Time-Inconsistent Preferences and Time-Inconsistent Policies

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First Draft: January 11, 2011

Abstract

In this note we show that a self-financing social security arrangement can help time-inconsistent consumers to achieve their initial consumption plan only if the arrangement itself is time inconsistent. In particular, the government would need to persistently mislead individuals, across the entire life cycle, into believing that future social security transfers are less generous than they truly are. Despite conventional wisdom that social security is a reasonable commitment device for hyperbolic discounters, it is problematic to justify social security arrangements by an appeal to hyperbolic discounting.

Key words: Time-Inconsistent Preferences, Time-Inconsistent Policy, Social Security

1. Introduction

Hyperbolic discounting leads to time-inconsistent behavior. Saving plans made today are abandoned tomorrow. The issue of how to commit hyperbolic discounters to behave more like their original saving plans is a fundamental topic in behavioral economics. Naturally, mandatory saving is conventionally thought to be a reasonable commitment device.

However, in this paper we show that a self-financing social security arrangement can help time-inconsistent consumers to achieve their initial consumption plan only if the arrangement itself is time inconsistent. It takes a time-inconsistent policy to combat time-inconsistent preferences. In particular, the government would need to persistently mislead individuals, across the entire life cycle, into believing that future social security transfers are less
generous than they truly are. Because such government behavior is impractical and unethical, conventional wisdom oversimplifies the role of social security as a commitment device.

In reaching this conclusion, we do not need to make any assumptions about the overall magnitude of discounting, the shape of the disposable income profile, the overall size of the social security arrangement, or the curvature of the period utility function. Thus, our result is quite robust along a variety of important dimensions. Furthermore, throughout the paper we assume individuals are naive and do not anticipate their own time inconsistency. This further strengthens our result because, intuitively, the assumption of naivete seems to bias the results in favor of finding a role for social security.

Our paper strengthens the result in Section 3 of İmrohoroğlu, İmrohoroğlu, and Joines (2003). They show that when social security has a net present value of zero and borrowing is allowed, social security has no effect on the consumption profile of a naive hyperbolic consumer. Caliendo (2011) generalizes their result to continuous time, to any discount function, to a variety of utility functions, and to other tax and transfer schemes beyond social security. The key technical difference between these two papers on the one hand and our paper on the other hand is that we expand the policy space to allow for time-inconsistent transfer policies. Taken together, these three papers tell a consistent story: despite conventional wisdom, it is problematic to justify social security arrangements by an appeal to hyperbolic discounting.

2. Model

To focus on the connection between time-inconsistent preferences and time-inconsistent policies, we abstract to the simplest possible model to make the results as sharp as possible.

Time is continuous and is indexed by $t$. The individual enters the workforce at $t = 0$ and passes away at $t = T$. Income $y(t)$ can be consumed $c(t)$ or saved in a zero-interest account $k(t)$. The individual starts and stops the life cycle with no savings $k(0) = k(T) = 0$. Period utility is of the CRRA variety $u(c) = c^{1-\sigma}/(1-\sigma)$ and is discounted according to a discount function $F(x)$, where $x$ is the length of the delay and $F(0) = 1$. It will be important to distinguish between planned and actual quantities of the variables $c(t)$ and $k(t)$. Actual quantities will be indicated with an asterisk.
Consider an individual standing at some age $t_0 \in [0, T]$. At this age, the government promises a transfer profile $T(t_0) \equiv \int_{t_0}^T \tau(t, t_0) dt$. Notice $\tau$ depends on the future age of the individual $t$ as well as the current age $t_0$, which expands the policy space to allow for time-inconsistent transfer policies.

The transfer that is actually paid (time-consistent social security arrangement) at age $t$ is $\tau^x(t)$, and the remaining transfer profile from the perspective of age $t$ that is actually paid is $T^x(t) \equiv \int_t^T \tau^x(z) dz$. If $\tau(t, t_0) = \tau^x(t)$ for all $t$ and for all $t_0$ then the promised transfer policy is time consistent. Otherwise the promised policy is time inconsistent. We expand the typical policy space to allow for both.

Proposition 1. For the case of hyperbolic discounting ($F(x) = (1 + \beta x)^{-1}$ with $\beta > 0$), the government can implement the individual’s initial consumption plan through a self-financing social security arrangement if and only if that arrangement is itself time inconsistent. In particular, the government would need to make individuals of all ages believe that future social security transfers are less generous than they truly are. (This result holds for any assumptions about the magnitude of $\beta$, the shape of $y(t)$, the overall size of the social security arrangement, and the magnitude of $\sigma$).

Proof. Standing at some age $t_0 \in [0, T]$, the individual solves a fixed endpoint control problem

$$\max : \int_{t_0}^T F(t - t_0) \frac{c(t)^{1-\sigma}}{1-\sigma} dt,$$

subject to

$$\frac{dk(t)}{dt} = y(t) + \tau(t, t_0) - c(t),$$

$$k(t_0) = k^*(t_0) = \int_{t_0}^t [y(t) + \tau^x(t) - c^*(t)] dt \text{ given, and } k(T) = 0.$$ 

The costate variable $\lambda$ is constant in this control problem and the Maximum Principle gives the consumption path the individual intends to follow from the perspective of $t_0$

$$c(t) = \left[ \frac{\lambda}{F(t - t_0)} \right]^{-1/\sigma}.$$
Using (2) and (3) we have

\[ 0 = k^*(t_0) + \mathcal{T}(t_0) + \int_{t_0}^T [y(t) - c(t)] \, dt. \] (5)

Insert (4) into (5) to pin down \( \lambda \)

\[ \lambda^{-1/\sigma} = \frac{k^*(t_0) + \mathcal{T}(t_0) + \int_{t_0}^T y(t) \, dt}{\int_{t_0}^T F(t - t_0)^{1/\sigma} \, dt}, \] (6)

which implies

\[ c(t) = \frac{k^*(t_0) + \mathcal{T}(t_0) + \int_{t_0}^T y(t) \, dt}{\int_{t_0}^T F(t - t_0)^{1/\sigma} \, dt} F(t - t_0)^{1/\sigma}, \text{ for } t \in [t_0, T]. \] (7)

Equation (7) is the path the individual intends to follow, from the perspective of age \( t_0 \). However, due to the time-inconsistent nature of his preferences (as long as \( F \) is other than exponential), the individual will deviate from (7) as time advances. Therefore, planned consumption (7) equals actual consumption only at the planning instant \( t_0 \). Thus, actual consumption is the envelope of infinitely many initial values from a continuum of planned timepaths. Actual consumption at time \( t_0 \) is found by replacing the \( t \) in (7) with \( t \). But then because the model is cast in continuous time and every instant is a planning instant, we can switch all \( t_0 \) back to \( t \) to obtain the actual consumption profile, which is a function of actual capital at time \( t \) and the promised transfer profile \( \mathcal{T}(t) \)

\[ c^*(t) = \frac{k^*(t) + \mathcal{T}(t) + \int_{t}^T y(z) \, dz}{\int_{t}^T F(z - t)^{1/\sigma} \, dz}. \] (8)

From the perspective of time 0, the initial consumption plan in a world without transfers is found by evaluating (7) at \( t_0 = 0 \) with \( \mathcal{T}(0) = 0 \)

\[ c(t) = \int_0^T y(t) \, dt \frac{F(t)^{1/\sigma}}{\int_0^T F(t)^{1/\sigma} \, dt}. \] (9)

We treat (9) as the target for which a commitment device would aim to replicate. The policymaker wants to design a transfer policy that causes
actual consumption to match the initial plan, so we set \( c(t) \) from (9) equal to \( c^*(t) \) from (8)

\[
\int_0^T y(t) dt \frac{F(t)^{1/\sigma}}{\int_0^T F(t)^{1/\sigma} dt} = k^*(t) + \int_t^T y(z) dz + T(t) \frac{F(t)^{1/\sigma}}{\int_0^T F(z-t)^{1/\sigma} dz},
\]

(10)

and solve for \( T(t) \)

\[
T(t) = \int_0^T y(t) dt \frac{\int_t^T F(z-t)^{1/\sigma} dz}{\int_0^T F(t)^{1/\sigma} dt} F(t)^{1/\sigma} - k^*(t) - \int_t^T y(z) dz.
\]

(11)

If (11) holds for all \( t \), then actual consumption will always coincide with the first plan.

Note that \( T(t) \) in (11) is a function of actual savings \( k^*(t) \), which is given by

\[
k^*(t) = \int_0^t y(z) dz - \int_0^t c^*(z) dz + \int_0^t \tau^{ss}(z) dz.
\]

(12)

And, if \( T \) is indeed chosen according to (11), then actual consumption always matches the first plan, and therefore we can rewrite (12) as

\[
k^*(t) = \int_0^t y(z) dz + \int_0^t \tau^{ss}(z) dz - \frac{\int_0^T y(t) dt}{\int_0^T F(t)^{1/\sigma} dt} \int_0^T F(z)^{1/\sigma} dz.
\]

(13)

Insert (13) into (11) to get the solution transfer profile (commitment device) in closed form

\[
T(t) = Y G(t) - \int_0^t \tau^{ss}(z) dz,
\]

(14)

\[
Y \equiv \int_0^T y(t) dt, \quad G(t) \equiv \frac{\int_t^T F(z-t)^{1/\sigma} dz F(t)^{1/\sigma} - \int_t^T F(z)^{1/\sigma} dz}{\int_0^T F(t)^{1/\sigma} dt}.
\]

(15)

Hence, if individuals are naive enough to consistently believe the government’s promises, then the government can restore the initial consumption profile by sequentially promising a continuum of transfer profiles \( T(t) \) given by (14) and (15) for all \( t \).

However, recall that what the government actually operates is a time-consistent social security arrangement \( T^{ss}(t) = \int_t^T \tau^{ss}(z) dz \). Define \( D(t) \) as the difference between the transfer necessary for actual consumption to
mimic the initial plan \((T(t) \text{ from } (14))\) and the actual transfer from the social security arrangement \((T^{ss}(t))\),

\[
D(t) \equiv T(t) - T^{ss}(t) = YG(t) - \int_0^t \tau^{ss}(z) dz - \int_t^T \tau^{ss}(z) dz.
\] (16)

But assuming social security is self-financing, that is \(T^{ss}(0) = \int_0^T \tau^{ss}(z) dz = 0\), then

\[
D(t) = YG(t).
\] (17)

For the case of the hyperbolic discount function \(F(x) = (1 + \beta x)^{-1}\),

\[
[F(z - t)F(t)]^{1/\sigma} - F(z)^{1/\sigma} = \left[ \frac{1}{1 + \beta z + \beta^2 (zt - t^2)} \right]^{1/\sigma} - \left[ \frac{1}{1 + \beta z} \right]^{1/\sigma},
\] (18)

and noting that \(z \geq t\), it must be the case that for \(t > 0\)

\[
[F(z - t)F(t)]^{1/\sigma} - F(z)^{1/\sigma} \begin{cases} < 0 & \text{for } z > t \\ = 0 & \text{for } z = t \end{cases}.
\] (19)

From (19), we have \(G(t) < 0\) for \(t \in (0, \bar{T})\) and hence \(D(t) < 0\) for \(t \in (0, \bar{T})\). (At the boundaries, \(D(0) = D(\bar{T}) = 0\) because \(G(0) = G(\bar{T}) = 0\).) Thus, from (16)-(19) we have shown the government can implement the initial consumption plan if and only if it pursues a time-inconsistent policy in which it makes individuals believe future social security transfers are consistently less generous than they truly are. We emphasize that in reaching this conclusion, we did not need to make any assumptions about the magnitude of \(\beta\), the shape of \(y(t)\), the overall size of social security, or the magnitude of \(\sigma\). Q.E.D.

3. Conclusion

The conventional wisdom is that mandatory saving (social security) is a reasonable commitment device for hyperbolic discounters. However, in this paper we show that a self-financing social security arrangement can help time-inconsistent consumers to achieve their initial consumption plan only if the arrangement itself is time inconsistent. In deriving this result we expand the typical policy space to allow for time-inconsistent transfer policies. In particular, we find that the government would need to persistently mislead
individuals, across the entire life cycle, into believing that future social security transfers are less generous than they truly are. This result is robust along a variety of important dimensions. Thus, in theory, conventional wisdom is valid in the sense that it is possible to engineer a transfer policy that acts as a commitment device, but this transfer policy is totally impractical. Our result reinforces İmrohoroğlu, İmrohoroğlu, and Joines (2003) and Caliendo (2011): a typical time-consistent social security arrangement is not a commitment device for hyperbolic discounters.

4. References
