A Dynamic Model of Search and Intermediation

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Abstract
This paper develops a dynamic model of an economy with search frictions in which homogeneous agents choose between specializing as producers or as merchants, and can change occupation at any time. Merchants operate alongside a decentralized search market and provide immediacy in trade in return for a price. Agents who discover the location of a merchant have the option of returning to the merchant in future periods, paying the merchant’s price and avoiding search. We characterize equilibria in symmetric strategies with one-period memory, and derive conditions under which merchants and their clients form a repeated relationship. We also investigate the prospect of an endogenous emergence of an institution of intermediation when the status quo ante is an economy with no merchants.

Keywords: Search, endogenous intermediation, repeated interaction.

JEL Classification: D02, D51, D83.
1. Introduction

Merchants and traders—agents who mediate the transfer of goods and services between producers and consumers—are central to the economic process. Specialization, the source of the wealth of nations, cannot flourish unless some agents mediate trade. Thus Hicks, who considered the merchant trader the “principal character” in economic history, wrote that “it is specialization upon trade that is the beginning of the new world” (Hicks, 1969, p.25). In an economy with any degree of specialization, agents must trade to acquire goods that they wish to consume in exchange for goods that they produce. Searching for trading partners can be costly and time-consuming. Some agents recognize that there is profit in facilitating trade and specialize as intermediaries or merchants. They reduce search costs and provide immediacy in exchange. Further, merchants derive profits from repeat business with returning clients rather than from random encounters with one-time clients.

The present paper focuses on these characteristics of the merchant. Our objective is a parsimonious but self-contained model of the merchant trader that reflects some essential features of an institution of intermediation in its “early and rude state”. We are especially interested in the prospect of an endogenous emergence of specialist merchant traders. We establish equilibria in which merchants price their services to encourage repeat business, and their clients prefer to return to them in future periods rather than search for other trading partners. We show that such intermediation is profitable for some ranges of values of critical technological and institutional parameters; in other ranges the economy may harbor brigands but not bona fide merchants. We fully characterize two classes of intermediation equilibria that may exist, as well as a class of equilibria with brigandry. In a closed-form example with reasonable search technologies, we show the existence of a unique equilibrium of each class.

The model is a variant of Diamond’s (1982) tropical island economy. In our version homogeneous agents choose to specialize as producers or merchants, and can change occupation at any time. In each period a producer must exchange his output before he can consume. Exchanges are made with other producers or with merchants—the producer finds these partners through search. A merchant sets up a trading post; producers arriving at her post can trade by paying a commission that the merchant sets each period. Locating a merchant through search may not be a priori easier than locating another producer. However, a merchant operates at a fixed location, whereas the pursuit of production takes producers to random locations. A producer who succeeds in finding a merchant may thus return to her in the next period, avoiding search. The viability of specialized intermediation is predicated on the ability of intermediaries to form such ongoing
relationships with their clients.

Producers sometimes forget the locations of their merchants, so an unmediated search market remains active in parallel. A producer may therefore credibly decline to return to his merchant and choose instead to search anew for a trading partner. The continued existence of a search market also affords incipient merchants a pool from which they can draw clients. The process by which a new merchant acquires clients is a part of the dynamics of the model.

An equilibrium determines the occupational choice of each agent, and the commissions charged by merchants. We find that there are three classes of equilibria in symmetric time-invariant strategies with one-period memory. In bandit equilibria, merchants act as bandits and claim the entire output of their clients as “commission”. In this case, producers understandably never return to these merchants, but search for trading partners in each period. This of course bears no resemblance to an institution of intermediation.\(^1\)

Of greater interest to us are intermediation equilibria in which merchants charge a commission that induces existing clients to return in succeeding periods. Intermediation equilibria exist only if producers remember the location of their merchants with sufficiently high probability. There are two distinct classes of intermediation equilibria. In one class, the price charged by merchants is the highest that is compatible with repeated interaction: were the price any higher, clients of merchants would be better off searching for trading partners each period. In the other class, the price is the lowest that is compatible with repeated interaction: were the price any lower, merchants would be better off acting as bandits instead of intermediaries that encourage clients to return.

An agent who specializes as a merchant facilitates exchange, but does not produce output. Moreover, as the measure of merchants increases, the unmediated search market gets thinner. The optimal measure of merchants in the economy must balance these effects. We find that, in general, an equilibrium is not optimal. We give conditions under which an intermediation equilibrium improves welfare compared to an economy with no merchants.

Finally, we discuss conditions under which an institution of intermediation can be expected to arise endogenously when the \textit{status quo ante} is an economy with no merchants. We provide a heuristic interpretation of the parameters in our model in terms of technological and socio-political conditions that determine when intermediation rises and flourishes and when it declines under threat of brigandry and disorder.

The investigation of the role of intermediaries in speeding up search was

\(^1\)It is perhaps no accident that, in many historical contexts, merchants and brigands possessed similar enforcement capabilities. Even today unscrupulous merchants may, if they choose, defraud an unsuspecting client once with ease.
initiated by Rubinstein and Wolinsky (1987). In their model, buyers, sellers, and intermediaries are randomly matched: trading with an intermediary is not a choice. In equilibrium, intermediaries are active if buyers and sellers encounter an intermediary at least as often as they encounter each other.

Gehrig (1993) presents a static model in which buyers and sellers, who differ in valuations and costs that are private information, can choose to search for trading partners and negotiate price, or access intermediaries whose locations and prices are publicly observable. Yavas (1994) allows heterogeneous agents to choose the intensity of private search, or to opt for the service of an intermediary. Spulber (1996) presents a dynamic model in which buyers, sellers, and intermediaries are heterogeneous in several respects, and intermediaries and their prices have to be found through search; there is no parallel unmediated search market in which buyers and sellers can trade directly. The focus is on deriving the equilibrium bid-ask spread and comparing the outcome with Walrasian prices. Rust and Hall (2003) extend the model of Spulber (1996) by adding a second type of intermediaries who post publicly observable prices. Howitt (2005) examines the role of fiat money in a search market where merchants organize exchange.

The papers cited above can accommodate rich heterogeneity among agents, but intermediaries are exogenously present in the economy and do not choose their calling. In contrast, a key concern of the present paper is to generate an endogenous distribution of occupational assignments in equilibrium starting from a homogeneous population.

We are aware of only a few papers that explicitly model endogenous occupational choice between production and intermediation. Li (1998) does so in the context of a friction quite different from ours: the function of intermediaries is to assess the quality of goods that are traded. In Bhattacharya and Hagerty (1987), producers may trade only with intermediaries; thus, the viability of intermediation is never in question. In Hellwig (2002) and Shevchenko (2004), the role of intermediaries is to resolve the problem of double coincidence of wants. Intermediaries achieve this by complementing money in Hellwig’s model, and by stocking a variety of goods in Shevchenko’s model. In both these models, there is also an unmediated search market, as in ours. Prices set by intermediaries in Hellwig’s model are publicly observable and the terms of trade with an intermediary are determined by Nash bargaining in Shevchenko’s model.

Our paper is closely related to Masters (2007). Masters also investigates the endogenous emergence of intermediaries in the context of Diamond’s tropical island economy. In his model, intermediaries enter the market with a unit of a good they have acquired after exchange. This gives them an advantage in Nash bargaining with producers because the intermediary has

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2 Various other functions of intermediaries have been investigated in the literature: Spulber (1999) has an extensive survey.
the option of consuming the good she holds whereas the producer does not. He finds that, when all producers have identical production costs, intermediaries uniformly reduce welfare in the economy; however, when production costs are \textit{ex ante} unequal intermediation can increase welfare. In contrast, the advantage of intermediaries in our model comes from the potential of repeated trade which Masters does not allow. As a consequence, in our model, intermediation can improve welfare even though all producers face identical costs (which we normalize to zero).

Intermediaries do not establish durable links with their clients in any of the papers above. In our paper, the benefit of establishing a trading link with an intermediary is that search costs can be avoided in \textit{future} periods. At any intermediation equilibrium, clients and their merchants form ongoing relationships.

Durable client relations is accommodated in some papers that investigate price-setting by sellers in a market where consumers search for prices. The pricing component of the model here is similar to Benabou (1997), but simpler as our agents are homogeneous while Benabou allows heterogeneous agents. Burdett and Coles (1997) presents a model of noisy search in which a searching producer can observe more than one price with positive probability.

The next section sets out the model. Equilibrium is derived in Sections 3 and 4 for general matching functions. We then adopt a specific form for the matching functions for the remainder of the paper. In Section 5 we derive closed-form characterizations of equilibria of each class and show these to be unique. Welfare is analyzed in Section 6. Section 7 discusses the possibility of an endogenous rise of merchants \textit{ab initio}. Section 8 concludes with comments. Some of the proofs have been placed in the Appendix.

2. Model

2.1. Context

The setting is a highly stylized model of production, search, and exchange, adapted from Diamond (1982). The economy operates over an infinite succession of discrete periods. Agents are homogeneous and risk-neutral; they live for ever; the set of agents is a continuum of unit measure.

Each agent has access to a technology for production that generates one unit of a homogeneous, divisible good at no cost in each period. A taboo against consuming the output of one’s own production ensures that exchange must precede consumption. This artifice allows, within a one-commodity framework, a representation of the reality that agents in an economy consume very little of their own output and the need for specialization and
trade is paramount.\(^3\)

We imagine that the pursuit of production takes producers to random locations so that the search for trading partners has to be undertaken anew after each episode of production. Thus, every period each agent sets out to trade his output with any other agent he may encounter. The probability that a given agent will meet a trading partner during the period is \(\lambda \in (0, 1)\). We interpret \(\lambda\) as a measure of the efficacy of unmediated search.\(^4\) Once two agents meet, the units are traded one for one,\(^5\) consumption takes place, and the agents are free to return to production, which will generate another unit of the good next period. Throughout the paper we assume that untraded good cannot be carried as inventory from one period to another.\(^6\) Thus, an agent unsuccessful in effecting exchange foregoes consumption and returns in the next period with a newly produced unit.

We normalize payoffs so that the utility of consuming \(x\) units in a period is \(x\). Letting \(\delta \in (0, 1)\) represent the common discount factor of the agents, the present value of expected payoff of any agent is given by

\[
v = \frac{\lambda}{1 - \delta}.
\]

Is there scope in such a setting for some agents to set up as specialist intermediaries and offer the service of immediate trade in return for a price?

2.2. Producers and Merchants

We next allow each agent, in every period \(t\), the choice of specializing either as a producer, or as a merchant. The choice of an occupation is endogenous but the technology of production and the technology of trade constrain what are feasible for a specialist in each occupation. There are three main differences. A specialist producer can access the production technology described above to produce output, but cannot commit to be available for trade at a fixed location,\(^7\) and cannot carry an inventory of untraded good from one period to the next. In contrast, a specialist merchant cannot produce output, but can commit to be available for trade at a fixed location,\(^3\) Although the model is in effect one of pure exchange, it will be convenient for terminological clarity to interpret it as a special model with production.

\(^4\)More generally, \(1 - \lambda\) may be interpreted as a measure of any transaction cost associated with coordinating trades without an organized market. In the language of search models the transaction cost is a lost trading opportunity.

\(^5\)As agents are symmetric, any reasonable bargaining solution would prescribe an equal share of the gains from trade.

\(^6\)Alternatively, we could assume that production cannot be carried out with a unit of the good already in hand. What we need is that a producer is not in possession of more than one unit at any given time.

\(^7\)Recall that the pursuit of production takes producers to random locations.
and carry an inventory from one period to the next. We elaborate on these below.

We suppose that a specialist merchant operates a trading post at a fixed location where producers can exchange their output. For this service, a merchant charges a price that she sets each period. (Since units are traded one for one in the unmediated search market, the merchant’s price is also her bid-ask spread or commission.)

A merchant’s location or price is not public information and must be initially found by a producer during search. Since a merchant has a fixed location, once a producer makes contact with a merchant, it opens for him the prospect of a long-term relationship for future trade without further search.

If a producer trades with a merchant in period \( t - 1 \), then he learns the merchant’s location. We suppose that he remembers this information at \( t \) with probability \( \gamma \in (0, 1) \); with the complementary probability \( 1 - \gamma \), he forgets this information before \( t \). An agent is informed in period \( t \) if he knows the location of a merchant’s trading post at the beginning of \( t \), and uninformed if he does not. We suppose that an informed producer can remember the location of at most one merchant at any given time—it is the location of the last merchant he traded with. Thus a producer who is informed at \( t \) of the location of a merchant \( \mu \) remains informed at \( t + 1 \) with probability \( \gamma \) regardless of whether he returns to merchant \( \mu \) at \( t \) or he searches at \( t \), as long as the search does not lead him to find a different merchant.

The non-persistence of memory embodied in \( \gamma \in (0, 1) \) reflects the unavoidable frictions in continuing business relations, perhaps more pervasive in a nascent market than in a mature one. For example, the pursuit of production may take a producer too far from the location of his merchant. In the model the assumption ensures that an unmediated search market remains viable.

An uninformed producer must search for a trading partner. The search may yield one of three outcomes in a given period: either he does not meet a trading partner, or he meets another producer who is also searching, or he comes upon a merchant’s trading post. If he fails to find a partner, he cannot trade. If he finds another producer, units are exchanged one for one. If he comes upon a trading post, he concludes trade at the merchant’s set price, and learns the merchant’s location.

An informed producer has two options: he may proceed directly to the trading post of the last merchant he traded with, pay the merchant’s price, and immediately conclude trade; alternatively, he may search anew for a trading partner. Since agents can switch occupation between periods, it is possible that an informed producer may return to a trading post to find that her merchant is absent because she has switched occupation in the
intervening time. Such a producer proceeds to the unmediated search market as well. We assume that informed producers on their way to a merchant are unavailable to other producers searching for a partner, and informed producers who have come to a merchant’s trading post cannot trade amongst each other without paying the merchant her commission.

A producer can meet at most one trading partner in a period. Thus, if he meets a merchant, he may as well trade—regardless of how adverse the price set by the merchant is—since his current unit of the good will become obsolete after the present period. However, a producer will not return to a merchant who offers an unacceptable price even if he remembers her location in the following period. Since a producer can meet at most one trading partner, an informed producer can search after visiting a merchant’s trading post if and only if the merchant is unavailable because she has switched occupation since their last transaction.

Let \( m_t \) denote the measure of agents who specialize as merchants in period \( t \) and \( s_t \) denote the measure of producers in the search market at \( t \). For a producer who searches for a trading partner, the probability of success is governed by two matching functions, \( \lambda^m \) and \( \lambda^s \): \( \lambda^m(m_t, s_t) \) is the probability that he finds a merchant and \( \lambda^s(m_t, s_t) \) the probability that he meets another producer who is also searching in period \( t \). Thus, the probability that he is able to trade within the period is given by \( \lambda^m(m_t, s_t) + \lambda^s(m_t, s_t) \). Throughout the paper we assume that \( \lambda^m \) (respectively, \( \lambda^s \)) is (weakly) increasing in \( m \) (respectively, \( s \)), \( \lambda^m \) and \( \lambda^s \) are continuous and satisfy

\[
\lambda^m(m, s) + \lambda^s(m, s) \in (0, 1),
\lambda^m(0, 1) = 0, \quad \lambda^m(m, s) > 0 \quad \text{for } m > 0,
\lambda^s(0, 1) = \lambda, \quad \lambda^s(m, s) > 0 \quad \text{for } s > 0.
\]

Observe that when \( m = 0 \) all agents are producers who search, i.e., \( s = 1 \), which is the primitive economy described in Section 2.1. In what follows, we often suppress the arguments of the functions \( \lambda^m \) and \( \lambda^s \) unless we are evaluating them at a particular point: no confusion should arise.

A merchant can serve multiple clients at her trading post within a period. The clients of a given merchant in a given period \( t \) come from two sources: informed clients who choose to return, and new clients—producers who discover her trading post during period \( t \) in the course of search. (Merchants do not meet other merchants; such meetings are inconsequential in this model.)

A merchant must start a period with sufficient inventory to conduct the first trade. She funds each subsequent trade out of the proceeds of the previous one. A merchant who plans to set a price \( p_t \) in period \( t \) must therefore carry from period \( t - 1 \) an inventory of \( 1 - p_t \) unit to offer her first client at \( t \) in return for the client’s single unit.
Since untraded output cannot be carried in inventory, only producers who have effected an exchange in the previous period can exercise the option of becoming merchants. A continuing merchant can carry inventory as needed. Thus pricing and occupational choice decisions must be made one period in advance. An agent who was a producer in \( t - 1 \) and wishes to switch occupation and become a merchant at \( t \) must carry over the necessary inventory by foregoing \( 1 - p_t \) units of consumption. Correspondingly, a merchant who decides to switch to production in period \( t \) can enjoy an extra \( 1 - p_{t-1} \) units of consumption in \( t - 1 \).

In an alternative formulation, the merchant could be modelled as a market-maker who organizes exchange between producers. Producers who show up at her trading post pay a commission to the merchant to trade with each other. A merchant then would not need to carry inventory between periods; nor would a new merchant need capital to start up business. Then all decisions—in particular, occupational choice and pricing—pertaining to period \( t \) could be made at the start of that period. The two formulations lead to almost identical results with only small differences in the explicit expressions for equilibrium values of some variables.

Agents can switch occupation between periods. At the end of each period, a producer may decide to switch occupation and become a merchant in the next period, and correspondingly, a merchant may decide to become a producer. Implementing a change in occupation, however, may be temporarily constrained by cost and feasibility considerations. We model this as a friction in the implementation of the decision: for any agent, an attempt to switch occupations is successful with an idiosyncratic probability \( \alpha \in [0, 1) \). With probability \( 1 - \alpha \in (0, 1] \), the agent finds that the decision to change occupation cannot be implemented in that particular period. In this model, this friction rules out a class of somewhat implausible equilibria that are separately characterized in Observation 4.1.

2.3. The Game and the Solution Concept

The interaction among the agents in this economy over time is modelled as a stochastic game. At the beginning of period \( t \), each agent knows his information state—whether he is informed, or uninformed—and his occupation—a producer or a merchant. The sequence of events unfolds as follows.

At the beginning of period \( t \), each merchant \( \mu \) posts a price \( p_{t}^{\mu} \in [0, 1] \) which she has decided upon at \( t - 1 \). Producers will learn this price only when (and if) they arrive at the merchant’s post. Each informed producer (having produced output) decides whether to return to the last merchant that he traded with or to undertake search for a trading partner. An uninformed producer has no option but to search. Informed producers whose erstwhile
merchants have changed occupation since their last trade also search. The outcomes of the search processes are realized; and trade takes place. An informed producer forgets his merchant’s location with probability $1 - \gamma$; those who do not will be the only informed agents in period $t + 1$. Next, agents choose their occupations for period $t + 1$. Decisions to change occupation are implemented subject to the friction described earlier (that the occupation switch may not be implementable with probability $1 - \alpha > 0$). Agents who will be merchants in the following period $t + 1$ choose the prices they will charge in $t + 1$. Finally, consumption takes place.

The history observed by an agent in period $t$ consists of (a) the prices she set and the size of the clientele she served in every period up to $t - 1$ that she operated as a merchant, (b) the prices he paid in periods up to $t - 1$ that he was a producer and traded with a merchant, and (c) the outcome in every period up to $t - 1$ that he searched for a trading partner.\footnote{In particular, a merchant does not observe the identity of an individual customer and thus cannot give discounts to returning customers. Merchants do not observe the history of prices set by other merchants. Producers do not observe the client size of any merchant.}

An agent’s strategy is a sequence of functions (indexed by time period) that prescribes, for every period $t$, an action commensurate with the agent’s information state as a function of the history observed by the agent at $t$ and the agent’s information state at the beginning of $t$. An agent’s strategy is called time-invariant with one-period memory if the sequence of functions is time-invariant, and if the action it prescribes in period $t$ is determined entirely by the outcomes observed by the agent at $t - 1$ and by the agent’s information state at the beginning of $t$. In particular, for a merchant $\mu$ following a time-invariant strategy with one-period memory, the choice of price at $t$, $p^\mu_t$, can depend only on the size of her clientele $k^\mu_{t-1}$ and her price $p^\mu_{t-1}$ at $t - 1$: we refer to this as the merchant’s pricing rule. Similarly, for an informed producer following a time-invariant strategy with one-period memory, the decision of whether to return to his merchant at $t$ can depend only on the price the merchant had charged at $t - 1$: we refer to this as the informed producer’s return rule.

We call a profile of time-invariant strategies symmetric if the functions prescribing the pricing rule is the same for every merchant and the functions prescribing the return rule are the same for every informed producer. The occupational choice decisions may vary across agents.

For a producer, the period-$t$ (Bernoulli) payoff is 1 if at $t$ he trades with another producer, $1 - p_t$ if he trades with a merchant who charges a price $p_t$, and 0 if he fails to execute a trade at $t$. The period-$t$ payoff of a merchant $\mu$ who sets a price $p^\mu_t$ and serves $k^\mu_t$ clients is $p^\mu_t k^\mu_t$.

**Definition 2.1.** An equilibrium is a profile of symmetric time-invariant strategies with one-period memory such that, given the strategy choices of other agents,
• the occupational choice of each agent in every period $t$ is optimal;
• the price set by each merchant in every period $t$ maximizes that merchant’s expected continuation payoff at $t$;
• for each informed producer, the return decision in every period $t$ maximizes his expected continuation payoff at $t$;
• $m_t = m$, $s_t = s$ for every period $t$.

The focus of the paper is the class of equilibria in which each merchant and her (informed) clients form a repeated relationship: we call these intermediation equilibria.

**Definition 2.2.** An *intermediation equilibrium* with $m^* \in (0, 1)$ merchants is an equilibrium such that an informed producer has no incentive to search for a trading partner: his optimal choice in every period $t$ is to return to the last merchant he had dealt with.

We have incorporated in the definition of an equilibrium the properties of symmetry, time-invariance and one-period memory of the strategy profile as well as the steady-state restriction that the measure of merchants and searchers remain constant over time. This is done to avoid adding these qualifiers repeatedly when discussing an equilibrium in this paper. We emphasize that do not *a priori* restrict an agent’s set of strategies to be symmetric or time-invariant or to have only one-period memory. However, we are able to characterize only the set of equilibria in which agents’ strategy choices are symmetric time-invariant and have one-period memory; we have not attempted a characterization of any equilibria involving either more general strategies or varying measures of merchants and searchers.

Note also that the class of symmetric Markov equilibria—in which a strategy profile is time-invariant and the actions of players in any period can depend only on the payoff-relevant part of the state observed in the preceding period—is a subset of the set of equilibria, as we have defined. Therefore, our analysis of equilibria subsumes an analysis of the class of symmetric Markov equilibria.

### 2.4. Preliminary Observations

**Observation 2.1.** It is important to note that the size of the clientele that a given merchant $\mu$ serves in period $t$, $k^\mu_t$, is stochastic. It is possible that most—or even all—of the clients served by a particular merchant at $t - 1$ forget her location at $t$. The realization of $k^\mu_t$ can vary across merchants at $t$, and over time for the same merchant $\mu$.

**Observation 2.2.** At an intermediation equilibrium, by definition, only uninformed producers search. Thus, at an intermediation equilibrium with $m \in (0, 1)$ merchants, the expected size of clientele for a given merchant $\mu$,
who had set a price $p_{t-1}^\mu$ and served $k_{t-1}^\mu$ clients at $t-1$, evolves according to the equation

$$E(k_t^\mu | k_{t-1}^\mu, p_{t-1}^\mu, m) = \gamma k_{t-1}^\mu + \frac{\lambda^m s}{m},$$

(2)

where $E$ is the expectation operator. A fraction $\gamma$ of a merchant’s clients from period $t-1$ retain the knowledge of her location. At an intermediation equilibrium, these informed clients return to deal with her. Moreover, each of the $s$ producers in the search market discovers a merchant with probability $\lambda^m$. Since there are $m$ merchants, the expected size of searching producers that arrive at the trading post of a given merchant in any period $t$ is $\lambda^m s/m$. This yields equation (2).

The above equation also shows how an incipient merchant—an erstwhile producer who sets up as a merchant in period $t$—can start from a base of $k_{t-1}^\mu = 0$ and acquire clients over time.

**Observation 2.3.** At an intermediation equilibrium with $m$ merchants, the expected continuation payoff at $t$ of a merchant $\mu$ who sets a constant price $p$ each period and who had $k_{t-1}^\mu$ clients at $t-1$ can be written as

$$V_t^\mu(k_{t-1}, p, m) = \sum_{\tau=t}^{\infty} \delta^{\tau-1} p \gamma^\tau k_{t-1} + V^\mu(0, p, m)$$

(3)

$$= p \gamma k_{t-1} \left( 1 - \gamma \delta \right) + V^\mu(0, p, m).$$

(4)

The first term in (3) reflects the discounted stream of profits from the pool of returning clients from $t-1$: of the $k_{t-1}$ clients, a fraction $\gamma$ returns at $t$, of whom a fraction $\gamma$ return at $t+1$ and so on. The second term reflects the (time-invariant) continuation value from clients who come upon merchant $\mu$ at $t$ for the first time: as argued in Observation 2.2, their measure is $\lambda^m(m, s)s/m$.

By a similar argument, the expected continuation payoff, $V^\mu(0, p, m)$, can be written as

$$V^\mu(0, p, m) = \frac{p \lambda^m s}{(1 - \gamma \delta)m} + \delta V^\mu(0, p, m),$$

(5)

so that we have

$$V^\mu(0, p, m) = \frac{p \lambda^m s}{(1 - \delta)(1 - \gamma \delta)m}.$$  

(6)

**Observation 2.4.** At an intermediation equilibrium with $m$ merchants, the expected measure of searching producers is given by

$$s = (1 - \gamma)(1 - m - s) + (1 - \gamma \lambda^m)s.$$  

(7)
In each period, a fraction $(1 - \gamma)$ of the $(1 - m - s)$ informed producers forget the location of their merchants and return as searchers. Of the $s$ searching producers, a fraction $\lambda^m$ discover a merchant’s trading post; of those, a fraction $\gamma$ return as informed; all others return as searchers. This gives (7), which simplifies to

$$s = \frac{(1 - \gamma)(1 - m)}{1 - \gamma(1 - \lambda^m)}.$$  

The next two sections develop the analysis of the class of equilibria in which the occupational choices of agents result in a measure $m \in (0, 1)$ of merchants.

3. Pricing

In this section, we take a fixed stationary occupational assignment with $m \in (0, 1)$ merchants as given, and characterize the sequence of prices that the merchants set and the return decision rules that the informed producers follow in any equilibrium. Section 4 analyzes the occupational choices in equilibrium.

For an informed producer, a symmetric return rule in period $t$ with one-period memory can be represented by a set $P_t^R \subset [0,1]$: by definition, an informed producer returns to his merchant in period $t$ if and only if the price charged by that merchant in their last transaction was in $P_t^R$; otherwise, the producer searches for a trading partner at $t$. If the return rule is also time-invariant, then $P_t^R = P^R$ for all $t$.

Our primary interest is in intermediation equilibria with an ongoing relationship between a merchant and her informed clients. First, we dispense with a case in which all merchants always charge a price of unity and producers never return. This is really a description of a search equilibrium with bandits, not of an institution of intermediation. These bandits live off appropriating the entire endowments of any searching producers who unluckily encounter them; the producers obviously do not return to trade with them.

**Proposition 3.1 (Bandit Pricing).** Each merchant always setting a price of 1 and each producer always choosing to search, even when informed, constitute mutual best responses for any given measure of merchants.

**Proof.** Since clients do not return, it is never optimal for a merchant to set a price below 1. Even if a merchant deviates and sets a price below 1 in some period, the pricing rule calls for her to revert to a price of 1 in the following period. Hence, it is never optimal for an informed producer to return to his merchant.\(^9\)

\(^9\)A producer is indifferent between trading at a price of 1 and declining trade and
The next lemma describes a merchant’s best response to a symmetric time-invariant return rule with one-period memory on the part of all informed producers, characterized by the set of prices $P^R$ for which a producer will return. A merchant will set her price at 1 or the highest price at which clients return ($\sup P^R$), depending on whether it is more profitable to act as a bandit and take her clients’ entire endowment, or to induce her clients to return when they remember her location.

**Lemma 3.1 (Optimal Pricing).** Suppose that all informed producers follow a symmetric time-invariant return rule with one-period memory represented by $P^R$. A merchant’s best response is to set price in each period $t$ as follows:

$$p_t = \begin{cases} 
\sup P^R & \text{if } \sup P^R > 1 - \gamma \delta \\
\sup P^R \text{ or } 1 & \text{if } \sup P^R = 1 - \gamma \delta \\
1 & \text{if } \sup P^R < 1 - \gamma \delta \text{ or } P^R = \emptyset.
\end{cases}$$

(9)

**Proof.** Fix a return decision rule $P^R$. The case when $P^R$ is empty is already covered by Proposition 3.1. Let $P^R$ be nonempty and let $\sup P^R = \hat{p}$. It is clear that no price other than $\hat{p}$ or 1 can be optimal: a merchant can increase her current period profit, without affecting the return-decision of any client, by raising her price slightly.

We start with the price sequence $p_t = \hat{p}$ for every period $t$, and show that the merchant cannot gain by deviating from this sequence if $\hat{p} > 1 - \gamma \delta$.

By the one-shot deviation principle, it suffices to show there is no profitable one-shot deviation.\(^\text{10}\) Consider a one-period deviation in some period $\tau$ in which the merchant sets $p_\tau = 1$ (the best one-period deviation). The merchant’s discounted sum of expected net gain from this deviation is:

$$k_\tau \left\{ (1 - \hat{p}) - \gamma \delta \hat{p} (1 + \gamma \delta + \ldots) \right\} = k_\tau \left\{ 1 - \frac{\hat{p}}{1 - \gamma \delta} \right\} < 0 \text{ since } \hat{p} > 1 - \gamma \delta.$$

The expressions within braces represent the merchant’s net gain from the deviation from each of her $k_\tau$ clients: she would gain $1 - \hat{p}$ from each client at $\tau$, but lose her client base.

Using an analogous argument, when $\hat{p} < 1 - \gamma \delta$, it is optimal to set $p_t = 1$ in each period. When $\hat{p} = 1 - \gamma \delta$, the merchant, in each period, is indifferent between setting $p_t = \hat{p}$ and setting $p_t = 1$.

**Observation 3.1.** Recall that our definition of an equilibrium (Definition 2.1) incorporates the restriction of symmetry, time-invariance and one-period memory of the strategy profile. It then follows from Lemma 3.1 that foregoing consumption. However, an outcome in which a producer declines trade cannot be sustained through mutual best responses since his merchant would be better off lowering the price slightly.

\(^\text{10}\) It is also easy to verify directly, without invoking the one-shot deviation principle, that there are no profitable multi-period deviations.
all merchants must charge the same price in a given period in equilibrium unless \( \sup P^R = 1 - \gamma \delta \). It is also evident from Lemma 3.1 that the optimal pricing rule of a merchant cannot vary with the size of the merchant’s clients at \( t - 1 \): this is intuitively clear since a merchant’s clients never observe the size of the merchant’s clientele. Finally, by Lemma 3.1, at an intermediation equilibrium, we must have \( \sup P^R \geq 1 - \gamma \delta \).

We next compute the optimal symmetric time-invariant return rule with one-period memory for informed producers in period \( t \) when all merchants follow a symmetric pricing rule with one-period memory and charge the price \( p_t \) at \( t \). Let \( V^R_t \) denote the continuation value of an informed producer who returns to his merchant (takes action \( R \)) at \( t \) and let \( V^S_t \) denote his continuation value if he searches (takes action \( S \)) at \( t \). Let \( V_{t+1}(R) \) denote an informed producer’s continuation value at \( t + 1 \) given that he had taken action \( R \) at \( t \). Similarly, let \( V_{t+1}(S) \) denote his continuation value at \( t + 1 \) given that he had taken action \( S \) at \( t \). We now argue that \( V^R_{t+1} = V^S_{t+1} \).

First, the probability that a producer informed at \( t \) will remain informed at \( t + 1 \) is \( \gamma \) whether his chosen action at \( t \) is \( R \) or \( S \). Second, after either action \( R \) or \( S \) at \( t \), the producer will have the same payoff-relevant information: he will know at \( t + 1 \) with probability \( \gamma \) the location of a merchant who charged \( p_t \) at \( t \). (It is possible that action \( S \) led him to a different merchant at \( t \) but, by hypothesis, all merchants charge \( p_t \) at \( t \).) Therefore, the set of options available at \( t + 1 \) to an informed producer at \( t \) will be the same no matter which action he chooses at \( t \). It follows that \( V_{t+1}(R) = V_{t+1}(S) \). Let \( V_{t+1} \) represent the common continuation value at \( t + 1 \). Then, \( V^R_t \) and \( V^S_t \) are given by

\[
V^R_t = 1 - p_t + \delta V_{t+1} \tag{10}
\]

\[
V^S_t = \lambda^m (1 - p_t) + \lambda^s (1) + \delta V_{t+1}. \tag{11}
\]

An informed producer who chooses to return to his merchant at \( t \) concludes trade immediately with his merchant at price \( p_t \), consumes \( 1 - p_t \), and returns again with another unit next period when his continuation value at \( t + 1 \) is \( V_{t+1} \). This yields equation (10). An informed producer who chooses to search at \( t \) finds another merchant (who is also charging \( p_t \)) with probability \( \lambda^m \), encounters another searching producer with probability \( \lambda^s \) and trades units one for one, and fails to find a trading partner and receives no payoff with probability \( 1 - \lambda^m - \lambda^s \). No matter what the search outcome is, his continuation value at \( t + 1 \) is \( V_{t+1} \). This yields equation (11). The optimal return rule of an informed producer in period \( t \) follows from a simple comparison of the expected continuation values \( V^R_t \) and \( V^S_t \) in (10) and (11).

**Lemma 3.2 (Optimal Return Rule).** Suppose all informed producers at \( t \) follow a symmetric time-invariant return rule with one-period memory and
all merchants follow a symmetric time-invariant pricing rule with one-period memory and charge the price \( p_t \) in period \( t \). Define

\[
p^* = \frac{1 - \lambda^m - \lambda^s}{1 - \lambda^m}. \tag{12}\]

The optimal return rule prescribes a return to the merchant at \( t \) if \( p_t < p^* \) and search at \( t \) if \( p_t > p^* \); either of these choices is optimal if \( p_t = p^* \).

Proof. By the definitions of \( V^R_t \) and \( V^S_t \), it is optimal for an informed producer to return to his merchant at \( t \) if \( V^R_t > V^S_t \) and it is optimal to search at \( t \) if \( V^R_t < V^S_t \); either choice is optimal when \( V^R_t = V^S_t \). It is immediate from (10) and (11) that

\[
V^R_t \succeq V^S_t \quad \text{according as} \quad p_t \succeq p^*. \tag{13}
\]

This concludes the proof. \( \square \)

As (13) shows, when all merchants charge the same price, \( p^* \) is precisely the price charged by a merchant at which the continuation value of an informed producer from transacting with his merchant is the same as the continuation value from searching for a trading partner. If an informed producer expects his merchant to charge a price higher than \( p^* \), he is better off searching; at any lower price, he is better off returning to the merchant.\(^{11}\)

Lemma 3.1 and Lemma 3.2 allow us to characterize the stationary price paths that can be supported at an intermediation equilibrium. We begin by ruling out intervals of prices that cannot be supported.

**Lemma 3.3.** With fixed occupational choices, no price \( p \in [0, 1 - \gamma \delta) \cup (1 - \gamma \delta, p^*) \cup (p^*, 1] \) can be supported at an intermediation equilibrium.

Proof. It is immediate that no pricing rule and return rule that are part of an intermediation equilibrium can support a price \( p \) if \( p < 1 - \gamma \delta \) or \( p \in (p^*, 1] \). By Lemma 3.1, a merchant is better off acting as a bandit than charging any price lower than \( 1 - \gamma \delta \), and by Lemma 3.2, an informed producer is better off searching than returning to his merchant at any price above \( p^* \).

Suppose that there is some price \( p' \in (1 - \gamma \delta, p^*) \) that can be supported at an intermediation equilibrium. It follows from Lemma 3.1 that a necessary condition is that the associated return rule for the informed producers must stipulate that they do not return to their merchants if they observe a price greater than \( p' \), i.e., the return rule must have \( p' = \sup D^R \). Now suppose that a producer observes a price \( p'' \in (p', p^*) \) in some period \( t \). Then, the associated return rule must prescribe non-return at \( t + 1 \) since \( p'' > p' \).

\(^{11}\)We note that the price \( p^* \) depends on the measure \( m \) of merchants as \( m \) enters the arguments of \( \lambda^m \) and \( \lambda^s \). We often suppress the dependence of \( p^* \) on \( m \) in our notation.
By Lemma 3.1 again, his merchant’s optimal price at \( t+1 \) is \( p' \) since, by hypothesis, \( p' = \sup P^R > 1 - \gamma \delta \). But then, by Lemma 3.2, the return rule cannot be optimal since, by hypothesis, \( p' < p^* \). This proves the lemma. □

The main result of this section is Proposition 3.2 below that characterizes the two stationary price paths that can arise as outcomes at an intermediation equilibrium when occupational choices are fixed. One is the highest (symmetric) price compatible with repeated interaction: namely, \( p^* \). At this price, the merchant appropriates the entire information rent from her clients. At any higher price, informed producers are better off searching than returning to their merchants. This can be thought of as the monopoly price in the context of a repeated relationship in the presence of a parallel unmediated search market: we refer to \( p^* \) as the monopoly intermediation price. The second is the lowest (symmetric) price compatible with repeated interaction: namely, \( 1 - \gamma \delta \). At this price, the clients appropriate the entire information rent. A merchant is better off acting as a bandit than charging a lower price to attract repeat clients. We refer to \( 1 - \gamma \delta \) as the competitive intermediation price.

It is clear from a glance at Lemma 3.1 and Lemma 3.2 how to support a constant price path of \( p^* \) at an intermediation equilibrium. A constant price path of \( 1 - \gamma \delta \) is supported by the expectations of the informed producers that any merchant who charges a price above \( 1 - \gamma \delta \) in any period will turn to banditry (i.e., charge a price of 1) next period onward. Based on this expectation, an informed producer never returns to such a merchant. Correspondingly, merchants charge \( 1 - \gamma \delta \) and any self-deviation to a higher price triggers a permanent switch to bandit pricing. Since a merchant’s payoff is the same from bandit pricing and pricing \( 1 - \gamma \delta \) with returning customers, the return rule and the pricing rule are indeed mutual best responses.

**Proposition 3.2** (Intermediation Pricing). *With fixed occupational choices and a given measure \( m \) of merchants, an intermediation equilibrium exists if and only if*

\[
\gamma \delta \geq \frac{\lambda^s}{1 - \lambda^m}. 
\]  

(14)

*Moreover, only two stationary price paths can arise as outcomes of intermediation equilibria:*

- \( p_t = p^*(m) = \frac{1 - \lambda^m - \lambda^s}{1 - \lambda^m} \quad \forall t \quad \text{and} \)
- \( p_t = 1 - \gamma \delta \quad \forall t. \)

*Proof.* We know, by Lemma 3.1, that it is optimal for a merchant to charge a price \( p \) in period \( t \) that induces clients to return (as required in an intermediation equilibrium) only if \( p \geq 1 - \gamma \delta \). Further, by Lemma 3.2, an optimal return rule of clients prescribes return to merchants at \( t \) only if the price \( p \)}
charged by the merchants at $t$ satisfies $p \leq p^*(m)$. Thus, an intermediation equilibrium can exist only if
\[ p^*(m) \geq 1 - \gamma \delta. \tag{15} \]
Substitution of (12) for $p^*(m)$ shows that (14) is a restatement of (15).

Conversely, suppose that (14) is satisfied. We want to demonstrate the existence of two intermediation equilibria, one with a stationary price path of $p^*(m)$ and the other with a stationary price path of $1 - \gamma \delta$.

To support the former equilibrium, let the pricing rule of each merchant prescribe a fixed price of $p^*(m)$ regardless of history and let the return rule of each informed producer stipulate $P^R = [0, p^*(m)]$, i.e., a return to his merchant at $t$ if the merchant had charged $p_{t-1} \leq p^*(m)$ and search at $t$ if $p_{t-1} > p^*(m)$. Since $p^*(m) \geq 1 - \gamma \delta$ by virtue of (14), by Lemma 3.1, the pricing rule of each merchant is a best response to the return rule of informed producers. Moreover, by Lemma 3.2, every informed producer’s return rule is a (weak) best response to the pricing rule of merchants along the equilibrium path. Finally, we observe that even off the equilibrium path, the return rule’s prescription of search at $t$ if an informed producer encounters a merchant with $p_{t-1} > p^*(m)$ is a weak best response to the pricing rule. This follows because an informed producer’s expected payoff from search at $t$ exactly equals that from transacting with his merchant at price $p^*(m)$ (to which the merchant will revert at $t$ in accordance with his pricing rule). This demonstrates that there is an intermediation equilibrium with a stationary price path of $p^*(m)$.

To support an intermediation equilibrium with a stationary price path of $1 - \gamma \delta$, let the pricing rule of each merchant be given by
\[ p_t = \begin{cases} 1 - \gamma \delta & \text{if } t = 1 \text{ or } p_{t-1} \leq 1 - \gamma \delta \\ 1 & \text{if } p_{t-1} > 1 - \gamma \delta, \end{cases} \tag{16} \]
and let the return rule of each informed producer be given by $P^R = [0, 1 - \gamma \delta]$. This pricing rule and the return rule are mutual best responses—on the equilibrium path (by Lemma 3.1) as well as off the equilibrium path (by Proposition 3.1).

It remains to show that no other stationary price paths can be supported at an intermediation equilibrium. By Lemma 3.3, no price in the set $[0, 1 - \gamma \delta) \cup (1 - \gamma \delta, p^*) \cup (p^*, 1]$ can be supported at an intermediation equilibrium, and the proof is complete.

\textbf{Observation 3.2.} Proposition 3.1 and Proposition 3.2 together establish that, with fixed occupational choices and a measure $m$ of merchants, altogether three stationary price paths can be sustained in equilibrium: the static pure monopoly price of 1 that we have termed bandit pricing, and the
two price paths associated with intermediation equilibrium—the monopoly intermediation price \( p^*(m) \) and the competitive intermediation price \( 1 - \gamma \delta \).

The analysis of the monopoly intermediation price extends the classic model of search by Diamond (1971) to a repeated environment and incorporates a parallel search market. As in Diamond’s model, each merchant enjoys local monopoly; but here the monopoly power is tempered by the coexistence of the search market. The highest price at which a merchant has repeat clients is less than the static pure monopoly price of unity, which would be the equilibrium in Diamond’s framework.\(^{12}\)

We have modelled the merchants as price-setters. In an alternative formulation, the price could be determined through Nash bargaining between a merchant and a producer as in Rubinstein and Wolinsky (1987). Each party would then retain some of the gains from the reduced search costs.

4. Occupational Choice and Equilibria

4.1. Occupational Choice

Consider a given configuration \((p_t, m)\) in which a measure \( m \) of merchants set a price \( p_t \) at \( t \). An agent who was a producer at \( t - 1 \), and decides to start up as a merchant at \( t \), will begin with no established client base and must acquire clients over time. Some of the searching producers in period \( t \) will chance upon her trading post, initiating the evolution of her client base. A continuing merchant may also find herself with no clients, since it is possible that all her clients may forget her location. Let the continuation value of such an incipient merchant, who starts with a client base of \( k_t = 0 \) and sets a price of \( p_t \) at \( t \) be \( V^\mu(0, p_t, m) \). Let the continuation value of a producer who searches for trading partners at \( t \) when all \( m \) merchants charge \( p_t \) be \( V^S_t(p_t, m) \).

**Lemma 4.1** (Occupational Choice). At a configuration \((p, m)\), the occupational choice of each agent is optimal in every period \( t \) if and only if

\[
\delta V^\mu(0, p_t, m) = \delta V^S_t(p_t, m) + (1 - p_t).
\]

**Proof.** A producer who decides at \( t - 1 \) to start up as a merchant at \( t \) can obtain the continuation value \( \delta V^\mu(0, p_t, m) \), but he must sacrifice \( 1 - p_t \) of

\(^{12}\)The model of competition among merchants here is qualitatively similar to models of sequential search without recall. In models of sequential search, an agent who encounters an unacceptable price simply defers consumption and continues to search—the cost of additional search may be a delay in consumption or represented as a fixed amount. In our model, the cost of a bad search outcome is a reduction in current period consumption. The agent (producer) trades at the unfavourable price, consumes, and resumes search next period with a new unit of the good.
consumption at $t-1$. Correspondingly, a merchant who decides at $t-1$ to switch to production at $t$ will start as an uninformed producer and obtain the continuation value $\delta V^S(p_t, m)$, but she saves the opportunity cost of carrying an inventory of $1 - p_t$ that would have funded her next trade.

Thus if $\delta V^\mu(0, p_t, m) - (1 - p_t) > \delta V^S_t(p_t, m)$, a positive measure of uninformed producers will want to start up as merchants but no merchant will want to switch to production.\footnote{It is not necessary to explicitly consider informed producers, since they must have a continuation value bounded below by $V^S$.} Conversely, if $\delta V^\mu(0, p_t, m) < \delta V^S_t(p_t, m) + (1 - p_t)$, a positive measure of merchants will have an incentive to switch to production, but no producer will have an incentive to become a merchant. In either case, the measure of merchants cannot remain constant. Condition (17) negates these two possibilities, and ensures that each agent’s occupational choice is optimal at any $t$. \hfill $\Box$

4.2. Equilibria

Proposition 4.1 below combines Proposition 3.1, Proposition 3.2 and Lemma 4.1 to provide a characterization of the equilibria in which some but not all agents are merchants.

Definition 4.1. We define three strategy profiles that will feature in the characterization of equilibria. The three profiles differ in their specifications of pricing and return decision rules. They share in common the part of the profile concerning occupational choice. For all three profiles,

\begin{itemize}
  \item $\star$ a subset of measure $m \in (0, 1)$ of agents always specializes as merchants;
  \item the remaining agents always specialize as producers.
\end{itemize}

A \textit{bandit profile} is one in which

- $\star$ holds,
- each producer always searches (i.e., $P^R$ is empty),
- each merchant always sets the price $p_t = 1$ in every period $t$.

A \textit{monopoly intermediation profile} is one in which

- $\star$ holds,
- the informed producers’ return rule is given by $P^R = [0, p^*(m)]$,
- each merchant always sets $p_t = p^*(m)$.

A \textit{competitive intermediation profile} is one in which

- $\star$ holds,
- the informed producers’ return rule is given by $P^R = [0, 1 - \gamma \delta]$,
- each merchant sets $p_t = 1 - \gamma \delta$ if she had set $p_{t-1} \leq 1 - \gamma \delta$, and sets $p_t = 1$ otherwise.
Proposition 4.1 (Equilibria). There are at most three classes of equilibria with a measure \( m \in (0, 1) \) of merchants in every period \( t \): they are described in (a), (b) and (c).

(a) A bandit profile constitutes an equilibrium if and only if

\[
\frac{\lambda^m(m,1-m)}{\lambda^s(m,1-m)} = \frac{(1-\delta)m}{1-m}.
\]  

(b) A monopoly intermediation profile constitutes an equilibrium if and only if (14) holds and

\[
1 - \gamma \delta = \frac{\delta (1 - \lambda^m - \lambda^s) \lambda^m s}{\lambda^s m}.
\]  

(c) A competitive intermediation profile constitutes an equilibrium if and only if (14) holds and

\[
\frac{\lambda^m s}{m} = \frac{\gamma \delta \lambda^m + (1 - \gamma \delta) \lambda^s}{(1 - \gamma \delta) (1 + \lambda^m)} + (1 - \delta) \gamma.
\]  

Proof. See Appendix. \[\square\]

An informal sketch of some of the steps in the proof of Proposition 4.1 may be useful. Verifying that the profiles in parts (a)–(c) are equilibrium profiles is straightforward. The occupational choices of agents of course must be optimal at an equilibrium so that Condition (17) in Lemma 4.1 must be satisfied. The definitions of the bandit profile, the monopoly intermediation profile and the competitive intermediation profile all call for invariant occupational choices. With invariant occupational choices, we can apply Propositions 3.1 and 3.2. Proposition 4.1 now follows from combining the implications of Propositions 3.1 and 3.2 with the implications of Condition (17). In particular, Propositions 3.1 and 3.2 show that the pricing decisions of merchants and the return decisions of producers given in (a)–(c) of Definition 4.1 are mutual best responses; moreover, they are the only ones that are mutual best responses under invariant occupational choices. It remains to evaluate, for each strategy profile in turn, the continuation payoffs of an incipient merchant and a searching producer at the price and the return rule given by the profile and then substituting the result in (17). Thus modified, the (17) gives the necessary and sufficient condition for the profile to be an equilibrium; it also implicitly determines the measure of merchants in equilibrium.

The first part of Proposition 4.1 asserts that there can be no equilibria except the bandit profile, the monopoly intermediation profile and the competitive intermediation profile. We cannot now simply invoke Lemma 3.3, since Lemma 3.3 was predicated on fixed occupational choices and here occupational choices are endogenous. Instead, we use the assumption that
switching occupation is subject to friction in that a decision to switch may not be implementable with probability $1 - \alpha > 0$ to make the argument. It is instructive to see the role of this friction in the argument by considering for a moment the counterfactual scenario where switching occupation is frictionless, i.e., where $\alpha = 1$. Observation 4.1 below shows that then a continuum of prices would have been supportable as equilibria.

**Observation 4.1.** Consider a stationary configuration $(\bar{p}, \bar{m})$ that solves (17), with $\bar{p} \in (1 - \gamma \delta, p^*(\bar{m}))$. Define the strategy profile $\bar{\sigma}$ as a profile in which informed producers set $P^R = [0, \bar{p}]$; merchants set $p_t = \bar{p}$ at each $t$ if he had set $p_{t-1} \leq \bar{p}$, and if a merchant sets $p_t > \bar{p}$ at any $t$, then at $t + 1$, the merchant switches occupation and becomes a producer. $\bar{\sigma}$ constitutes an equilibrium if and only if $\alpha = 1$.

**Proof.** See Appendix.

A scrutiny of the strategy profile used in Observation 4.1 reveals the role of friction in the implementation of a decision to switch occupation. The return rule of the informed producers is based on the expectation that a merchant who charges a price higher than $\bar{p}$ at $t$ is a “fly-by-night” operator who will switch occupation and become a producer at $t + 1$ (and therefore be unavailable for trade at $t + 1$). This expectation rationalizes the decision of informed producers to not return to such a merchant. Against this return rule, it is indeed (weakly) optimal for a merchant who had set a price higher than $\bar{p}$ at $t$ to decide to switch occupation and start as an uninformed producer at $t + 1$: (17) ensures that the expected payoff of an uninformed producer is the same as that of a merchant who has lost his entire client base. Were occupational switches frictionless, the expectation implicit in the return rule of the informed producers would have been confirmed so that the profile would be an equilibrium. In contrast, in the presence of friction in implementing occupational switch, it can never be optimal for an informed producer to not return to his merchant if he expects the merchant to charge a price less than $p^*(\bar{m})$. The merchant must be available at his trading post with probability at least $1 - \alpha > 0$, and the producer has the option of proceeding to the unmediated search market should the merchant be absent. Therefore, his expected payoff from returning to his merchant exceeds his expected payoff from searching without returning to the merchant’s trading post (recall that these two expected payoffs are equal at $p^*(\bar{m})$). Step 1 in the proof of Proposition 4.1 makes these informal arguments precise and shows that the friction eliminates the possibility that any profile other than those in the statement of the proposition can be an equilibrium.

**Remark.** The arguments in Sections 3 and 4 can be used to characterize the set of symmetric Markov equilibria—in which a strategy profile is time-invariant and the actions of players in any given period can depend only on
the payoff-relevant part of the state observed in the preceding period—as a simple corollary of Proposition 4.1. Since we permit actions of players in a given period to depend on the observed outcomes in the previous period—whether payoff-relevant or not—the set of symmetric Markov equilibria is a subset of the set of equilibria identified in Proposition 4.1. In particular, the bandit profile and the monopoly intermediation profile are easily seen to be equilibria in symmetric Markov strategies; they are the only such equilibria. The competitive intermediation profile does not qualify as a Markov equilibrium as it is supported by a pricing rule in which a merchant’s price in a period depends on the price she set in the preceding period.

5. Closed-Form Solutions

In our analysis so far, the matching functions $\lambda^m$ and $\lambda^s$ were quite arbitrary. In the remainder of the paper, we focus on a specific pair of matching functions in the interest of gaining further insight and obtaining closed-form expressions for the equilibrium values of some key variables. The matching functions we focus on are given by

$$\lambda^m(m,s) = \frac{\lambda m^{1/2}}{m^{1/2} + s^{1/2}},$$

$$\lambda^s(m,s) = \frac{\lambda s^{1/2}}{m^{1/2} + s^{1/2}}, \quad \lambda \in (0,1).$$

Observe that merchants and producers are treated symmetrically by the matching technology: merchants enjoy no \textit{a priori} advantage in this respect. The matching functions are also 0-homogeneous so that there are no thick-market externalities.

Using the matching functions in (21) and (22), we find existence conditions for each of the three types of equilibria outlined in Proposition 4.1 in terms of the parameters of the model ($\gamma$, $\delta$, and $\lambda$). We also determine the equilibrium price at these equilibria and the equilibrium measure of merchants at the bandit and monopoly intermediation equilibria.\footnote{We omit the long and uninformative closed-form expression for $m$ in the competitive intermediation equilibrium.}

\textbf{Proposition 5.1 (Closed-Form Solutions).} Let the matching functions $\lambda^m$ and $\lambda^s$ be given by (21) and (22). Then,

(a) A unique bandit equilibrium always exists. At this equilibrium, the price is unity every period, and the measure of agents specializing as merchants is given by $m = 1/2$.

(b) A monopoly intermediation equilibrium exists if and only if

$$\gamma \delta^2 (1 - \lambda)^2 \geq (1 - \gamma \delta)(\lambda - \gamma \delta).$$

$$\square$$
If it exists, it is unique. The measure of agents specializing as merchants is given by

\[ m^* = \frac{(1 - \gamma)(1 + a^{1/2})}{(1 - \gamma)(1 + a^{1/2})(1 + a) + \gamma \lambda a}, \]  

(24)

where \( a = \frac{(1 - \gamma \delta)^2}{\delta^2(1 - \lambda)^2} \).  
(25)

and the price set by each merchant in every period is given by

\[ p^* = \frac{\delta(1 - \lambda)^2 + (1 - \lambda)(1 - \gamma \delta)}{\delta(1 - \lambda)^2 + (1 - \gamma \delta)}. \]  

(26)

(c) If \( \gamma \delta \geq \lambda \), a competitive intermediation equilibrium exists. If it exists, it is unique. At this equilibrium, the price is \( 1 - \gamma \delta \) in every period.

**Proof.** See Appendix.

The proof consists of using the particular matching functions (21) and (22) in the conditions derived in Proposition 4.1—in particular, inequality (14), and the equations (18), (19) and (20).

**Observation 5.1.** (i) There is a threshold value \( \lambda^* \in (\gamma \delta, 1) \) such that a monopoly intermediation equilibrium exists if and only if \( \lambda \leq \lambda^* \).

(ii) The condition in part (c) is sufficient but not necessary.

(iii) It follows, from parts (b) and (c) of Proposition 5.1, that both a monopoly and a competitive intermediation equilibrium exist when \( \gamma \delta \geq \lambda \).

**Proof of (i).** Rewrite (23) as

\[ \phi(\lambda) \equiv \gamma \delta^2(1 - \lambda)^2 - (1 - \gamma \delta)(\lambda - \gamma \delta) \geq 0 \]  

(27)

Inequality (27) is strict for \( \gamma \delta \geq \lambda \). Also, we have \( \phi' < 0 \) for all \( \lambda \in [0, 1] \), and \( \phi(1) = -(1 - \gamma \delta)^2 < 0 \). Since \( \phi \) is continuous, the existence of \( \lambda^* \) in the remark is assured.

The intuition is transparent: for intermediation equilibria to exist, the rate at which searchers can be found cannot be too high compared to the rate at which former clients return to their merchants. Otherwise, it would be more profitable for merchants to act as bandits than to induce clients to return.
6. Welfare

Merchants in this model provide a beneficial trading externality: an encounter with a merchant opens up the prospect of a long-term relationship for future trade, and potentially reduces search costs. However, specialization by merchants in the service of exchange comes at the expense of the production of the physical good. Merchants also create negative externalities: as the measure of merchants rises, the search market gets thinner, affecting the trading prospects of the searching population. Also, the clientele of a new merchant in steady-state is not drawn entirely from the hitherto searching population; some of her clients would otherwise have been clients of the merchants already in the market.

A natural measure of social welfare here is the expected aggregate consumption per period. How does welfare at an equilibrium with merchants compare with an economy with no merchants? What is the optimal measure of merchants in the economy? Is the equilibrium measure of merchants optimal? This section addresses these questions with the particular matching functions described in Section 5.

It is obvious that the bandit equilibrium outcome is worse for welfare than an economy with no merchants: bandits do not produce; nor do they reduce search cost for other agents.

Suppose that the economy is in steady-state with \( m \) merchants who set a price \( p \in P^R \). Then, the size of the set of uninformed agents is given by equation (8). With \( m \) merchants, \( 1 - m \) units are produced in a period; of these, a fraction \( (1 - \lambda^m - \lambda^s) s \) fails to get traded and consumed. Letting \( W \) denote the welfare, we have

\[
W(m) = 1 - m - (1 - \lambda^m - \lambda^s) s(m). \tag{28}
\]

Proposition 6.1 and Observation 6.1 demonstrate that the welfare associated with an intermediation equilibrium may be higher or lower than the welfare associated with a pure-search economy.

**Proposition 6.1 (Welfare).** Let the matching functions be given by (21) and (22).

(a) Welfare is maximized at an interior measure \( \tilde{m} \) of merchants. At \( \tilde{m} \),

\[
s'(\tilde{m}) = -\frac{1}{1 - \lambda}. \tag{29}
\]

(b) The welfare associated with the monopoly intermediation equilibrium characterized in Proposition 5.1(b) is greater than the welfare in the pure-search economy if and only if

\[
\gamma(1 - \gamma\delta)^2 > (1 - \gamma)[(1 - \gamma\delta) + (1 - \lambda)]. \tag{30}
\]
Observation 6.1. Condition (30) holds for a wide range of parameter values that are consistent with Proposition 5.1 (b). For example, try $\gamma = 9/10$, $\delta = 5/8$, $\lambda = 9/16$. Note that $\lambda = \gamma \delta$, so the existence condition is satisfied. Similarly, there are also ranges of values for which an intermediation equilibrium exists, but condition (30) does not hold. Thus, in general, there is no correspondence between equilibria and optima, or even a presumption that welfare at an equilibrium is necessarily greater than in the pure-search economy.

7. The Emergence of Merchants

Suppose that we start with an economy in which all agents specialize as producers. Under what conditions can we expect an institution of intermediation to endogenously arise in this economy? We show that, if $\gamma$ is sufficiently large, then it will be strictly profitable for an arbitrarily small positive measure of producers to deviate and set up as merchants. Below we interpret an increase in $\gamma$ as a consequence of increasing maturity and stability in civil society. In this interpretation, as society progresses from its “early and rude state”, intermediation arises endogenously and merchants replace bandits and pirates.

Consider therefore an economy in which each agent specializes as a producer every period, and searches for trading partners. Each agent’s period payoff is $\lambda s(0,1) = \lambda$. Let each producer’s return rule, when informed, be given by $P_R = [0, 1 - \lambda]$.

Now, suppose a small measure of agents deviates, starts up as merchants, and sets a price less than $1 - \lambda$. Producers who come upon their trading posts will want to return. Thus, the merchants, beginning with a client base of zero, will acquire clients over time. Proposition 7.1 below shows that this deviation is profitable provided that a merchant’s retention rate of clients, $\gamma$, is sufficiently high relative to $\lambda$.

Proposition 7.1 (Emergence of Merchants). Let the matching functions be specified by (21) and (22), and let $\gamma \delta > \lambda$. Suppose that each agent’s strategy is to specialize as producer in every period and, if informed, use the return rule $P_R = [0, 1 - \lambda]$.

(a) There exists $\hat{m} \in (0,1)$ such that, for all $m' \in (0, \hat{m})$, any subset of agents of measure $m'$ would find it profitable to start up as merchants and set a price $\tilde{p} \in [1 - \gamma \delta, 1 - \lambda]$.

(b) The payoffs of deviating merchants increases without bound as $m' \to 0$.

Proof. Let a subset of agents of measure $m$ simultaneously start up as merchants and set a price $\tilde{p} \in [1 - \gamma \delta, 1 - \lambda]$ every period. Since $\gamma \delta > \lambda$, $\tilde{p} \in P_R$. 

Proof. See Appendix.
Then, the continuation value of this deviation for an individual agent is given by

\[ V^\mu_\mu(0, \tilde{p}, m) = \tilde{p} \lambda^m (m, 1 - m) (1 - m) \frac{(1 - \delta)(1 - \gamma \delta) m}{(1 - m)} \text{, [see (35) in Appendix]}
\]

\[ = \frac{\tilde{p} \lambda (1 - m)}{(1 - \delta)(1 - \gamma \delta) \left[ m + m^{1/2}(1 - m)^{1/2} \right]}, \text{ by (21) and (22)}
\]

\[ \geq \frac{\lambda (1 - m)}{(1 - \delta) \left[ m + m^{1/2}(1 - m)^{1/2} \right]}, \text{ since } \tilde{p} \geq 1 - \gamma \delta. \quad (31)
\]

\[ > \frac{\lambda}{1 - \delta}, \text{ for } m < 1/4, \text{ since } m^{1/2}(1 - m)^{1/2} \leq 1/2. \quad (32)
\]

The condition \( \gamma \delta > \lambda \) also ensures that \( \sup P^R > 1 - \gamma \delta \); thus, the payoff for the deviating agents is higher than their payoff if they were to become bandits (see Lemma 3.1). This, in conjunction with inequality (32), establishes part (a).

Moreover, from (31), \( V^\mu_\mu(0, \tilde{p}, m) \) increases without bound as \( m \to 0 \) which is part (b). \( \square \)

**Observation 7.1.** If \( \tilde{p} \in (1 - \gamma \delta, 1 - \lambda) \), the deviation makes all agents—not only those in the deviating subset—strictly better off.

The prospect of an endogenous rise of an institution of intermediation *ab initio* thus depends on the relative values of the parameters \( \gamma \) and \( \lambda \) (for a fixed \( \delta \)). These parameters, in turn, are arguably determined by social and technological conditions.

Exchange for personal consumption between producers has occurred since prehistory within local circles, and formed the basis for division of labor and specialization in village economies. The ambit of such exchange, for which \( \lambda \) is a proxy, is likely to remain limited and evolve slowly in the absence of professional traders.

The parameter \( \gamma \), which captures the ability of merchants to communicate with their clients and of the clients to return to their merchants, is likely to be more sensitive to social, political, and technological conditions. Communication and commerce may be rendered impossible between one period and the next by natural calamities or bandits or unreliable transportation; rulers may prevent access or impose tolls; local wars may intervene. Viewed in this way, \( \gamma \) is likely to rise with improvements in law and order and in the technology of communication and transport. Thus, farsighted merchants are unlikely to thrive in primitive and unmoderated societies; they appear only when some modicum of public security has already been established.

When order deteriorates in established societies, disrupting communication and transportation networks, even erstwhile reliable merchants may turn to
banditry; but professionally mediated trade arises again as the rule of law is restored and communication improves.\textsuperscript{15}

8. Conclusion

Intermediaries perform many roles in facilitating trade. They may variously exploit advantages in the technology of transaction and trade, costs of storing inventory, aggregating information, assessing quality of goods or some attributes of agents or a market, matchmaking, and so forth. We focused on only two aspects that are interrelated in our model—reducing the cost of search, and fostering long-term trading relationships with clients. Our primary objective was to develop a self-contained, if rudimentary, account of an emergent institution of intermediation. Thus, the important modelling concern was to start with a homogeneous population, endogenize the choice between the two occupations of production and intermediation, and investigate the configuration of parameters that predicate the rise of intermediation as a sustainable occupation.

In focusing on these, we have marginalized several other concerns that may legitimately claim attention in the context of this paper. We briefly comment on some of these below.

We suppose that the price at which a producer trades with a merchant is set by the merchant. In our model, this results in either the merchant or the producer extracting all the rent in equilibrium. In an alternative formulation, the price could be determined through Nash bargaining between a merchant and a producer, with the consequence that both parties would retain some of the gains from reduced search costs.

Our treatment of competition among merchants is minimalist. In particular, a producer knows at most one merchant; he cannot maintain his link with a merchant and simultaneously search for a better price. It may be of interest to investigate the consequences of allowing producers to randomly observe a second price, as in Burdett and Coles (1997). It is worth reiterating, however, that even the simple model elaborated here incorporates the full extent of competition that is afforded by standard models of sequential search without recall (see Footnote 12).

The present model is one of pure exchange: the production process is entirely mechanistic in that it involves no choice variable. As Diamond (1982) has shown in a model of search, reducing anticipated delays in exchange can influence production decisions. In future work, we plan to extend the

\footnotetext{15}{This interpretation is not inconsistent with European history. In the second half of the first millennium of the Common Era there was a general decline of law and order, accompanied by a contraction of trade. As stability was re-established and the rule of law gained ascendancy in the second millennium, professional merchants flourished and trade expanded as well, both within Europe and across the Mediterranean.}
present model by incorporating production to yield richer general equilibrium interactions. This would also provide the bridge between the analysis of the microstructure of trade and the formulation of macroeconomic policy, which was the intention of Diamond’s original article.

Appendix

Proof of Proposition 4.1. We will complete the proof in two steps.

Step 1. We first argue that no price other than the prices associated with the bandit profile, the monopoly intermediation profile and the competitive intermediation profile can be supported in equilibrium.

Suppose that, at an equilibrium, \( \bar{p} \) is the price in period \( t \) and \( \bar{m} \) is the associated measure of agents who are merchants at \( t \). Since occupational choices must be optimal in equilibrium, Condition (17) must be satisfied.

It is immediate that, regardless of any decision concerning occupational choice, no pricing rule and return rule can support the price \( \bar{p} \) at \( t \) at an equilibrium if \( \bar{p} < 1 - \gamma \delta \) or \( \bar{p} \in (p^*(\bar{m}), 1) \): a merchant charging such a price can unilaterally increase her payoff by charging a price of 1 if the return rule is optimal.

Now suppose \( \bar{p} \in (1 - \gamma \delta, p^*(\bar{m})) \). Let \( \bar{P}^R \) denote the associated return rule of informed producers. We know that a necessary condition for \( \bar{p} \) to be sustainable in equilibrium is that \( \bar{p} = \sup \bar{P}^R \) (since \( \bar{p} \neq 1 \)). Let a producer observe a price \( p' > \bar{p} \) charged by some merchant in period \( t - 1 \). Since \( \bar{p} > 1 - \gamma \delta \), optimality of merchant’s pricing rule against \( \bar{P}^R \) requires that she charge precisely the price of \( \bar{p} \) at \( t \) if she operates as a merchant at \( t \). Therefore, a return to his merchant at \( t \) after observing \( p' \) at \( t - 1 \) will get an informed producer the price \( \bar{p} \) with probability at least \( 1 - \alpha > 0 \) (in the event that the merchant is still operating) and the payoff from search with the complementary probability (in case the merchant did successfully switch occupation). Since \( \bar{p} < p^*(\bar{m}) \), the expected payoff from returning to the merchant, even after observing a price of \( p' \), is higher than that from directly proceeding to search, regardless of the merchant’s decision on occupational choice. But then the return rule \( P^R \) cannot be optimal.

Therefore, we have established that, even if occupational choices are endogenous, no price other than the prices identified in Proposition 4.1 can be supported in equilibrium.

Step 2. We now verify parts (a), (b) and (c) in the proposition. Begin by observing that the strategy profile in each of the parts (a)–(c) stipulates invariant occupational choices. Therefore, we can apply Propositions 3.1 and 3.2.

If Condition (17) is satisfied, no agent can benefit by changing occupation so that the invariant occupational choices are indeed (at least weakly)
optimal. Propositions 3.1 and 3.2 show that the pricing decisions of merchants and the return decisions of producers given in (a)–(c) of Definition 4.1 are mutual best responses; moreover, they are the only ones that are mutual best responses under invariant occupational choices. The remainder of the proof consists of evaluating, for each strategy profile in turn, the continuation payoffs of an incipient merchant and a searching producer at the price and the return rule given by the profile and then substituting the result in (17). Thus modified, the (17) gives the necessary and sufficient condition for the profile to be an equilibrium; it also implicitly determines the measure of merchants in equilibrium.

(a) If agents follow the bandit strategy profile, we have
\[ s = 1 - m \]
as at every \( t \), and a merchant extracts \( 1 \) from each of the \( \lambda^m(m, 1 - m)(1 - m)/m \) of searching producers who come upon her trading post. Further, since informed clients do not return, every merchant is in the same position as one who served no clients in the previous period. Using this information in (6) yields the continuation payoff for any merchant \( \mu \) at any \( t \) given by
\[ V^\mu(0, 1, m) = \frac{\lambda^m(m, 1 - m)(1 - m)}{(1 - \delta)m}. \] (33)
Moreover, at the given strategy profile, since the expected payoff of a searching producer each period is \( \lambda^s(m, 1 - m) \), the continuation payoff at any \( t \) for any producer, whether informed or uninformed, is given by
\[ V^S_t(1, m) = \frac{\lambda^s(m, 1 - m)}{1 - \delta}. \] (34)
Substituting the bandit price \( p = 1 \) and the continuation payoffs from (33) and (34) in (17) gives (18).

(b) If agents follow the monopoly intermediation strategy profile, the price \( p^*(m) \) is constant. Then, using the constant price (12) in (6), we have
\[ V^\mu(0, p^*(m), m) = \frac{(1 - \lambda^m - \lambda^s)\lambda^m s}{(1 - \delta)(1 - \gamma \delta)(1 - \lambda^m)m}. \] (35)
By (10), (11) and (12), we note that \( V^R_t = V^S_t = V_{t+1}^S \) when the price is \( p^*(m) \) every period. It now follows from (10), (11) and (12) that
\[ V^S_t(p^*(m), m) = \frac{\lambda^s}{(1 - \delta)(1 - \lambda^m)}. \] (36)
Substituting the price from (12) and the continuation payoffs from (35) and (36) in (17) gives (19). Finally, by Proposition 3.2, an intermediation equilibrium exists if and only if Condition (14) is satisfied.

(c) The proof again involves computations similar to parts (a) and (b): determine the continuation payoffs \( V^\mu(0, 1 - \gamma \delta, m) \) and \( V^S_t(1 - \gamma \delta, m) \) at
price \(1 - \gamma \delta\) and substitute the price and these continuation values in (17) to derive (20). Condition (14) ensures that the price \(1 - \gamma \delta\) does not exceed the monopoly intermediation price derived in (12), a necessary and sufficient condition for the existence of such an intermediation equilibrium by Proposition 3.2.

Finally, we observe that all of the strategy profiles given in the proposition are stationary, time-invariant, symmetric and involve one-period memory. This concludes the proof of the proposition. \(\square\)

**Proof of Observation 4.1.** If \(\alpha = 1\), according to the pricing rule in \(\bar{\sigma}\), merchants are unavailable for trade after they charge a price higher than \(\bar{p}\). Therefore, for an informed producer, not returning to his merchant after observing such a price is a best response to the pricing rule. On the other hand, if a merchant charges a price no higher than \(\bar{p}\), the pricing rule calls for charging \(\bar{p}\) in the following period. Therefore a prescription to return after observing a price no higher than \(\bar{p}\) is also a best response since \(\bar{p} < p^*(\bar{m})\). Correspondingly, the pricing rule in \(\bar{\sigma}\) is easily seen to be a best response to the return rules of informed producers. Since (17) is satisfied at \(\bar{\sigma}, \bar{\sigma}\) is indeed an equilibrium.

The necessity of \(\alpha = 1\) follows from Step 1 of the proof of Proposition 4.1. \(\square\)

**Proof of Proposition 5.1.** (a) Using (21) and (22) in (18) and recognizing that \(s = 1 - m\) at a bandit equilibrium, we get

\[
\frac{\lambda m^{1/2}(1 - m)}{m^{1/2} + (1 - m)^{1/2}} = \frac{\lambda (1 - m)^{1/2}}{m^{1/2} + (1 - m)^{1/2}},
\]

which reduces to \(m = 1/2\). This proves part (a).

(b) Simplify (19) to obtain

\[
\delta(1 - \lambda^m - \lambda^s)\lambda^m s - (1 - \gamma \delta)\lambda^s m = 0 \tag{37}
\]

Substitute (21) and (22) in (37) to get

\[
\frac{m^{1/2}}{s^{1/2}} = \frac{\delta(1 - \lambda)}{1 - \gamma \delta}. \tag{38}
\]

Using (21) and (22) in (14), we get

\[
\gamma \delta \geq \frac{\lambda s^{1/2}}{(1 - \lambda) m^{1/2} + s^{1/2}}, \quad \text{or,} \quad \frac{m^{1/2}}{s^{1/2}} \geq \frac{\lambda - \gamma \delta}{\gamma \delta(1 - \lambda)}. \tag{39}
\]
Combining (38) and (39) gives (23).
By (38),
\[
\frac{s}{m} = \frac{(1 - \gamma \delta)^2}{\delta^2(1 - \lambda)^2}.
\] (40)
Defining \( a = \frac{s}{m} \) gives the value of \( a \) in (25).
Using \( a = \frac{s}{m} \), (21) reduces to
\[
\lambda^m = \frac{\lambda}{1 + a^{1/2}},
\] (41)
and (8) becomes
\[
am = \frac{(1 - \gamma)(1 - m)}{1 - \gamma \left(1 - \frac{\lambda}{1 + a^{1/2}}\right)},
\]
which yields the value \( m^* \) in terms of the parameters given in (24). It is easily seen that \( m^* \in (0, 1) \).
Finally, (26) follows from substituting (21), (22) and (38) in (12).
\( c \) Since \( \lambda = \lambda^s + \lambda^m \geq \lambda^s \), we have
\[
\frac{1 - \lambda}{\lambda^s} \geq \frac{1 - \lambda}{\lambda}
\] \Rightarrow
\[
\frac{1 - \lambda + \lambda^s}{\lambda^s} \geq \frac{1}{\lambda}
\] \Rightarrow
\[
\lambda \geq \frac{\lambda^s}{1 - \lambda m^*}.
\]
Thus, whenever \( \gamma \delta \geq \lambda \), (14) is satisfied.
Using (21) and (22) and letting \( b = \frac{m^{1/2}}{\sigma_1^{1/2}} \), (20) reduces to
\[
\lambda \frac{b}{b(b + 1)} - \frac{\gamma \delta \lambda b + (1 - \gamma \delta)\lambda}{(1 - \gamma \delta)(b + 1) + \gamma \delta \lambda b} - \gamma(1 - \delta) = 0.
\] (43)
Which is a function of \( b \) and the parameters. Cross-multiplying and collecting terms, (43) can be written as
\[
\psi(b) \equiv Ab^3 + Bb^2 + Cb + D = 0,
\] (44)
where
\[
A = - [\gamma \delta \lambda + (1 - \gamma \delta)(1 - \delta)\gamma + (1 - \delta)\gamma^2 \delta \lambda] < 0,
\]
\[
B = - [(1 - \gamma \delta)\lambda + \gamma \delta \lambda + 2(1 - \gamma \delta)(1 - \delta)\gamma + (1 - \delta)\gamma^2 \delta \lambda] < 0,
\]
\[
C = \gamma \delta \lambda^2 - (1 - \delta)(1 - \gamma \delta)\gamma,
\]
\[
D = \lambda(1 - \gamma \delta) > 0.
\]
Since \( \psi''(b) = 6Ab + 2B < 0 \) for \( b \geq 0 \), \( \psi \) is strictly concave for \( b \geq 0 \). Moreover, \( \psi(0) = D > 0 \), and it is easy to verify that \( \psi(1) < 0 \). It follows that (44) has a unique solution for \( b \geq 0 \), and the solution occurs in the range \( b \in (0, 1) \). Observe that in this range, \( 0 < m < s(m) \leq (1-m) \) which implies that \( m \in (0, \frac{1}{2}) \). Thus, (20) always has a unique positive solution which occurs for some \( m \in (0, \frac{1}{2}) \). Further, \( \gamma \delta \geq \lambda \) is sufficient to ensure that this is a competitive equilibrium.

**Proof of Proposition 6.1.** (a) Using equations (21) and (22) in (28),

\[
W(m) = 1 - m - (1 - \lambda) s(m).
\]  

(45)

\( W(0) = \lambda \) and \( W(m) \) must fall below \( \lambda \) for values of \( m \) in excess of \( 1 - \lambda \): at least \( \lambda \) units of output must be produced in the economy for welfare to exceed \( \lambda \). Since \( W \) is continuous in \( m \), it attains a maximum over \([0, 1-\lambda]\).

We now verify that the derivative of \( W(m) \) is positive at \( m = 0 \). From (45), we have

\[
W'(m) = -1 - (1 - \lambda) s'(m)
\]

so that \( W'(m) > 0 \) if \( s'(m) \) is negative and larger in absolute value than \( 1/(1 - \lambda) \). Some tedious algebra yields

\[
s'(m) = -\frac{(1 - \gamma)(m^{1/2} + s^{1/2})^2 + (1/2)\gamma \lambda s^{2/3}m^{-1/2}}{(1 - \gamma)(m^{1/2} + s^{1/2})^2 + \gamma \lambda m^{1/2}(m^{1/2} + s^{1/2}) + (1/2)\gamma \lambda m^{1/2}s s^{1/2}},
\]

which is negative for all values of \( s \) and \( m \) between 0 and 1. Using the fact that \( s \to 1 \) as \( m \to 0 \), we find that \( s'(m) \) increases without bound in absolute value as \( m \to 0 \). Thus, \( W \) attains an interior maximum and (29) follows from the first-order condition.

(b) For welfare in the monopoly intermediation equilibrium with \( m^* \) merchants, identified in Proposition 5.1(b), to be greater than that in the pure-search economy, we need

\[
1 - m^* - (1 - \lambda)s(m^*) > \lambda,
\]

or,

\[
\frac{1}{m^*} - \frac{s(m^*)}{m^*} > \frac{1}{1 - \lambda}.
\]  

(46)

Recalling that we defined \( a = \frac{s}{m} \) and substituting the value of \( m^* \) from (24), (46) reduces to

\[
\gamma a(1 - \lambda) > (1 - \gamma)(1 + a^{1/2}).
\]  

(47)

Substituting \( a = \frac{(1-\gamma \delta)^2}{(1-\lambda)^2} \) from equation (38) in (47) and simplifying yields condition (30).
References


