] + \tilde{HR} \xi F_\lambda(\lambda, \cdot) + F_\lambda(\lambda, \cdot) \right) \]

### B.1 Hazard Rate

We will show now that \( \tilde{HR}_\lambda < 0 \):

\[
\frac{\partial \tilde{HR}}{\partial \lambda} = \frac{\partial}{\partial \lambda} \left( \frac{f(\lambda, I(1+r(\lambda)) - G)}{1 - F(\lambda, I(1+r(\lambda)) - G) - \xi f(\lambda, I(1+r(\lambda)) - G)} \right)
\]

\[ = \frac{\partial}{\partial \lambda} \left( \frac{f(\lambda, I(1+r(\lambda)) - G)}{1 - F(\lambda, I(1+r(\lambda)) - G) - \xi f(\lambda, I(1+r(\lambda)) - G)} \right) \]

\[ = \frac{\partial}{\partial \lambda} \left( \frac{\tilde{HR}}{1 - \xi \tilde{HR}} \right) \]

\[ = \frac{\tilde{HR}_\lambda (1 - \xi \tilde{HR}) + \xi \tilde{HR}_\lambda \tilde{HR}}{(1 - \xi \tilde{HR})^2} \]

\[ = \frac{\tilde{HR}_\lambda}{(1 - \xi \tilde{HR})^2} \]

\[ = \frac{1}{(1 - \xi \tilde{HR})^2} \left[ \left( \frac{f(\lambda, \cdot)}{1 - F(\lambda, \cdot)} \right)' + \left( \frac{f(\lambda, \cdot)}{1 - F(\lambda, \cdot)} \right)' \right]_{x=I(1+r(\lambda))} \tilde{I}_R(\lambda) \quad (27) \]

The first term in square brackets in 27 is the derivative of hazard rate w.r.t. news signal. It is assumed to be negative in the proposition. Intuition behind this assumption is simple — given that firm has not defaulted we assume that firm is less likely to be on the default boundary if news are good.

Second term in the square brackets is the derivative of hazard rate. Many distributions, as discussed by Froot et al. [1993], have increasing hazard ratios (derivative is positive).
Proposition assumes that $F(\lambda,\nu)$ satisfies that property. Then it is clear that $27$ is less than zero. That is, $\tilde{HR}_\lambda < 0$.

**B.2 Second Derivative of the Cost function**

Let us go back to the first derivative of the expected cost of default:

$$C_\lambda = \xi \left( \tilde{HR} \left[ \int_0^{I(1+r(\lambda))-G} F_\lambda(\lambda, x) \, dx \right] + \tilde{HR} \xi F_\lambda(\lambda, \cdot) + F_\lambda(\lambda, \cdot) \right)$$

Now differentiate expression $28$ with respect to the news signal once again:

$$C_{\lambda\lambda} = \xi \left( \tilde{HR}_\lambda \left[ \int_0^{I(1+r(\lambda))-G} F_\lambda(\lambda, x) \, dx \right] + \tilde{HR} \left[ \int_0^{I(1+r(\lambda))-G} F_{\lambda\lambda}(\lambda, x) \, dx \right] + \tilde{HR} F_\lambda(\lambda, \cdot) I\lambda \right)$$

$$+ \tilde{HR}_\lambda F_\lambda(\lambda, \cdot) + (\tilde{HR} + 1) \left\{ F_{\lambda\lambda}(\lambda, \cdot) + f_\lambda(\lambda, \cdot) I\lambda \right\}$$

Expression in the third line can be simplified to:

$$\tilde{HR} F_\lambda(\lambda, \cdot) + (\tilde{HR} + 1)f_\lambda(\lambda, \cdot)$$

$$= \frac{f(\lambda, \cdot) F_\lambda(\lambda, \cdot)}{1 - F(\lambda, \cdot) - \xi f(\lambda, \cdot)} + \frac{\xi f(\lambda, \cdot) + 1 - F(\lambda, \cdot) - \xi f(\lambda, \cdot)}{1 - F(\lambda, \cdot) - \xi f(\lambda, \cdot)} f_\lambda(\lambda, \cdot)$$

$$= \frac{1}{1 - F(\lambda, \cdot) - \xi f(\lambda, \cdot)} \left[ f(\lambda, \cdot) F_\lambda(\lambda, \cdot) + (1 - F(\lambda, \cdot)) f_\lambda(\lambda, \cdot) \right]$$

$$= \frac{(1 - F(\lambda, \cdot))^2}{1 - F(\lambda, \cdot) - \xi f(\lambda, \cdot)} \left[ \left( \frac{f(\lambda, \cdot)}{1 - F(\lambda, \cdot)} \right)' \right] < 0$$  

(32)
Hence,

\[
C_{\lambda\lambda} = \xi \left( \overline{HR}_{\lambda} \left[ \int_0^{I(1+r) - G} F_{\lambda}(\lambda, x) \, dx \right] + \overline{HR} \left[ \int_0^{I(1+r) - G} F_{\lambda\lambda}(\lambda, x) \, dx \right] \right) \\
+ \xi \overline{HR}_{\lambda} F_{\lambda}(\lambda, \cdot) + (\xi \overline{HR} + 1) F_{\lambda\lambda}(\lambda, \cdot) \quad (33)
\]

\[
+ Ir_{\lambda} \left( \overline{HR} F_{\lambda}(\lambda, \cdot) + (\xi \overline{HR} + 1) f_{\lambda}(\lambda, \cdot) \right) \right) > 0 \quad (34)
\]

\[
\square
\]

C Proposition 2.3

Proof. To start with, we have \( V = V_{sh} + V_B + V_D \), that is the value of the company is split between shareholders, senior and junior bondholders. This notation refers to the period 2 values - that is, terminal values realized in period 2. Taking the expectation we can note that \( E(V_D) \) represents the market value of senior debt at the moment it is issued. Denote it \( \delta \). Analogously, \( E(V_B|\lambda) = P \) and we have a condition \( I = \delta + P \), that is, the market value of senior and junior bond holders should constitute the required investment \( I \).

\[
E(V) = E(V_{sh}) + \delta + P = E(V_{sh}) + I
\]

Naturally, the interests of shareholders are aligned with the interest of the company overall. Financial policy (how to borrow \( I \) — mainly later or earlier) does not affect the expected payoff to the bondholders since it is exactly \( I \) but might affect the welfare of shareholders\(^{10}\).

\[
\frac{\partial V}{\partial D} = -\xi \frac{\partial}{\partial D} \left( E^\lambda [F(\lambda, B(\lambda)) + D - G)] \right) \\
= -\xi E^\lambda \left( f(\lambda, B(\lambda) + D - G) \left[ 1 + \frac{\partial B(\lambda)}{\partial D} \right] \right) \quad (35)
\]

\(^{10}\)Think about the two ways to write the value of the company as of period 1: \( V = V_{sh} + \delta + P \) and \( V = E(v + G - \xi 1_{v + G < B(\lambda) + D}|\lambda) \). In both cases can take expectation with respect to distribution of news signal and then take derivative w.r.t \( D \).
Hence, in order to determine if borrowing earlier (increasing $D$) is beneficial for the company or not, need to determine the $1 + \frac{\partial B(\lambda)}{\partial D}$. Intuitively expect that expression to be less than zero when news are high — we loose money by borrowing earlier (at average terms) when news turn out to be good after all. And greater than zero when news are bad — firm saved some money by borrowing at average terms earlier. We then weight those cases by $f(\lambda, \cdot)$.

C.1 Deriving $\frac{\partial B(\lambda)}{\partial D}$

We will use equation $\delta + P = I$ to derive the effect of increase in early borrowing on necessary amount of later borrowing. By implicit function theorem,

$$\frac{\partial B(\lambda)}{\partial D} = -\frac{\partial \delta/\partial D + \partial P/\partial D}{\partial P/\partial B(\lambda)}$$

Start with definition of $P$:

$$P = \int_{D-G+\xi}^{D+B(\lambda)-G} (v + G - \xi - D)f(\lambda, v) \, dv + B(\lambda)[1 - F(\lambda, D + B(\lambda) - G)] \quad (36)$$

$$\frac{\partial P}{\partial D} = (B(\lambda) - \xi)f(\lambda, D + B(\lambda) - G) + \left( F(\lambda, D - G + \xi) - F(\lambda, D + B(\lambda) - G) \right)$$

$$-B(\lambda)f(\lambda, D + B(\lambda) - G)$$

$$= F(\lambda, D - G + \xi) - F(\lambda, D + B(\lambda) - G) - \xi f(\lambda, D + B(\lambda) - G) \quad (37)$$

$$\frac{\partial P}{\partial B(\lambda)} = (B(\lambda) - \xi)f(\lambda, D + B(\lambda) - G) + 1 - F(\lambda, D + B(\lambda) - G) - B(\lambda)f(\lambda, D + B(\lambda) - G)$$

$$= 1 - F(\lambda, D + B(\lambda) - G) - \xi f(\lambda, D + B(\lambda) - G) \quad (38)$$

Next, look at the definition of $\delta$:

$$\delta = E^\lambda \left( \int_0^{D-G+\xi} (v + G - \xi)f(\lambda, v) \, dv + D\left(1 - F(\lambda, D - G + \xi)\right) \right)$$
\[ \frac{\partial \delta}{\partial D} = E^\lambda \left(Df(\lambda, D - G + \xi) + 1 - F(\lambda, D - G + \xi) - Df(\lambda, D - G + \xi)\right) \]
\[ = E^\lambda \left(1 - F(\lambda, D - G + \xi)\right) \quad (39) \]

Now, we can plug those values in \( \partial B(\lambda)/\partial D \):

\[ \frac{\partial B(\lambda)}{\partial D} = \frac{E^\lambda \left(1 - F(\lambda, D - G + \xi)\right) + F(\lambda, D - G + \xi) - F(\lambda, D + B(\lambda) - G)}{1 - F(\lambda, D + B(\lambda) - G) - \xi f(\lambda, D + B(\lambda) - G) + \xi f(\lambda, D + B(\lambda) - G)} \quad (40) \]

\[ 1 + \frac{\partial B(\lambda)}{\partial D} = \frac{1 - F(\lambda, D - G + \xi) - E^\lambda \left(1 - F(\lambda, D - G + \xi)\right)}{1 - F(\lambda, D + B(\lambda) - G) - \xi f(\lambda, D + B(\lambda) - G)} \quad (41) \]

Numerator of that expression has zero mean (w.r.t. \( \lambda \) distribution). That illustrates the intuition that on average debt issued before news arrival is no cheaper no more expensive than the debt issued after news arrival. The next step would be to look at the weights that are attached to this zero mean term. Were those weights constant, we would get zero in expectation.

**C.2 Covariance**

Recall the expression for derivative of value of the firm with respect to early issue of debt and plug the expression that was found for change in face value of debt issued due to change in early issued debt \((1 + \partial B(\lambda)/\partial D)\):

\[ \frac{\partial V}{\partial D} = -\xi E^\lambda \left(f(\lambda, B(\lambda) + D - G) \frac{1 - F(\lambda, D - G + \xi) - E^\lambda \left(1 - F(\lambda, D - G + \xi)\right)}{1 - F(\lambda, D + B(\lambda) - G) - \xi f(\lambda, D + B(\lambda) - G)} \right) \]
\[ = -\xi E^\lambda \left[HR \left[1 - F(\lambda, D - G + \xi) - E^\lambda \left(1 - F(\lambda, D - G + \xi)\right)\right]\right] \quad (42) \]
\[
\tilde{HR} = \frac{f(\lambda, B(\lambda) + D - G)}{1 - F(\lambda, D + B(\lambda) - G) - \xi f(\lambda, D + B(\lambda) - G)}
\]  

(43)

Since expression on square brackets in 42 is zero mean, we can re-write 42:

\[
\frac{\partial V}{\partial D} = -\xi \text{Cov}^\lambda \left( \tilde{HR}, 1 - F(\lambda, D - G + \xi) - E^\lambda \left( 1 - F(\lambda, D - G + \xi) \right) \right)
\]  

(44)

In order to find the sign of covariance, need to take derivatives of expressions inside w.r.t to \( \lambda \):

\[
\frac{\partial}{\partial \lambda} \left( 1 - F(\lambda, D - G + \xi) - E^\lambda [1 - F(\lambda, D - G + \xi)] \right) = -F_\lambda(\lambda, \cdot) > 0
\]  

(45)

The proof exactly repeats the argument made in the proof of proposition 2.2. The sign of 46 depends on the sign of \( B_\lambda(\lambda) \). Just as was established in Lemma 2.1, \( B_\lambda(\lambda) < 0 \) — the better the news the smaller is the interest expense on the debt. We can show that formally by applying implicit function theorem to 36:

\[
H(\lambda, B) = \int_{D - G + \xi}^{D + B - G} (v + G - \xi - D) f(\lambda, v) \, dv + B[1 - F(\lambda, D + B - G)]
\]  

(47)
\[ \frac{\partial H(\lambda)}{\partial B} = (B - \xi)f(\lambda, D + B - G) + 1 - F(\lambda, D + B - G) - Bf(\lambda, B + D - G) \]
\[ = 1 - F(\lambda, D + B - G) - \xi f(\lambda, B + D - G) > 0 \quad (48) \]

\[ \frac{\partial H(\lambda)}{\partial \lambda} = (B - \xi)F_\lambda(\lambda, B + D - G) - \int_{D-G+\xi}^{D+B-G} F_\lambda(\lambda, v) \, dv - BF_\lambda(\lambda, D + B - G) \]
\[ = - \int_{D-G+\xi}^{D+B-G} F_\lambda(\lambda, v) \, dv - \xi F_\lambda(\lambda, D + B - G) > 0 \quad (49) \]

The sign in (49) is positive due to exactly the same reason as in 20. The optimal policy of the firm is to borrow only if the marginal benefit to debt holders from the last promised dollar of debt is positive. Hence,

\[ B_\lambda(\lambda) = - \frac{\partial H(\lambda)}{\partial B} < 0 \quad (50) \]

That makes covariance in expression 44 negative. And therefore, the derivative of value of the firm with respect to early issue of debt positive.

\[ \square \]

### D  Empirical Evidence

**Table 1:** Proportion of firms with significant cash and debt balance.

<table>
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<tr>
<th>Cash/Assets &gt; 5%, Debt/Assets &gt; 5%</th>
<th>Year</th>
<th>1980</th>
<th>1995</th>
<th>2010</th>
<th>1980-2010</th>
</tr>
</thead>
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<tr>
<td>Cash/Assets &gt; 20%, Debt/Assets &gt; 20%</td>
<td></td>
<td>0.41826</td>
<td>0.39429</td>
<td>0.50617</td>
<td>0.42728</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cash/Assets &gt; 20%, Debt/Assets &gt; 20%</th>
<th>Year</th>
<th>1980</th>
<th>1995</th>
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<th>1980-2010</th>
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<td>0.042877</td>
<td>0.062852</td>
<td>0.081735</td>
<td>0.072958</td>
</tr>
</tbody>
</table>

**References**


Figure 7: Distribution of Cash and Short Term and Long Term debt for US firms in 1980-2010. The mass of firms with co-existing cash and debt is larger for Long Term debt. Hence, Cash is more likely not to be a negative of long term debt.
Figure 8: Joint distribution of Cash and Debt to Asset ratios. As you can see from the top left hand diagram, firms in 1980 were not as levered as in 2010 (lower left had diagram) and fewer proportion of firms had significantly positive balances of cash and debt. The lower right hand diagram incorporates 30 years of data and emphasizes the fact there is non-zero mass of firms that have large cash and debt holdings at the same time.


Francisco Covas and Wouter J. den Haan. The role of debt and equity finance over the business cycle, 2006.


