1 Introduction
Climate change can have a significant effect on daily life and economic activity. Adaptation to new climatic conditions is, hence, a necessary strategy for those living in the affected parts of the world. Adaptation comprises measures of prevention as well as measures of adopting a change in a traditional way of life. While measures to prevent human-caused climate change are being sought and implemented, it is not clear whether they will prove sufficiently effective and eliminate all negative consequences of global warming. Adoption of new techniques and technologies will have to complement preventive strategies. In many cases, adaptation will require the adoption of new technologies by individuals and households. These may range from replacing existing energy resources by new ones, such as solar energy, to changing farming practices or crop varieties.

In this paper, we will study the process of adaptation to climate change via choice of technology. Though the analysis can be applied to many problems of introducing new technologies in...
traditional societies, for concreteness’ sake, we will conduct the analysis in the context of the adoption of new agricultural technologies. Moreover, in developing countries where farming at the household level still represents a significant proportion of economic activity, adopting new methods of farming will be of particular importance.

Burton and Lim (2005), Howden et al. (2007), Hassan and Nhachena (2008), and others have stressed the severe impact that climate change can have on societies which depend for their existence on traditional agricultural methods. While farming methods have evolved over time so as to optimally utilize local climate conditions, changes in temperature, precipitation and an increased variability of weather conditions reduce expected returns, lead to hunger and endanger the chances of households to survive. Learning how to adapt traditional farming methods to the new environment can be a long and complex process, see Quiggin and Horowitz (2003). To ensure sustainability, new technologies are being developed which can reduce the risk of famine and increase returns, see Howden et al. (2007). Adapting planting dates, using different seed varieties which are more resistant to droughts, water management and irrigation, tree planting and soil preservation, are some of the methods which have been suggested and used to counteract the negative consequences of climate change. Whereas some of these efforts have been successful, see for instance, Yang et al. (2007), two important barriers to adaptation have come to the fore: lack of money and lack of information. While high poverty rates and the lack of available credit have been discussed in the literature on technology adoption in developing countries, see Udry (2010), in this paper, we will focus on the role of ambiguity arising from a lack of information for the process of adaptation.

Ambiguity arises on several levels. First, the scientific processes behind climate change are not well understood, making it difficult to predict future regional weather conditions and their exact impact on local agriculture, see, e.g., the Report of the Australian Academy of Science (2010, pp. 2-3). Even though new technologies might have been tested under laboratory conditions, they eventually have to be adapted to local circumstances — social, biological and environ-
mental, see Lee (2005). Second, climate change may lead to irreversible effects which render invalid currently accepted scientific truths. Finally, there is uncertainty about the reaction of economic agents to changes in the environment. Behavioral predictions depend on costs and benefits of adaptation, which might be very difficult to quantify, see Stern (2006) and Parry et al. (2007). Furthermore, short-run motives often overpower long-run considerations, thus leading to suboptimal decisions, Lee (2005). Such ambiguities pose a severe problem in the process of adoption of new technologies, rendering their evaluation difficult and highly subjective, see O’Neill (2008). Moreover, with incomplete markets and highly subjective assessments of the likelihoods of ambiguous events, insurance for this type of uncertainty is, in general, unavailable. Hence, the agents’ attitude towards ambiguity will play an important role in their decisions to adopt new and better technologies.

In a more formal sense, ambiguity refers to situations, in which decision makers lack knowledge about the probability distribution over outcomes. Ambiguity is distinct from situations of risk, where objective probabilities are known. Following Ellsberg’s (1961) seminal work, a large number of subsequent studies show that people behave differently in situations of ambiguity compared to situations of risk, for an overview see, e.g., Mukerji and Tallon (2004). Traditional subjective expected utility theory (SEU) as developed by Savage (1954), is ill-suited for dealing with data that are not related to a prespecified and commonly known state of the world. Notions such as precision and relevance of information for a particular choice situation are difficult to model in the SEU approach. Moreover, there is no room for attitudes towards ambiguity like optimism and pessimism.

In contrast, case-based decision theory (CBT), which was pioneered by Gilboa and Schmeidler (2001), is designed to model the impact of the amount and precision of the available data on a decision makers’ evaluation of actions. Recently, Eichberger and Guerdjikova (2011) introduced a special form of CBT which allows one to distinguish optimistic and pessimistic attitudes towards ambiguity due to insufficient data. This framework can be used to operationalize Knight’s (1921) notion of willingness to engage in new activities with highly uncertain
outcomes, which he considers to be the main characteristic of the entrepreneurial personality. Incorporating both pessimism and optimism towards the performance of technologies, we can study the effects of these attitudes on the process of technology adoption.

The framework presented here is rather general. It is applicable to any scenario in which successful adaptation to an external shock requires the adoption of a new technology. In this paper, we apply our model specifically to the problem of changing technologies in response to climate change. For ease of exposition, we consider a population of agents who, in each period, choose between two technologies. One is a "traditional" technology, for which extensive data are available. The second technology is an "innovation", which promises increased expected returns, but for which little evidence about its performance is available. Successful adaptation to climate change occurs in our model only if the new and better performing technology is adopted by the population.

An important feature of our model is the assumption that the decision to adopt a new technology is reversible, i.e., individuals are not constrained to the new technology forever once they have chosen it. This is a reasonable assumption when switching the technology is not too costly, as in the case of adopting a new farming technology, but may be inadequate when huge investment costs are involved. In the concluding section of the paper, we will further discuss how this assumption affects our results.

We adopt a partial equilibrium approach, assuming that the choices made in each period do not influence the returns of the two technologies. This would be the case, e.g., if agents were farming the same crop for their own consumption by using different seed varieties or different farming methods. The community is assumed to be small, so that choices as well as realized returns can be observed by all agents. The set of past observations is the data base available to all agents. We assume that preferences and beliefs can be represented by the CBT as axiomatized in Eichberger and Guerdjikova (2011). In order to focus on attitudes towards ambiguity, we assume that the population is divided into two extreme groups: optimists and pessimists. These groups differ with respect to their ex-ante expectations. In absence of any information, optimists
behave as if the best possible outcome of the technology were guaranteed, while pessimists act as if the technology were certain to result in the worst possible outcome. When the actual performance of the technology becomes known, the beliefs of both types are updated based on the observed frequency of payoffs.

In this framework, we establish an important difference in the behavior of the two types of agents. Given equal empirical frequencies of successes and failures for both technologies, pessimists favor less ambiguous technologies for which a larger amount of data is available (the "traditional" technology), whereas optimists prefer to experiment with "innovations" and choose more ambiguous technologies, for which data are scarce. This has important implications for the process of technology adoption: optimists will be the first to experiment with new technologies, whereas pessimists will only adopt the new technology once there is sufficient positive evidence in favor of it. Hence, optimists bear the ambiguity and provide the public good of information about the profitability of the new technology. The presence of optimists in the population is therefore crucial for the adoption of a new technology. This result is consistent with field and experimental evidence on technology adoption in developing countries, which shows that ambiguity-averse (pessimistic) agents are less likely to invest in new technologies, see Carlsson and Naranjo (2009) and Engle-Warnick, Escobar and Laszlo (2007).

Though optimists are the driving force of innovation, their desire to experiment eventually leads to suboptimal outcomes. As the frequency with which the new technology is chosen grows, the inclination to choose technologies based on little evidence leads optimists to abandon the new technology prematurely in response to a few bad observations and to switch back to the traditional technology. In a dynamic equilibrium, depending on the share of pessimists in the society, some or all of the optimists will eventually switch back to the traditional technology. This is due to the fact that the decision to adopt the new technology is not irreversible. In contrast, the conservatism of pessimists makes them choose the new and better technology forever.

This result highlights the role of both types of agents in the process of technology adoption: a
positive share of optimists is crucial for jump-starting the process, a positive share of pessimists ensures that the optimal technology is persistently chosen. Formally, we show that the system always has a steady state, in which pessimists choose the innovation and the optimists are either indifferent between the two technologies or opt for the old one. In this steady state, the average payoff of the population weakly decreases in the share of optimists. For a large share of optimists, this steady state is globally stable. As the share of optimists decreases, two new steady states emerge. The three steady-states can be ordered with respect to the average payoff of the population. In particular, there is a worst steady state, in which only the optimists adopt the new technology and the pessimists stay with the traditional one. Examining the stability properties of these steady states, we find that, for a small share of optimists, starting from the traditional technology in the initial period, the economy will converge to the worst steady state with a probability close to 1. Hence, the worst steady-state allocation can be avoided only if the share of optimists in society is sufficiently high. Increasing the share of optimists beyond this threshold, however, will not increase the average payoff of the community.

The specific assumptions in this application of our model, i.e., the existence of only two types of technology with returns on a common support, symmetric information of agents, just two extreme types of ambiguity attitude, were chosen to highlight important prerequisites for the willingness to innovate in a society where information on the new technology is relatively scarce compared to the amount of data about the existing, old technology. In particular, there is nothing "irrational" in being conservative with respect to new untested technologies. Our specific assumptions show that it is sensible to switch to new technologies only based on sufficient evidence. However, in order to generate evidence for new technologies, the presence of individuals who are willing to act on weak evidence is necessary. This is the role of the optimists in our model. On the other hand, their tendency to act on little evidence makes them abandon even successful technologies prematurely on a small sample of bad evidence.

Embedding this story in a formal model has its price in "realism". Since there are just two types of technology, technology choice can only oscillate between these two options. Similarly, in
many real world applications, assuming an exogenous process of returns for the technologies and symmetric information of agents appears questionable. Relaxing these assumptions in order to increase realism of the model is straightforward, but the analysis of the dynamics becomes more complex and less transparent.

1.1 Related literature

Our paper stresses the impact of ambiguity due to a lack of data and attitude towards ambiguity for the process of technology adoption. Hence, learning occurs as new data are accumulated. In contrast, most models in the literature use either Bayesian updating, see Foster and Rosenzweig (2010), Foster and Rosenzweig (1995), Bandiera and Rasul (2006), or frequentism, as in Udry and Conley (2010) in order to model the decision makers’ response to new information. Hence, these models consider only risk and neglect the role of ambiguity, see Ulph and Ulph (1997), Ingham et al. (2007), Baker (2005).

Only a few papers on technology adoption and climate change deal with ambiguity. Lange and Treich (2008) study ambiguity in a model of environmental pollution. Ambiguity aversion induces agents to become more cautious towards pollution and to reduce emissions in the current period. Lempert and Collins (2007) apply the precautionary principle to the process of technology adoption and show that a strong degree of ambiguity aversion may slow down innovation.

In a similar spirit, Grant and Quiggin (2009) show that unawareness of potential consequences in combination with the precautionary principle can reduce incentives to innovate.

Our paper considers not only ambiguity aversion (pessimism), but also ambiguity loving behavior (optimism). It studies the interaction between these two types of attitudes and their effects on data generation and learning. Thus, it can be viewed as a stylized version of Roger’s (1962) theory of innovation diffusion — early innovators correspond to optimists, while pessimists play the role of late adopters.

The literature on policy options for stimulating early adoption of technologies has tried to identify factors that promote adoption and increase its speed. Standard policies, such as preferential loans and subsidies for early adopters, have been discussed by Howden et al (2007). It has been
stressed, however, that individual producers rely strongly on their own experience and judgement, as well as on the experience of other members of their social network. Public awareness campaigns, collaboration between local authorities and farmers, training and practical demonstrations have a positive effect on adoption, Lee (2005), Bandiera and Rasul (2006), Yang et al. (2007). Marshall et al. (2011) study the willingness of farmers to purchase climate forecasts and to act on it. They report that 40% of the farmers would find such information useful, while the others feel that they do not have sufficient control over their resources to make use of such information.

In view of this literature, we compare two policies intended to stimulate early adoption: first, a monetary subsidy paid to early adopters of the new technology and, second, an information policy that provides farmers with additional data on the performance of the technologies. We can show that pessimists, the conservative agents in our model, would be willing to pay for additional information if it (i) reflects the performance of the technologies, (ii) is sufficiently relevant (i.e., generated in conditions sufficiently similar to the conditions in the region under consideration), and (iii) is sufficiently precise (consists of sufficiently many observations). In contrast, a subsidy sufficient to induce pessimists to adopt the new technology may be prohibitively expensive, i.e., exceed the expected additional revenue obtained from switching to the more profitable technology.

1.2 Organization of the paper

The next section presents the model of technology adoption. Section 3 describes the behavior of optimists and pessimists. In Section 4, we derive the steady states of the system, discuss the dynamics of the system and the issues of convergence as well as the optimality of the steady state allocations. Section 5 studies the implications of two policy options: (i) subsidies for early adopters and (ii) provision of additional information. Section 6 provides some concluding remarks. All proofs, and a more detailed analysis of the dynamics, can be found in the Appendix.
2 The model

We consider a small economy where the farmers of a (finite) community $I$ have to choose which crop variety to plant \((a\ technology)\) from the set $A$, $A = \{a_O; a_N\}$. The action $a_O$ can be viewed as the choice of a traditional technology, \("old\" crop variety), while $a_N$ denotes an alternative technology, \("new\" crop variety). In each period, the output is either a monetary success $\bar{r}$ or a failure $r$, $R = \{\bar{r}; r\}$.

Decisions are based on observations of cases $c = (a, r) \in A \times R$ which inform the decision maker about the outcome $r$ obtained by the technology $a$ in the past. In order to avoid problems of asymmetric information, we will assume that the data set of the economy $D = ((a_i^t, r_i^t)_{i \in I})_{t \in T}$ is common knowledge. Hence, all cases $c_j^t = (a_j^t, r_j^t)$ generated by activities of any other player $j \in I$ in any past period $t \in T$ will represent data available to all players. Thus, after $T$ periods, the length of the available data set is given by $I \times T$.

Two special assumptions will simplify our analysis. Let $q_k \in [0, 1]$ denote the probability of a success $\bar{r}$ if technology $a_k$ is used, where $k \in \{N, O\}$.

**Assumption 1** We assume that $q_N = 1 - q_O =: q$ and consider two regimes:

(i) in the first regime, the old technology is more successful than the new one, i.e., $q_N < q_O$ and, thus, $q < \frac{1}{2}$, whereas

(ii) in the second regime, the new technology outperforms the old one, i.e., $q_N > q_O$ and, thus, $q > \frac{1}{2}$.

Assumption 1 expresses the idea that as climate conditions change, i.e., a switch from regime 1 to regime 2 occurs, the performance of the traditional technology $O$ deteriorates and technology $N$ becomes objectively superior. For instance, diminishing precipitation might render a crop that requires a lot of water inferior to a crop that grows well in drier conditions. The symmetry assumption, $q_N = 1 - q_O$, is made for simplicity and allows us to model the process of returns using a single parameter $q$.

Note that agents in the population do not know the underlying probability distributions, nor the fact that a regime switch occurs. Instead, as we describe below, they rely on evidence from data when choosing a technology.
Assumption .2 We assume that the population of agents can be split into two groups, optimists $I_o$ and pessimists $I_p$ (with $I_o \cup I_p = I$ and $I_o \cap I_p = \emptyset$). We denote by $\omega := \frac{I_o}{I}$ the proportion of optimists in the population and assume further that this proportion is constant over time.

Assumption .2 captures the fact that people in the population might differ with respect to their attitude towards ambiguity. Grant, Kaji and Polak (1998, p. 234) quote the New York Times: "There are basically two types of people. There are “want-to-knowers” and there are “avoiders.” There are some people who, even in the absence of being able to alter outcomes, find information of this sort beneficial. The more they know, the more their anxiety level goes down. But there are others who cope by avoiding, who would rather stay hopeful and optimistic and not have the unanswered questions answered." To keep the model tractable, we assume that the shares of the two types are constant over time.

Due to these two assumptions, the set of exogenous parameters which drive this economy can be reduced to $(q, \omega) \in [0, 1]^2$. We next describe the decision process of the agents.

We assume that agents use the data in order to evaluate the two technologies according to the case-based decision criterion introduced in Eichberger and Guerdjikova (2011). For a given data set $D$, an agent of type $i \in \{o; p\}$ evaluates technology $a \in A$ by an $\alpha_i$-max-min expected utility:

$$V^i(a; D) = \alpha_i \max_{p \in H_a(D)} \sum_{r \in R} u(r) p(r) + (1 - \alpha_i) \min_{p \in H_a(D)} \sum_{r \in R} u(r) p(r),$$

where $\alpha_i$ is the decision maker’s degree of optimism, $(1 - \alpha_i)$ is his degree of pessimism and $H_a(D)$ is a set of probability distributions over outcomes associated with action $a$ when the information is given by a data set $D$. The set $H_a(D)$ captures the decision maker’s uncertainty about the distribution of outcomes of $a$ given the information contained in the data.

When evaluating action $a$, the decision maker assigns a weight $\alpha_i$, his degree of optimism, to the "best" (in terms of expected utility) probability distribution in the set $H_a(D)$ and a weight $(1 - \alpha_i)$, his degree of pessimism, to the "worst" probability distribution. Optimists have a degree of optimism $\alpha_i = 1$ and pessimists of $\alpha_i = 0$. According to Assumption .2, we consider
only the extreme cases,

\[ \alpha_i := \begin{cases} 1 & \text{if } i = o \\ 0 & \text{if } i = p \end{cases} . \]

To describe the set \( H_a(D) \), we introduce some notation. For a given case \( c \), \( a_c \) denotes the action chosen in case \( c \), i.e., \( a_c \) is the action for which \( c = (a_c, r) \). The set \( H_a(D) \) is given by:

\[ H_a(D) =: \left[ \gamma_T + \left( 1 - \gamma_T \right) \sum_{\{c|a_c \neq a\}} f_D(c) \right] \Delta + \left( 1 - \gamma_T \right) \sum_{r \in R} f_D(a;r) \delta_r . \]  

Here, \( \Delta \) denotes the interval \([0;1]\) and \( \delta_r \) is the probability distribution which assigns probability 1 to outcome \( r \). \( T \) is the length of the data set \( D \), \( f_D(c) \) is the frequency of observations of a given case \( c \) in the data set \( D \) and \( \gamma_T \) is the perceived degree of ambiguity due to the fact that the data set contains a limited number of cases \( T \). We assume that \( \gamma_0 = 1 \) and that \( \gamma_T \) is strictly decreasing in \( T \) with \( \lim_{T \to \infty} \gamma_T = 0 \), i.e., ambiguity is maximal whenever no information is available and decreases as data accumulate.

Without information about the return of technology \( a \), decision makers believe that the probability of success may take any value in the interval \([0;1]\). This happens either if the data set \( D \) is empty \( (T = 0) \), or, if \( D \) contains only observations of the alternative technology, i.e., \( \sum_{\{c|a_c \neq a\}} f_D(c) = 1 \) and \( f_D(a;r) = 0 \) for all \( r \in R \). In these two cases, the set of beliefs is maximal and includes all possible probability distributions in the simplex \( \Delta \). As information about a technology \( a \) accumulates, the degree of ambiguity \( \gamma_T \) decreases and beliefs converge to the empirically observed distribution of outcomes of \( a \). If the decision maker observes a very large data set containing only observations of action \( a \), he will be almost certain that the empirical distribution coincides with the actual probability distribution of returns. In this sense beliefs are based on evidence. For small and heterogeneous data sets, however, beliefs reflect the ambiguity resulting from a limited number of observations and from observations of alternative technologies.

---

4 This is a special case of the representation axiomatized in Eichberger and Guerdjikova (2011).
3 The Optimal Choice of Technology

For the set of probability distributions $H_a (D)$ given in Equation (2), one computes easily that agent $i$ will choose the technology which maximizes

$$V^i (a; D) = \left[ \gamma_T + (1 - \gamma_T) \sum_{c \neq a} f_D (c) \right] [\alpha_i u (\bar{r}) + (1 - \alpha_i) u (r)] + (1 - \gamma_T) \sum_r u (r) f_D (a; r).$$

Since we consider only two outcomes, $\bar{r}$ and $r$, one can normalize the utility function so that $u (\bar{r}) = 1$ and $u (r) = 0$ without any loss of generality. For a given data set $D$ and a given type of agents, $i \in \{o; p\}$, the utility derived from technology $a_O$ can be written as

$$V^i (a_O; D) = \gamma_T \alpha_i + (1 - \gamma_T) \left[ f_D (a_O; \bar{r}) + \alpha_i [f_D (a_N; \bar{r}) + f_D (a_N; r)] \right],$$

whereas the utility associated with $a_N$ is

$$V^i (a_N; D) = \gamma_T \alpha_i + (1 - \gamma_T) \left[ f_D (a_N; \bar{r}) + \alpha_i [f_D (a_O; \bar{r}) + f_D (a_O; r)] \right].$$

The choice between $a_O$ and $a_N$ will depend on the comparison of these two expressions.

**Proposition 1** For a given data set $D$, the optimal technology chosen by the optimists, $a^o (D)$, is given by

$$a^o (D) = \begin{cases} a_N & \text{iff } f_D (a_N; \bar{r}) < f_D (a_O; r) \\ \{a_O; a_N\} & \text{iff } f_D (a_N; \bar{r}) = f_D (a_O; r) \\ a_O & \text{iff } f_D (a_N; \bar{r}) > f_D (a_O; r) \end{cases}$$

and the optimal technology chosen by the pessimists, $a^p (D)$, satisfies:

$$a^p (D) = \begin{cases} a_N & \text{iff } f_D (a_N; \bar{r}) > f_D (a_O; \bar{r}) \\ \{a_N; a_O\} & \text{iff } f_D (a_N; \bar{r}) = f_D (a_O; \bar{r}) \\ a_O & \text{iff } f_D (a_N; \bar{r}) < f_D (a_O; \bar{r}) \end{cases}.$$

We note that the technology choice is independent of the total number of observations in the data set $T$ and, hence, also of ambiguity due to the total number of observations which is represented by $\gamma_T$. This appears intuitive, since ambiguity due to the size of the data set concerns both technologies. What matters for the optimal choice is the relative frequency $f_D (a_k; r)$ with which each of the alternatives $a_k$ has been observed to result in a specific outcome $r$.

In order to understand the choice behavior of optimists and pessimists, it is useful to consider
the empirically observed rates of success $\rho_k^D (\bar{r})$ and failure $\rho_k^D (r)$ for technology $a_k$:

$$
\rho_k^D (\bar{r}) = \frac{f_D (a_k; \bar{r})}{f_D (a_k; \bar{r}) + f_D (a_k; r)},
$$

$$
\rho_k^D (r) = \frac{f_D (a_k; \bar{r})}{f_D (a_k; \bar{r}) + f_D (a_k; r)} = 1 - \rho_k^D (\bar{r}),
$$

with $k \in \{O; N\}$.

The behavior of optimists and pessimists differs in the type of comparisons they use to choose among technologies. Optimists prefer the technology which has resulted in the smallest number of failures $r$. For an optimist, observing a technology yielding the high return just confirms his initial beliefs that the best possible outcome will obtain. Observation of a failure, however, represents "news" for the optimist. It makes him revise downwards the prior probability of success. Hence, an optimist assigns a higher probability of success to the technology with the smaller number of failures. In particular, if both technologies have the same empirically observed rate of failure, $\rho_O^D (\bar{r}) = \rho_N^D (\bar{r})$, optimists will prefer the action which has been observed less frequently in the data. In this sense, optimists will be prone to experimentation, choosing (ceteris paribus) the technology with the fewest observations in the data set.

In contrast, pessimists choose the technology with the most successes $\bar{r}$ in the data. Pessimists’ choices are affected only if they observe that a certain technology results in a success. Hence, if both technologies have the same empirically observed rate of success, $\rho_O^D (\bar{r}) = \rho_N^D (\bar{r})$, pessimists will prefer the one more frequently chosen in the past. This means that pessimists may opt for an established technology which has been frequently chosen in the past even if its success rate is lower than that of a new technology with few observations. Thus, pessimists exhibit conservative behavior, preferring technologies for which an ample amount of evidence is available.

4 Dynamics and Steady States

The model presented in the previous sections induces natural dynamics of decision making and data generation. The observed data determine the population’s choice of technologies. In turn, agents’ choices, together with the exogenous process of returns, drive the data generation.
process.

More formally, suppose that at a given point in time $T$ the previous choices of technology and their results have been recorded in a data set $D$. These data determine the choices of optimists and pessimists, $a^o(D)$ and $a^p(D)$ at time $(T + 1)$. The technologies chosen at time $(T + 1)$ together with their outcomes form a new set of cases which augments the initial data set $D$. Since the outcome realizations are random, it follows that both the data set and the choices in a given period are stochastic and endogenous. Hence, investigating the evolution of the system means studying a stochastic process. Rather than considering the stochastic development of the system, in this section, we will focus on its long-run behavior.

### 4.1 Steady states

At any point in time, a state of the dynamic system is characterized by the proportions of optimists and pessimists who have adopted the new technology, $\phi_o^N$ and $\phi_p^N$, respectively. The associated proportions choosing the old technology are $\phi_o^o = 1 - \phi_o^N$ and $\phi_p^o = 1 - \phi_p^N$. We will be looking for steady states of the system.

A steady state is described by the proportions of optimists and pessimists holding each of the two technologies, $((\phi_o^o; \phi_o^N); (\phi_p^o; \phi_p^N))$. These proportions define, combined with the process of returns, the long-run frequencies with which each of the four possible cases is observed in the data. Since returns are independently distributed across time and are also independent of individuals’ choices, we can express the frequencies of observations in terms of the proportions $\phi_o^N$ and $\phi_p^N$. According to the LLN, the frequency of cases, e.g., of $(a_N; \bar{r})$, will almost surely be given by the product of the probability with which $a_N$ results in $\bar{r}$ and the proportion of the population choosing $a_N$. The latter is the sum of the proportions of each type of agent choosing $a_N$, $\phi_o^N$ and $\phi_p^N$, weighted by their shares, $\omega$ and $(1 - \omega)$. Thus, one obtains:

$$f (a_N; \bar{r}) = [\omega \phi_o^N + (1 - \omega) \phi_p^N] q,$$

$$f (a_N; r) = [\omega \phi_o^N + (1 - \omega) \phi_p^N] (1 - q),$$

$$f (a_O; \bar{r}) = [\omega (1 - \phi_o^N) + (1 - \omega) (1 - \phi_p^N)] (1 - q),$$

$$f (a_O; r) = [\omega (1 - \phi_o^N) + (1 - \omega) (1 - \phi_p^N)] q.$$
Given these long-run frequencies of observations, one can check whether a given combination of choice proportions is optimal. Assuming that all individuals in the society follow their respective choice rules and aggregating across individuals yields

\[
\phi^p_N = \begin{cases} 
1 & \text{iff } f(a_N; \bar{r}) > f(a_O; \bar{r}) \\
[0;1] & \text{iff } f(a_N; \bar{r}) = f(a_O; \bar{r}) \\
0 & \text{iff } f(a_N; \bar{r}) < f(a_O; \bar{r})
\end{cases}
\]

and

\[
\phi^o_N = \begin{cases} 
1 & \text{iff } f(a_N; \bar{r}) < f(a_O; \bar{r}) \\
[0;1] & \text{iff } f(a_N; \bar{r}) = f(a_O; \bar{r}) \\
0 & \text{iff } f(a_N; \bar{r}) > f(a_O; \bar{r})
\end{cases}
\]  

A steady state of the system is defined by a time-invariant proportion of optimists and pessimists adopting the new technology, \(\phi^o_N\) and \(\phi^p_N\), such that (i) the aggregate behavior of optimists and pessimists is optimal given the long-run frequencies of observations \(f(c)\), i.e., (8) is satisfied, and, (ii) the long-run frequencies of observations are consistent with the frequencies in the data as in (7). The proportions of agents choosing the old technology are \(\phi^o_O = 1 - \phi^o_N\) and \(\phi^p_O = 1 - \phi^p_N\).

**Definition 1**  
A steady state of the system is a tuple \((\phi^o_N; \phi^p_N)\) which simultaneously satisfies conditions (7) and (8).

Intuitively, a steady state has the property that, for given frequencies of observations, no agent in the society has an incentive to change his choice of technology. In turn, these optimal choices generate exactly those frequencies of observations that make them optimal. Hence, for a large number of observations such that a set of new observations has little impact on the frequencies in the data, starting from a steady state, the economy will almost surely remain in this state indefinitely.

Our next proposition lists the steady-states of the economy for different values of the parameters \(\omega\) and \(q\).

**Proposition 2**  
Depending on the values of the parameters \(\omega\) and \(q\), the following steady-states may arise:

**Case 1**  
If \(\omega > 1 - q\), the unique steady-state is given by:

\[
\phi^o_N = \frac{\omega + q - 1}{\omega}, \quad \phi^p_N = 1.
\]
Case 2 If $\omega \leq 1 - q < q$, the system has three steady states:

1. $\phi_N^o = 0$, $\phi_N^{sp} = 1$;
2. $\phi_N^o = 1$, $\phi_N^{sp} = \frac{1-\omega-q}{1-\omega}$;
3. $\phi_N^o = 1$, $\phi_N^{sp} = 0$.

Depending on the parameter values $\omega$ and $q$, there are two possible equilibrium constellations. If the share of optimists in the society is higher than the probability of failure for the new technology, $\omega > 1 - q$, then the economy has a unique steady state in which all pessimists choose the new technology, whereas only a subset of the optimists holds the new technology. Note that the new technology is on average more profitable than the old one. Moreover, the new technology is chosen by a larger fraction of the population than the old one. Since the new technology is chosen more often and is more successful than the old one, all pessimists choose it. On the other hand, optimists choose both technologies with positive probability. If the old technology were never chosen, $f(a_O;r) < f(a_N;r)$ would obtain and all optimists would prefer the old technology. The result that a positive share of the optimists switches back to the old technology is due to the fact that their "overly-optimistic" expectations eventually make them dissatisfied with any available alternative and induce them to choose the technology which has been observed least frequently in the past. This behavior is an artifact of our simplifying assumption of just two technologies. If we would allow for constant arrival of new technologies, an admittedly more realistic assumption, then, instead of reverting to the old technology, optimists would switch to the newest available one, for which no or few observations are available. Such a modification would re-enforce the role of optimists as market leaders, always "reaching for the sky", trying out the newest technologies, and providing valuable information for the rest of the population.

In the second case, when the share of optimists in the society is relatively low, $\omega \leq 1 - q$, the economy has three possible steady states. The steady state in Case 2a is a limiting case of Case 1, $\omega = 1 - q$. In this case, all pessimists choose the new technology and all optimists, the old one. The proportion of optimists choosing the new technology in the steady state of Case 1 has disappeared. The two steady states characterized in Case 2b and Case 2c are qualitatively
different. In both cases, all optimists adopt the new technology and either all, Case 2c, or a positive proportion, Case 2b, of the pessimists retain the old technology. These steady states rely on the fact that the proportion of optimists, i.e., those people adopting the new technology and experiencing the better return rate $q$, is low relative to the success rate of the old technology, $1 - q$. These cases illustrate that pessimism may prevent the introduction of a better new technology when optimists fail to generate a sufficient amount of evidence for its superiority.

4.2 Stability of Steady States

In this section, we study the stability of the steady states. In Case 1, i.e., when the share of optimists in the population $\omega$ exceeds $1 - q$, the system almost surely converges to the unique steady state.

**Proposition 3** Suppose that $\omega > 1 - q$. Then the system almost surely converges to the steady state

$$(\phi^*_N; \phi^*_N) = \left(\frac{\omega + q}{\omega} - 1; 1\right).$$

According to Proposition 3, convergence to the steady state occurs regardless of the initial state of the system, provided the share of optimists is sufficiently high. The system will approach the equilibrium even if the new technology has never been adopted before. This property captures an essential feature of technology adoption.

Since the new technology has never been applied before in this community, optimists strictly prefer it to the traditional one, whereas all pessimists continue to use the old technology. Since the share of optimists in the population is relatively high, they will eventually generate sufficiently many observations of the new technology being successful. As information about successful applications of the new technology accumulates, pessimists learn about the superiority of the new technology and begin using it as well. The optimists, who were the initial adopters, will eventually split up into two groups, one staying with the new technology and another switching back to the old one, in proportions indicated in Proposition 3. In the steady state, the proportion of people adopting the new technology will be such that the data about success and failure of the technologies generated by the population will make the optimists indifferent.
between the two technologies\(^5\).

We now turn to the analysis of stability in Case 2, when the share of optimists in the population is relatively small. In this case, the system has three steady states. Which steady-state is reached by the system will in general depend on the initial conditions. The initial conditions are characterized by the proportion of optimists and pessimists who have adopted the new technology in the past and by the data set available at this stage.

A complete analysis of the dynamics from arbitrary initial conditions is beyond the scope of this paper. Moreover, given the application to technology adoption in the wake of a climate change, we will focus on situations where the new technology has never been chosen in the past. In order to characterize the data set available in the initial period \(T\), we will assume that

\begin{itemize}
  \item[(i)] no observations of the new technology are available, i.e., \(f_T(a_N; r) = 0\) for all \(r \in R\), and
  \item[(ii)] the regime shift has occurred in the past at \(T_0 < T\).
\end{itemize}

In periods up to \(T_0\), the old technology had been successful, yielding the high outcome \(\bar{r}\) with probability \(q\). After the climate change took place in \(T_0\), the probability of a good result \(\bar{r}\) from the old technology has fallen to \((1 - q)\). Denoting by \(T_1 = T - T_0\) the number of periods after the climate change, and assuming a sufficiently long history with the old technology before and after the climate change, the relative frequencies of cases in the initial period \(T+1\) are

\[
\begin{align*}
  f_{T+1}(a_O; \bar{r}) & = \frac{T_0 q + T_1 (1 - q)}{T_0 q + T_1}, \\
  f_{T+1}(a_O; \bar{r}) & = \frac{T_0 (1 - q) + T_1 q}{T_0 + T_1}, \\
  f_{T+1}(a_N; \bar{r}) & = f_{T+1}(a_N; \bar{r}) = 0.
\end{align*}
\]

Our first result concerns the worst steady state \((\phi^o_N = 1, \phi^p_N = 0)\) where all pessimists hold on to the old technology and only the small proportion of optimists adopts the new technology.

**Proposition 4** Suppose that at \(T+1\), the new technology has never been chosen in the past. Furthermore, let the frequencies of observations satisfy condition (9). Then, for each \(\xi > 0\), there exists a critical minimal proportion of optimists \(\bar{\omega} \in (0; 1 - q]\) such that for all \(\omega < \bar{\omega}\), the system will converge to the steady state \((\phi^o_N = 1, \phi^p_N = 0)\) with probability of at least \((1 - \xi)\).

Hence, after a long history of experience with the old technology both before and after the

\(^5\) Recall, that there are only two technologies. Hence, in a situation where the new technology has been widely adopted, optimists have no other option than experimenting with the “old technology”.
climate change, if the share of optimists in the society, $\omega$, is sufficiently small, then, with a high probability, only the small group of optimists will hold the new technology, while all pessimists will retain the old one.

For our next result we assume that the new technology arrives after the old technology has been exclusively used for $T$ periods. In the initial period, the available data set records a strictly positive frequency of high outcomes from the old technology, i.e.,

$$
f_{T+1}(a_O; \bar{r}) > 0, 

f_{T+1}(a_N; \bar{r}) = f_{T+1}(a_N; \bar{r}) = 0. 
\tag{10}
$$

Given these assumptions, we can show that one cannot expect the pessimists to adopt the new technology in finite time.

**Proposition 5** Suppose that at time $T+1$, the new technology has never been chosen in the past and that condition (10) is satisfied. Let $T^*$ denote the first period in which the new technology is adopted by pessimists. Then, for $\omega < 1 - q$, the expected value of $T^*$ is infinite.

Proposition 5 does not exclude the possibility that pessimists adopt the new technology. Unless the share of optimists in the society is sufficiently high (higher than $(1 - q)$), however, there is a positive probability that this will never happen and that the system will reach a steady state in which only optimists adopt the new technology.

### 4.3 Optimality of Steady States

In this section, we will discuss welfare properties of the steady states.

The average payoff of the society in a steady state $(\phi_{N}^{*o}; \phi_{N}^{*p})$ is given by:

$$
U(\phi_{N}^{*o}; \phi_{N}^{*p}) = 
[(\omega\phi_{N}^{*o} + (1 - \omega) \phi_{N}^{*p}) q + (\omega (1 - \phi_{N}^{*o}) + (1 - \omega) (1 - \phi_{N}^{*p})) (1 - q)] \bar{r}

+ [(\omega\phi_{N}^{*o} + (1 - \omega) \phi_{N}^{*p}) (1 - q) + (\omega (1 - \phi_{N}^{*o}) + (1 - \omega) (1 - \phi_{N}^{*p})) q] \bar{r}.
$$

The following proposition derives the average payoffs (per individual) obtained in the steady states.

**Proposition 6** Average payoffs obtained in the steady states:
Case 1: For $\omega > 1 - q$, the average payoff in the unique steady-state is given by:

$$U \left( \frac{\omega + q - 1}{\omega}; 1 \right) = (2q^2 - 2q + 1) \bar{r} + (2q - 2q^2) r.$$ 

Case 2: For $\omega \leq 1 - q < q$, the average payoffs in the three steady states is given by:

a) $U (0; 1) = (q + \omega - 2\omega q) \bar{r} + (1 - q - \omega + 2\omega q) r$;

b) $U \left( 1; \frac{1 - \omega - q}{1 - \omega} \right) = 2q (1 - q) \bar{r} + (1 - 2q + 2q^2) r$;

c) $U (1; 0) = (2\omega q + 1 - \omega - q) \bar{r} + (\omega + q - 2\omega q) r$.

From a welfare point of view, the steady states in Case 2 can be ordered in terms of their average payoffs: Case 2a yields the highest average utility and Case 2c the worst.

Examining how the average payoff of these steady states varies with the share of optimists, $\omega$, it is easily seen that, in Case 2a, $\phi_N^o = 0, \phi_N^p = 1$, the average payoff is maximized for $\omega = 0$. This corresponds to a situation, in which the society is composed entirely of pessimists, all of whom choose the new technology. After the climate change, the new technology is assumed to be superior to the old one. Since optimists choose the inferior old technology in Case 2a, welfare is improved if there are no optimists, $\omega = 0$.

We know from Proposition 4, however, that for values of $\omega$ close to 0 and an initial state where only the old technology is used, the system converges with probability close to 1 to the steady state with the lowest average payoff, Case 2c. Hence, $\omega = 0$ cannot be optimal when the dynamics of the system are taken into account.

The unique steady state in Case 1 dominates both Cases 2b and 2c in terms of average payoff. For these cases, the optimal proportion of optimists in society can be computed comparing the average payoff obtained in Case 1 to the expected average payoff in Case 2c for values of $\omega \leq 1 - q$, where the expectation is taken with respect to the probabilities with which each of the three possible equilibria is reached. Although these probabilities are hard to compute, the analysis so far allows us to reach the conclusion that the optimal share of optimists in the society will be strictly above 0.

Optimists play an important role in the adoption process: they are the driving force of innovation, they experiment with new technologies and provide the public good of information to the conservative pessimists. However, their desire to experiment eventually forces them to switch
back to the old technology and leads to suboptimal outcomes. Hence, increasing the share of optimists above the level of \((1 - q)\) will not change the average payoff of the population. In contrast, pessimists are slow to adopt new technologies, but having discovered that the new technology is superior to the traditional one, they will continue using it.

In a more realistic model, in which innovation activities ensure that new technologies arrive continuously, the different behavior of optimists and pessimists will ensure that new technologies are sampled sufficiently often by optimists, so as to reveal their expected returns, while the pessimists choose the technology that has so far been credibly shown to perform best. This implies that both types of agents have to be present in a society in order to guarantee successful adoption of new technologies in the long run.

5 Policies Designed to Stimulate Adoption

The analysis of the previous section describes scenarios in which successful adoption may fail. When the share of optimists in the society is too low, the economy converges to the worst possible steady state, in which only optimists choose the new technology, while the majority of the population consisting of pessimists opts for the traditional method of farming. Following Ellsberg’s (1961) seminal paper, experimental evidence shows that a majority of people (more than 60%) dislike making choices under ambiguity due to a lack of information. This attitude is associated with pessimism in our framework. Given that ambiguity aversion is a wide-spread phenomenon, adoption of new technologies may be slow and intervention necessary in order to ensure a socially optimal allocation.

In this section, we will explore the effects of two policy measures which have been suggested in the literature as means to stimulate the adoption of new technologies: (i) subsidies for early adopters and (ii) additional information about the performance of the new technologies.

We will focus on Case 2, i.e., \(\omega < 1 - q\), in which regulation can influence which of the three steady states the economy will attain. We will concentrate on the initial period of the economy, after the climate change has occurred and the new technology has become available. In this
situation, without government intervention, all optimists in the population will choose the new technology, while pessimists will continue to choose the traditional farming method. Hence, in order to increase public welfare, the government would be interested in providing incentives for the pessimists to adopt the new technology earlier and to promote the socially optimal steady state.

For assessing the two interventions, we first identify the necessary conditions for the pessimists to choose the new technology. In a second step, we compare the increase in the average payoff of the society due to the policy with the cost of the policy in order to determine whether the policy will have positive net benefits.

To simplify the analysis, we assume a specific functional form for the parameter of perceived ambiguity, $\gamma_T$.

**Assumption .3** The parameter of perceived ambiguity $\gamma_T$ satisfies: $\gamma_T = \frac{1}{\tau+1}$.

Assumption .3 satisfies the conditions assumed in Section 2. For exploring the potential of government intervention improving welfare, it is without loss of generality.

### 5.1 Subsidizing Early Adoption

We begin with the case of a subsidy for early adoption. Assume that the data set $D$ consists only of observations of the traditional technology. This would be the case if the government intervention takes place in the first period in which the new technology becomes available\(^6\).

Assuming further that climate change took place before the new technology becomes available, the realized frequency of success of the old technology in the initial data set $D$ will satisfy $f_D(a_O; \bar{r}) \in (1-q; q)$.

Suppose that the government cannot distinguish optimists from pessimists and, hence, has to pay a monetary subsidy $G$ to all early adopters of the new technology. In order to induce pessimists to adopt the new technology, it has to compensate them for their loss in perceived

---

\(^6\) This case constitutes the worst possible scenario. The subsidy will be smaller if observations of the new technology were already available, i.e., if the government decided to subsidize the new technology several periods after it becomes available. This does not change the validity of our results, as long as the government wishes to implement the policy relatively soon after the new technology has been introduced.
utility due to switching from the old to the new technology:

\[ V^p (a_O; D) - V^p (a_N; D) = (1 - \gamma_T) [f_D (a_O; \bar{r}) - f_D (a_N; \bar{r})] = \left(1 - \frac{1}{T+1}\right) f_D (a_O; \bar{r}). \]

Hence, the subsidy \( G \) in period \( T + 1 \) has to be chosen so that

\[
u(G) = \left(1 - \frac{1}{T+1}\right) f_D (a_O; \bar{r}). \tag{11}\]

For convenience of computation, assume that the utility function \( u \) is linear\(^7\), i.e., \( u(r) = r \) for \( r \in R \). Then, \( G = \left(1 - \frac{1}{T+1}\right) f_D (a_O; \bar{r}) \). If the government cannot distinguish between optimists and pessimists, \( G \) will be also the average subsidy paid to the population.

The average expected increase in returns from pessimists adopting the new technology is

\[(1 - \omega) (q - (1 - q)) = (1 - \omega) (2q - 1),\]

since the subsidy only changes the behavior of the pessimists whose share in the population is \( (1 - \omega) \).

**Proposition 7** Suppose that \( u(r) = r \) for all \( r \in R \).

(i) A subsidy of \( G \) will induce the pessimists to adopt the new technology only if

\[ G \geq \left(1 - \frac{1}{T+1}\right) f_D (a_O; \bar{r}). \]

(ii) The average increase in returns generated by this policy is given by

\[(1 - \omega) (2q - 1).\]

(iii) There is an open set of parameter values \((\omega; q; T)\) for which the required subsidy exceeds the average increase in returns it induces, i.e.,

\[
\left(1 - \frac{1}{T+1}\right) f_D (a_O; \bar{r}) > (1 - \omega) (2q - 1). \tag{12}\]

Proposition 7 shows that a policy which subsidizes early adopters may be prohibitively costly, especially if the government cannot distinguish between the two types of agents. The reason for this is that the government has to compensate the pessimistic agents for the ambiguity they would bear when choosing the new technology. For some parameter values, this premium will be higher than the expected increase in returns obtained from choosing the more profitable farming method. Moreover, in order to ensure convergence to the optimal steady state, the gov-

---

\(^7\) The assumption that \( u(\cdot) \) is linear can be relaxed in favor of low levels of risk-aversion, without affecting the result of the proposition.
ernment will have to pay the subsidy until the data-set generated by the agents in the population correctly reflects the probabilities of success for the two technologies.

We next consider a variation of this policy — income subsidies. This is an alternative subsidy scheme, which consists in an income guarantee in case of a bad outcome. Suppose individuals adopting the new technology will be guaranteed a minimal income of \( \tilde{r} \in (\underline{r}; \bar{r}) \) if the new technology pays off \( \bar{r} \). No subsidy is paid if the outcome of the new technology is \( \bar{r} \), or if the individual chooses the old technology. Evaluating the new technology, pessimists now consider the worst case scenario to be \( \tilde{r} \), whereas, the worst outcome of the old technology continues to be \( \underline{r} \). As before, assuming \( u(r) = r \), we obtain the pessimists’ evaluations for the two technologies:

\[
V^p(a_O; D) = (1 - \gamma_T) f_D(a_O; \bar{r})
\]
\[
V^p(a_N; D) = [\gamma_T + (1 - \gamma_T) [f_D(a_O; \bar{r}) + f_D(a_O; \underline{r})]] u(\tilde{r}) = \tilde{r}.
\]

Pessimists will prefer the new technology, whenever

\[
V^p(a_N; D) - V^p(a_O; D) = \tilde{r} - (1 - \gamma_T) f_D(a_O; \bar{r}) \geq 0,
\]

or

\[
\tilde{r} \geq \left(1 - \frac{1}{T + 1}\right) f_D(a_O; \bar{r}).
\]

The decision of the optimists remains unchanged by the introduction of the subsidy.

After the climate change, the probability of the new technology obtaining a result \( \underline{r} \) is \( (1 - q) \). Hence, the expected subsidy paid by the government will be \( \tilde{r}(1 - q) \).

**Proposition 8** Suppose that \( u(r) = r \) for all \( r \in R \).

(i) An income subsidy of \( \tilde{r} \) will induce the pessimists to adopt the new technology only if

\[
\tilde{r} \geq \left(1 - \frac{1}{T + 1}\right) f_D(a_O; \bar{r}).
\]

(ii) The average increase in returns generated by this policy is given by

\[
(1 - \omega)(2q - 1).
\]

(iii) There is an open set of parameter values \( (\omega; q; T) \) for which the required subsidy exceeds the average increase in returns it induces, i.e.,

\[
(1 - q) \left(1 - \frac{1}{T + 1}\right) f_D(a_O; \bar{r}) > (1 - \omega)(2q - 1).
\]  

(13) We are greatful to Nicholas Vonortas for suggesting this variation.
The results derived in this section demonstrate that subsidies may not be cost-effective in inducing early adoption of a technology if the individuals perceive ambiguity and differ in their attitudes towards it.

5.2 Providing Additional Information

A second method to induce agents to adopt a new technology, which has been advanced in the literature, consists of the provision of additional information about the technology. Such information is usually provided in aggregate form, such as case studies, expert opinions, or practical advice. Staying within the framework of this paper, we will assume that the government can provide additional information directly in the form of raw data. The data may be obtained from controlled experiments, practical demonstrations, or outcome distributions of the technologies gathered at a different locations.

Assume that, in addition to the freely available information from past choices of the population, the government can offer access to an additional data set \( \tilde{D} \). We suppose that the frequencies of cases in the additional data set \( \tilde{D} \) are representative of the performance of the two technologies. This is an assumption about the quality of the additional data provided by the government.

**Assumption 4.** The data set \( \tilde{D} \) is of length \( \tilde{T} \) and contains the following frequencies of cases:

- **old technology:** \( f_{\tilde{D}}(a_O;\tilde{r}) = \frac{(1-q)^2}{2} \) and \( f_{\tilde{D}}(a_O;\tilde{r}) = \frac{q^2}{2} \),
- **new technology:** \( f_{\tilde{D}}(a_N;\tilde{r}) = \frac{q^2}{2} \) and \( f_{\tilde{D}}(a_N;\tilde{r}) = \frac{(1-q)^2}{2} \).

An agent who obtains the additional information contained in \( \tilde{D} \) combines this information with his own observations in data set \( D \). He predicts a set of probability distributions based on the combined data set \( D \cup \tilde{D} \), as in (2), where \( T + \tilde{T} \) is the length of the combined data set. We will assume, however, that the agents perceive a difference between the information provided by the government, \( \tilde{D} \), and the information in \( D \) obtained by personal experience. This may reflect that agents do not trust the government completely, or that the additional data refer to a location which is similar, but not identical to the one under consideration, or that the data were obtained from an experiment conducted by scientists who are more skilled than the agents themselves.

In order to model this difference in weighting the evidence from the data sets \( D \) and \( \tilde{D} \), the
additional data will be discounted by a factor $\tilde{s} \in (0; 1)$, while cases obtained from personal experience are assigned a relevance factor of 1.

**Assumption .5** The relevance of cases in the data set $\tilde{D}$ is discounted with $\tilde{s} \in (0; 1)$, whereas the relevance of all cases in the data set $D$ is weighted by 1.

Given Assumption .5, one obtains the following modified payoff function:

$$\tilde{V}_i (a; D \cup \tilde{D})$$

$$= \left( \gamma_{T+\tilde{T}} + (1 - \gamma_{T+\tilde{T}}) \left[ \frac{T \sum_{\{c|a_c \neq a\}} f_D (c) + \tilde{s}\tilde{T} \sum_{\{c|a_c \neq a\}} f_{\tilde{D}} (c)}{T + \tilde{s}\tilde{T}} \right] \right) \cdot \left[ \alpha_i u (\bar{r}) + (1 - \alpha_i) u (\bar{r}) \right] + \left( 1 - \gamma_{T+\tilde{T}} \right) \sum_{r \in R} u (r) \frac{T f_D (a; r) + \tilde{s}\tilde{T} f_{\tilde{D}} (a; r)}{T + \tilde{s}\tilde{T}}.$$  \hspace{1cm} (14)

The payoff function $\tilde{V}_i$ can be viewed as a weighted average of the evaluation of the technology $a$ with respect to the data sets $D$ and $\tilde{D}$, respectively. The weights $T$ and $\tilde{s}\tilde{T}$ reflect the fact that first, the two data sets may differ in length and hence, the evidence from the longer data set should be given more consideration, and, second, that information provided by the government is considered less relevant or reliable and thus is discounted by a factor $\tilde{s}$. Note that the degree of perceived ambiguity of the data set $D \cup \tilde{D}$ is determined by its entire length, $T + \tilde{T}$. Hence, as the government provides more information, it reduces the ambiguity perceived by the agents in the population.

The following proposition derives conditions, which guarantee that (i) pessimists will switch from the old to the new technology when informed by the combined data set $D \cup \tilde{D}$ rather than by $D$ alone, and (ii) the perceived utility derived from choosing the new technology given $D \cup \tilde{D}$ will be higher than the utility from choosing the old technology given $D$. The first condition means that the information policy is successful, while the second condition implies that pessimists would be willing to pay a positive amount of money in order to obtain the additional information.

**Proposition 9** Given Assumptions .4 and .5, a pessimist
(i) will choose \( a_O \) given \( D \) and \( a_N \) given \( D \cup \tilde{D} \) if

\[
\frac{s\tilde{T}}{2} (2q - 1) > T [f_D (a_O; \bar{r}) - f_D (a_N; \bar{r})] > 0
\]

and

(ii) will be willing to pay for the additional information in \( \tilde{D} \) if

\[
\frac{s\tilde{T}}{2} > \frac{T [f_D (a_O; \bar{r}) - f_D (a_N; \bar{r})]}{[q - f_D (a_O; \bar{r})]}
\]

Proposition 9 provides conditions for a self-financing policy to provide additional information. Indeed, if conditions (i) and (ii) are satisfied, the government can sell the data-set \( \tilde{D} \) to pessimists at a positive price, and, thus, implement adoption of the new technology by the pessimists at no cost to the public. Observe that conditions (i) and (ii) are more likely to be satisfied if the relevance \( \tilde{s} \) and the precision \( \tilde{T} \) of the additional information increase.

**Proposition 10** Given Assumptions .4 and .5, an optimist will

(i) choose \( a_N \) given \( D \) and \( a_N \) given \( D \cup \tilde{D} \) if

\[
[f_D (a_N; \bar{r}) - f_D (a_O; \bar{r})] < 0
\]

and

(ii) will refuse to pay for the additional information in \( \tilde{D} \) if \( f_D (a_N; \bar{r}) \) is sufficiently small.

Proposition 10 indicates the circumstances under which the choice of optimists will remain unaffected by the additional information provided by the government. Note also that optimists will not wish to obtain the additional information in \( \tilde{D} \) if the data set \( D \) contains no observations of the new technology. Intuitively, acquiring more evidence about the new technology means that the optimists, who without evidence about the new technology evaluate it favorably, may learn about bad outcomes from the new technology, which would reduce their evaluation of the new technology.

In summary, providing more information can be an effective policy for faster adoption of new technologies: it will not affect the (already optimal) choice of optimists, but will induce pessimists to adopt the new technology. Furthermore, if the data set \( \tilde{D} \) is sufficiently large, the government will have to provide information only once. Thereafter, observations generated by
the population will confirm the frequencies observed in $\tilde{D}$. Thus, pessimists will continue to choose the new technology in each period.

6 Conclusion

We presented a model of technology adoption in response to a change in climate conditions. The main feature of the model is that the returns of new technologies are not just risky, but also ambiguous. Pessimistic agents may be averse to trying out a new technology, even though its observed performance is superior to that of a traditional technology. Data generation by optimists, who are willing to try out technologies about which little evidence is available, induces learning by the pessimists. Thus, optimists provide the public good of information about a new technology which, in turn, creates incentives for the pessimists to adopt it, provided that it has generated satisfactory outcomes. In this sense, the presence of optimists helps to solve the problem of ambiguity. While optimists are crucial to induce the learning process, pessimists choose the new technology persistently and guarantee high average returns in the long-run. If the share of optimists in the population is small, the process of technology adoption may be slow and pessimists may never choose the new technology.

We study two policies which can alleviate this inefficiency: the government could either subsidize early adoption of the new technology until sufficient information is available so that everyone else wants to adopt it, or provide more information about the new technologies. We identify when a subsidy might be prohibitively expensive and show that pessimists will be willing to pay for more precise and relevant information, Hence, providing information can be a more efficient, self-financing policy resulting in better social outcomes.

Several important problems are beyond the scope of this paper. First, we assume that the shares of optimists and pessimists in the population are exogenously given and remain constant over time. Note, however, that in the socially optimal equilibrium, some of the optimists switch back to the inefficient technology and, thus obtain lower average revenues than the pessimists. In an evolutionary model, in which the shares of optimists and pessimists change in response
to past performance, we would observe that the share of optimists declines over time. If only two alternatives are available, this might lead to a socially optimal allocation over time, as optimists choosing the old technology disappear and only the part of the population using the new technology survives.

In a more realistic model, in which new technologies arrive continuously, optimists who bear the risk of testing new technologies might also earn lower returns than pessimists who adopt a technology only if they are certain of its superior performance. Furthermore, migration could also play a role: since optimists prefer ambiguous alternatives, they are more likely to migrate to different locations, which further reenforces the decline in the share of optimists in the society.

As shown in the paper, this may prevent society from adopting new technologies. It would also increase the need for governmental policies that alleviate the problem of pessimism in the population.

Another important assumption of our model is the reversibility of adoption decisions. Assuming that adoption decisions are irreversible, would prevent optimists from switching back and forth between the old and the new technology. Hence, in the case in which adoption is irreversible, the entire population will successfully adapt to climate change provided that the share of optimists in the society is sufficiently large. Note that if the share of optimists is low, the irreversibility assumption would not change our main conclusion that only optimists will adopt the new technology.

In this paper, we also neglected the impact of social networks on learning and attitudes towards ambiguity. Case-based decision theory provides tools to model distance between agents in the population by a similarity function. A similarity function depends on the individual characteristics of the agent and his adherence to different social groups and communities. The more similar two agents are, the more relevant the experience of one for the decisions of the other. Studying the dynamics of technology adoption for different patterns of social networks is a question of foremost interest.

Beyond these modelling questions, more research is needed to understand the institutions which
determine the choice of technology in traditional societies. Increased efforts in education for rural communities, better dissemination of research results, direct collaboration with scientists, and adaptation of successful technologies to regional requirements need to be studied in this context. The impact of these measures on the adoption of new technologies and the adaptation to climate change deserves more research.

Finally, our paper points to the importance of information provision in the process of optimal adaptation to climate change. Collecting detailed and precise information about the environment may be just as important as collecting data on the performance of the economy. Too little investment in collecting such information may prevent timely successful innovation and have irreversible consequences for societies which depend on climate conditions for their survival.

7 Appendix

Proof of Proposition 1:

The choice of an individual of type $i$ depends on the sign of the expression $V^i(a_O; D) - V^i(a_N; D)$. Substituting into expressions (3) and (4), we obtain:

$$V^i(a_O; D) - V^i(a_N; D) = \gamma_T \alpha_i + (1 - \gamma_T) [f_D(a_O; \bar{r}) + \alpha_i [f_D(a_N; \bar{r}) + f_D(a_N; \bar{r})]$$

$$-\gamma_T \alpha_i - (1 - \gamma_T) [f_D(a_N; \bar{r}) + \alpha_i [f_D(a_O; \bar{r}) + f_D(a_O; \bar{r})]]$$

$$= (1 - \gamma_T) [f_D(a_O; \bar{r}) + \alpha_i [f_D(a_N; \bar{r}) + f_D(a_N; \bar{r})]] - (1 - \gamma_T) [f_D(a_N; \bar{r}) + \alpha_i [f_D(a_O; \bar{r}) + f_D(a_O; \bar{r})]]$$

$$= (1 - \gamma_T) [f_D(a_O; \bar{r}) - f_D(a_N; \bar{r}) + \alpha_i [f_D(a_N; \bar{r}) + f_D(a_N; \bar{r})] - f_D(a_O; \bar{r}) - f_D(a_O; \bar{r})]$$

Hence, $a_O$ is chosen if and only if:

$$(1 - \alpha_i) [f_D(a_O; \bar{r}) - f_D(a_N; \bar{r})] + \alpha_i [f_D(a_N; \bar{r}) - f_D(a_O; \bar{r})] \geq 0.$$ 

Substituting $\alpha_i = 1$, we obtain condition (5), whereas setting $\alpha_i = 0$, gives the optimality condition for pessimists, (6). $\blacksquare$

Proof of Propositions 2 and 6:

We prove the claim of Proposition 2 by considering all possible constellations of $\phi_N^o$ and $\phi_N^p$ and eliminating those which cannot constitute a steady state. For each of the steady states, we
then compute the average payoff as defined in the statement of Proposition 6.

(i) Let $\phi_{N}^{o} = 1$, $\phi_{N}^{p} = 1$. This requires that:

\[
\begin{align*}
&f(a_{N}; \bar{r}) < f(a_{O}; \bar{r}) \\
&[\omega \phi_{N}^{o} + (1 - \omega) \phi_{N}^{p}] (1 - q) < [\omega (1 - \phi_{N}^{o}) + (1 - \omega) (1 - \phi_{N}^{p})] q
\end{align*}
\]

which is excluded by the assumption $q < 1$.

(ii) Let $\phi_{N}^{o} = 0$, $\phi_{N}^{p} = 0$. This requires that:

\[
\begin{align*}
&f(a_{N}; \bar{r}) > f(a_{O}; \bar{r}) \\
&[\omega \phi_{N}^{o} + (1 - \omega) \phi_{N}^{p}] (1 - q) > [\omega (1 - \phi_{N}^{o}) + (1 - \omega) (1 - \phi_{N}^{p})] (1 - q)
\end{align*}
\]

which is excluded by $q > 0$.

(iii) Let $\phi_{N}^{o} \in [0; 1]$, $\phi_{N}^{p} = 1$. This requires that:

\[
\begin{align*}
&f(a_{N}; \bar{r}) > f(a_{O}; \bar{r}) \\
&[\omega \phi_{N}^{o} + (1 - \omega) \phi_{N}^{p}] q > [\omega (1 - \phi_{N}^{o}) + (1 - \omega) (1 - \phi_{N}^{p})] (1 - q)
\end{align*}
\]

which would always be satisfied if $\omega < q$. More generally, we require that:

\[
\phi_{N}^{o} > \frac{\omega - q}{\omega}.
\]

Furthermore, we have that:

\[
\begin{align*}
&f(a_{N}; \bar{r}) = f(a_{O}; \bar{r}) \\
&[\omega \phi_{N}^{o} + (1 - \omega) \phi_{N}^{p}] (1 - q) = [\omega (1 - \phi_{N}^{o}) + (1 - \omega) (1 - \phi_{N}^{p})] q
\end{align*}
\]

which satisfies:

\[
\phi_{N}^{o} \in (0; 1) \text{ iff } \omega > 1 - q.
\]

and

\[
\phi_{N}^{o} = 0 \text{ iff } \omega \leq 1 - q.
\]

(this automatically becomes point (iv)). To conclude, we have to check that

\[
\frac{\omega + q - 1}{\omega} > \frac{\omega - q}{\omega},
\]

which is satisfied as long as $q > 1 - q$ (by assumption). Hence, for $\omega > 1 - q$, $\phi_{N}^{o} = \frac{\omega - (1 - q)}{\omega}$, $\phi_{N}^{p} = 1$ is a steady state. The average payoff is given by:

\[
U \left( \phi_{N}^{o} = \frac{\omega - (1 - q)}{\omega}, \phi_{N}^{p} = 1 \right) = [f(a_{N}; \bar{r}) + f(a_{O}; \bar{r})] \bar{r} + [1 - [f(a_{N}; \bar{r}) + f(a_{O}; \bar{r})]] \bar{r}.
\]

Note that

\[
[f(a_{N}; \bar{r}) + f(a_{O}; \bar{r})] = [\omega \phi_{N}^{o} + (1 - \omega) \phi_{N}^{p}] q + [\omega (1 - \phi_{N}^{o}) + (1 - \omega) (1 - \phi_{N}^{p})] (1 - q)
\]

\[
= \omega [2q - 1] \phi_{N}^{o} + q + \omega - 2\omega q = 2q^{2} - 2q + 1
\]
So, the average payoff of the society is given by
\[ U \left( \phi_N^o = \frac{\omega + q - 1}{\omega}; \phi_N^p = 1 \right) = (2q^2 - 2q + 1) \bar{r} + (2q - 2q^2) \bar{r}. \]

(iv) Let \( \phi_N^o = 0, \phi_N^p = 1 \). As shown in the discussion of point (iii), this requires \( \omega \leq 1 - q \). Furthermore, it is necessary that
\[ \phi_N^o = 0 > \frac{\omega - q}{\omega}, \]
which is true, whenever \( \omega \leq 1 - q < q \). Hence, for \( \omega < 1 - q, \phi_N^o = 0, \phi_N^p = 1 \) is a steady state. Here, the average payoff is given by:
\[
U (\phi_N^o = 0; \phi_N^p = 1) = [f (a_N; \bar{r}) + f (a_O; \bar{r})] \bar{r} + (1 - [f (a_N; \bar{r}) + f (a_O; \bar{r})]) \bar{r}.
\]

Note that
\[
[f (a_N; \bar{r}) + f (a_O; \bar{r})] = [\omega \phi_N^o + (1 - \omega) \phi_N^p] q + [\omega (1 - \phi_N^o) + (1 - \omega) (1 - \phi_N^p)] (1 - q) = (1 - \omega) q + \omega (1 - q) = q + \omega - 2\omega q
\]
and, hence, the average payoff is given by:
\[
U (\phi_N^o = 0; \phi_N^p = 1) = (q + \omega - 2\omega q) \bar{r} + (1 - q - \omega + 2\omega q) \bar{r}.
\]

(v) Let \( \phi_N^o \in [0; 1], \phi_N^p = 0 \). This requires that:
\[ f (a_N; \bar{r}) < f (a_O; \bar{r}) \]
\[ \omega \phi_N^o < (1 - q) \]
which will always be satisfied if \( \omega < 1 - q \). More generally, we require that:
\[ \phi_N^o < \frac{1 - q}{\omega}. \]
Furthermore, we have that:
\[
\begin{align*}
[f (a_N; \bar{r})] &= \frac{f (a_O; \bar{r})}{\omega} \\
[\omega \phi_N^o + (1 - \omega) \phi_N^p] (1 - q) &= [\omega (1 - \phi_N^o) + (1 - \omega) (1 - \phi_N^p)] q \\
\phi_N^o &= \frac{q}{\omega}
\end{align*}
\]
which satisfies:
\[ \phi_N^o \in (0; 1) \text{ iff } \omega > q, \]
in contradiction with the assumption \( \omega < 1 - q < q \). Hence, this cannot be a steady state.

(vi) Let \( \phi_N^o = 1, \phi_N^p \in [0; 1] \). This requires that:
\[
\begin{align*}
[f (a_N; \bar{r})] &= f (a_O; \bar{r}) \\
[\omega \phi_N^o + (1 - \omega) \phi_N^p] (1 - q) &= [\omega (1 - \phi_N^o) + (1 - \omega) (1 - \phi_N^p)] q \\
[\omega + (1 - \omega) \phi_N^p] (1 - q) &= (1 - \omega) (1 - \phi_N^p) q \\
\phi_N^p &= \frac{q - \omega}{1 - \omega}
\end{align*}
\]
Note that \( \frac{q - \omega}{1 - \omega} \in (0; 1) \) for all \( q \in (\omega; 1) \). However, if \( q < \omega \), this requires \( \phi_N^p < 0 \), a contradiction. So assume that \( \omega < q \), then the condition is simply \( \phi_N^p < \frac{q - \omega}{1 - \omega} \). \( \phi_N^p \) further has to
satisfy:

\[ f (a_N; \bar{r}) = f (a_O; \bar{r}) \\
[\omega \phi^o_N + (1 - \omega) \phi^p_N] q = [\omega (1 - \phi^o_N) + (1 - \omega) (1 - \phi^p_N)] (1 - q) \\
[\omega + (1 - \omega) \phi^p_N] q = (1 - \omega) (1 - \phi^p_N) (1 - q) \\
\phi^p_N = \frac{1 - \omega - q}{1 - \omega} \]

For \( \phi^p_N < \frac{q - \omega}{1 - \omega} \) to hold, it must be that \( q - \omega > 1 - \omega - q \), or \( q > 1 - q \), which is true by assumption. Furthermore, we also need \( \omega < 1 - q \), to ensure that \( \frac{1 - \omega - q}{1 - \omega} > 0 \). Hence, for \( \omega < 1 - q \), we have that \( \phi^{*o}_N = 1, \phi^{*p}_N = \frac{1 - \omega - q}{1 - \omega} \) is a steady state. Finally, we can compute the average payoff for the society for this case:

\[ U \left( \phi^{*o}_N = 1; \phi^{*p}_N = \frac{1 - \omega - q}{1 - \omega} \right) = [f (a_N; \bar{r}) + f (a_O; \bar{r})] \bar{r} + (1 - [f (a_N; \bar{r}) + f (a_O; \bar{r})]) \bar{u}. \]

Since

\[ [f (a_N; \bar{r}) + f (a_O; \bar{r})] = [\omega \phi^{*o}_N + (1 - \omega) \phi^{*p}_N] q + [\omega (1 - \phi^{*o}_N) + (1 - \omega) (1 - \phi^{*p}_N)] (1 - q) = [1 - q] q + q (1 - q) = 2q (1 - q), \]

we conclude that the average payoff is given by

\[ U \left( \phi^{*o}_N = 1; \phi^{*p}_N = \frac{1 - \omega - q}{1 - \omega} \right) = 2q (1 - q) \bar{r} + (1 - 2q + 2q^2) \bar{u}. \]

(vii) Let \( \phi^{*o}_N = 0, \phi^{*p}_N \in [0; 1] \). This requires that:

\[ f (a_N; \bar{r}) > f (a_O; \bar{r}) \]

\[ [\omega \phi^o_N + (1 - \omega) \phi^p_N] (1 - q) > [\omega (1 - \phi^o_N) + (1 - \omega) (1 - \phi^p_N)] q \]

\[ \phi^p_N > \frac{q}{1 - \omega} \]

For this to be feasible, we need that \( q < 1 - \omega \), or \( \omega < 1 - q \). The second condition (which determines \( \phi^{*p}_N \)) is:

\[ f (a_N; \bar{r}) = f (a_O; \bar{r}) \]

\[ (1 - \omega) \phi^p_N q = [\omega + (1 - \omega) (1 - \phi^p_N)] (1 - q) \]

\[ \phi^p_N = \frac{1 - q}{1 - \omega} \]

so that we also need \( 1 - q < 1 - \omega \), or \( \omega < q \), which is satisfied, since \( \omega < 1 - q < q \). However, \( \frac{1 - q}{1 - \omega} < \frac{q - \omega}{1 - \omega} \), in contradiction to the condition above.

(viii) Let \( \phi^{*o}_N \in [0; 1], \phi^{*p}_N \in [0; 1] \). This requires that:

\[ f (a_N; \bar{r}) = f (a_O; \bar{r}) \]

\[ [\omega \phi^o_N + (1 - \omega) \phi^p_N] (1 - q) = [\omega (1 - \phi^o_N) + (1 - \omega) (1 - \phi^p_N)] q \]

\[ \omega \phi^o_N + (1 - \omega) \phi^p_N = q \]

\[ f (a_N; \bar{r}) = f (a_O; \bar{r}) \]

\[ [\omega \phi^o_N + (1 - \omega) \phi^p_N] q = [\omega (1 - \phi^o_N) + (1 - \omega) (1 - \phi^p_N)] (1 - q) \]

\[ \omega \phi^o_N + (1 - \omega) \phi^p_N = (1 - q) \]
This can obviously only hold if \( q = 1 - q = \frac{1}{2} \), which is not an interesting case for our purposes.

Just for the reference, the social payoff equals \( \frac{1}{2} \bar{r} + \frac{1}{2} \bar{L} \).

(ix) Let \( \phi_{N}^{o} = 1, \phi_{N}^{p} = 0 \). This requires that:

\[
\frac{\omega \phi_{N}^{o} + (1 - \omega) \phi_{N}^{p}}{\omega} < \frac{\omega (1 - \phi_{N}^{o}) + (1 - \omega) (1 - \phi_{N}^{p})}{1 - q}.
\]

Furthermore, we have that:

\[
\frac{\omega \phi_{N}^{o} + (1 - \omega) \phi_{N}^{p}}{\omega} < \frac{\omega (1 - \phi_{N}^{o}) + (1 - \omega) (1 - \phi_{N}^{p})}{q}.
\]

Hence, \( \phi_{N}^{o} = 1, \phi_{N}^{p} = 0 \) is a steady state, whenever \( \omega < 1 - q < q \). The average payoff for the society is given by:

\[
U (\phi_{N}^{o} = 1; \phi_{N}^{p} = 0) = \left( \frac{2 \omega q + 1 - \omega - q}{q} \right) \bar{r} + (\omega + q - 2 \omega q) \bar{L}.
\]

Summarizing the findings of cases (iii), which corresponds to Case 1 and (iv), (vi) and (ix), which correspond to Cases 2 a), 2 b) and 2 c), respectively, we obtain the result of the two propositions.

**Proof of Proposition 3:**

We start by introducing some notation. Let \( \phi_{N,t}^{o} \) and \( \phi_{N,t}^{p} \) describe the empirical frequencies with which optimists and pessimists have been choosing the new technology \( a_{N} \) up to period \( t \).

The \( t + 1 \)-period frequencies depend on the choices made by the optimists and the pessimists in period \( t + 1 \) in the following way:

\[
\phi_{N,t+1}^{o} = \begin{cases} 
\frac{t_{N}^{o} + 1}{t_{N}^{o} + t_{N}^{p} + 1} & \text{if } a_{t+1}^{o} = a_{N} \\
\frac{t_{N}^{p} + 1}{t_{N}^{o} + t_{N}^{p} + 1} & \text{if } a_{t+1}^{o} = a_{O}
\end{cases}
\]

and

\[
\phi_{N,t+1}^{p} = \begin{cases} 
\frac{t_{N}^{p} + 1}{t_{N}^{o} + t_{N}^{p} + 1} & \text{if } a_{t+1}^{p} = a_{N} \\
\frac{t_{N}^{o} + 1}{t_{N}^{o} + t_{N}^{p} + 1} & \text{if } a_{t+1}^{p} = a_{O}
\end{cases}
\]

Hence, \( \phi_{N,t}^{i} \) grows if \( a_{N} \) is chosen by the agents of type \( i \) and falls, otherwise.

We will show that for almost every path \( \sigma \) and every \( \xi > 0 \), there is a \( t \) such that \( (\phi_{N,t}^{o}; \phi_{N,t}^{p}) \in \)
$B_\xi (\phi^N_0; \phi^N_1)$ for all $t \geq \hat{t}$, which implies the statement of the proposition.

Take a given path $\sigma$. Observe that the set of paths on which a given technology is chosen for a finite number of times is of measure 0. Indeed, suppose that on a path $\sigma$, one of the technologies, say $a_N$, is never chosen after a time $\tilde{t}$. The argument used in the proof of Proposition 2, Case (ii) shows that this can only occur if $f_T (a_O; 0) = 0$ for all $T > \tilde{t}$, which is a 0-probability event. Similarly, the derivations in Case (i) in the proof of Proposition 2 show that the set of paths on which $a_O$ is chosen for a finite number of times has measure 0.

Thus, it suffices to concentrate on paths $\sigma$ such that both technologies are chosen for an infinite number of periods. On such paths, we can choose a (path-dependent) time period $\bar{t}$ such that for all $t > \bar{t}$, the average returns are within $\epsilon$ of their corresponding expected values, where $\epsilon = \xi \frac{\omega}{2}$ and the empirically observed rates of success and failure for the two technologies reflect almost correctly the probabilities of failure and success, i.e.

$$\rho^N_t (\bar{r}) =: \frac{f_t (a_N; \bar{r})}{f_t (a_N; r) + f_t (a_N; \bar{r})} \in (q - \epsilon; q + \epsilon) \quad (15)$$

and

$$\rho^O_t (\bar{r}) =: \frac{f_t (a_O; \bar{r})}{f_t (a_O; r) + f_t (a_O; \bar{r})} \in (q - \epsilon; q + \epsilon). \quad (16)$$

Since the optimal choice of optimists is given by (5) and, thus, reduces to the comparison $f_t (a_N; \bar{r}) \geq f_t (a_O; \bar{r})$, and since the frequencies $f_t (a_O; \bar{r})$ and $f_t (a_N; \bar{r})$ satisfy (7), (15) and (16), we obtain that for sufficiently large $t$’s, the optimists choose $a_O$ if

$$[\omega \phi^o_{N,t} + (1 - \omega) \phi^p_{N,t}] (1 - q - \epsilon) > [\omega (1 - \phi^o_{N,t}) + (1 - \omega) (1 - \phi^p_{N,t})] (q + \epsilon)$$

and $a_N$, whenever

$$[\omega \phi^o_{N,t} + (1 - \omega) \phi^p_{N,t}] (1 - q + \epsilon) < [\omega (1 - \phi^o_{N,t}) + (1 - \omega) (1 - \phi^p_{N,t})] (q - \epsilon)$$

Hence, the optimal choice of the optimists is given by:
\[ a_{t+1}^o = \begin{cases}  
 a_N & \text{if } \phi_{N,t}^o < \frac{q-\epsilon}{\omega} - \frac{(1-\omega)\phi_{N,t}^p}{\omega} \\
 a_O & \text{if } \phi_{N,t}^o > \frac{q+\epsilon}{\omega} - \frac{(1-\omega)\phi_{N,t}^p}{\omega}  
\end{cases} \] (17)

Note that there is an interval of values of \( \phi_{N,t}^o \) and \( \phi_{N,t}^p \), in which we are unable to determine the optimal choice of the optimists as a function of \( \phi_{N,t}^o \) and \( \phi_{N,t}^p \) only, since in this case it will depend on the specific values \( f_t(a_N;\bar{r}) + f_t(a_N;r) \) and \( f_t(a_O;\bar{r}) + f_t(a_O;r) \) obtain in the interval \( (q-\epsilon; q+\epsilon) \). Similarly, combining (6) with (7), (15) and (16), we obtain that for sufficiently large \( t \)'s, the optimal choice of the pessimists is given by:

\[ a_{t+1}^p = \begin{cases}  
 a_N & \text{if } \phi_{N,t}^o > \frac{1-q+\epsilon}{\omega} - \frac{1-\omega}{\omega} \phi_{N,t}^p \\
 a_O & \text{if } \phi_{N,t}^o < \frac{1-q-\epsilon}{\omega} - \frac{1-\omega}{\omega} \phi_{N,t}^p  
\end{cases} \] (18)

We can now use these derivations to illustrate the dynamics of the system for different parameter constellations. In particular, if \( \omega > 1 - q \), we obtain the following three cases (the cases of equalities are suppressed and we have to choose, of course \( \epsilon = \xi \frac{\omega}{2} < \frac{2q-1}{2} \)):

<table>
<thead>
<tr>
<th>Case</th>
<th>( \phi_{N,t+1}^o &lt; \phi_{N,t}^o )</th>
<th>( \phi_{N,t+1}^p &gt; \phi_{N,t}^p ) if</th>
<th>( \phi_{N,t}^o &gt; \frac{q+\epsilon}{\omega} - \frac{(1-\omega)\phi_{N,t}^p}{\omega} )</th>
<th>( \phi_{N,t}^p = \frac{1-q+\epsilon}{\omega} - \frac{1-\omega}{\omega} \phi_{N,t}^p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case A</td>
<td>( \phi_{N,t+1}^o &gt; \phi_{N,t}^o )</td>
<td>( \phi_{N,t+1}^p &gt; \phi_{N,t}^p ) if</td>
<td>( \phi_{N,t}^o &lt; \frac{q-\epsilon}{\omega} - \frac{(1-\omega)\phi_{N,t}^p}{\omega} )</td>
<td>( \phi_{N,t}^p = \frac{1-q-\epsilon}{\omega} - \frac{1-\omega}{\omega} \phi_{N,t}^p )</td>
</tr>
<tr>
<td>Case B</td>
<td>( \phi_{N,t+1}^o &gt; \phi_{N,t}^o )</td>
<td>( \phi_{N,t+1}^p &gt; \phi_{N,t}^p ) if</td>
<td>( \phi_{N,t}^o &lt; \frac{q-\epsilon}{\omega} - \frac{(1-\omega)\phi_{N,t}^p}{\omega} )</td>
<td>( \phi_{N,t}^p = \frac{1-q-\epsilon}{\omega} - \frac{1-\omega}{\omega} \phi_{N,t}^p )</td>
</tr>
<tr>
<td>Case C</td>
<td>( \phi_{N,t+1}^o &gt; \phi_{N,t}^o )</td>
<td>( \phi_{N,t+1}^p &lt; \phi_{N,t}^p ) if</td>
<td>( \phi_{N,t}^o &lt; \frac{q-\epsilon}{\omega} - \frac{(1-\omega)\phi_{N,t}^p}{\omega} )</td>
<td>( \phi_{N,t}^p = \frac{1-q-\epsilon}{\omega} - \frac{1-\omega}{\omega} \phi_{N,t}^p )</td>
</tr>
</tbody>
</table>

Table 1: Dynamics of the system for the case \( \omega > 1 - q \).

Graph 1 provides an illustration.
Figure 1. Dynamics of the system for the case $\omega > 1 - q$.

Examining the graph, one sees that if

$$\phi_{o,N,t}^{o} \in \left(1 - q - \epsilon \frac{1}{\omega} - \frac{(1 - \omega)}{\omega} \phi_{p,N,t}^{p}; 1 - q + \epsilon \frac{1}{\omega} - \frac{(1 - \omega)}{\omega} \phi_{N,t}^{p}\right),$$

we have that on the path $\sigma$, $\phi_{N,t+1}^{o} > \phi_{N,t}^{o}$ (while the dynamics of $\phi_{N,t}^{p}$ is ambiguous — it depends on the exact position of $\phi_{N,t}^{o}$ in this interval). Suppose that initially, $\phi_{N,t+1}^{o} = 0$ and consider the maximal time after which it will exceed $1 - q + \epsilon \frac{1}{\omega}$. Note that the size of the step is given by:

$$\phi_{N,t+1}^{o} - \phi_{N,t}^{o} = \frac{t \phi_{N,t}^{o} + 1}{t + 1} - \phi_{N,t}^{o} = \frac{1 - \phi_{N,t}^{o}}{t + 1}.$$

Since $\phi_{N,t}^{o} \leq 1 - q + \epsilon$, we have that

$$\phi_{N,t+1}^{o} - \phi_{N,t}^{o} \geq \frac{1 - \frac{1 - q + \epsilon}{\omega}}{t + 1} = \frac{\omega - (1 - q) - \epsilon}{\omega (t + 1)}.$$

So, after $\kappa$ steps,

$$\phi_{N,t+\kappa}^{o} - \phi_{N,t}^{o} \geq \sum_{i=1}^{\kappa} \frac{\omega - (1 - q) - \epsilon}{\omega (t + i)} = \sum_{i=1}^{t+\kappa} \frac{\omega - (1 - q) - \epsilon}{\omega t} - \sum_{i=1}^{t} \frac{\omega - (1 - q) - \epsilon}{\omega t} \geq \frac{\omega - (1 - q) - \epsilon}{\omega} \left[\ln (t + \kappa) + \gamma - \ln t - 1\right],$$

where $\gamma$ is the Euler-Mascheroni constant ($\gamma \sim 0.577$). The last inequality follows from the
The fact that:

\[ \lim_{\kappa \to \infty} \sum_{i=1}^{\kappa} \frac{1}{i} = \ln \kappa + \gamma \]

and the fact that the difference \( \sum_{i=1}^{\kappa} \frac{1}{i} - \ln \kappa \) is monotonically decreasing in \( \kappa \), and thus, \( \sum_{i=1}^{t+\kappa} \frac{1}{i} \geq \ln (t + \kappa) + \gamma \) and \( \sum_{i=1}^{t} \frac{1}{i} - \ln t \leq 1 - \ln 1 = 1 \). It follows that choosing \( \kappa = \kappa_B \) such that \( \kappa_B \) is the minimal integer value of \( \kappa \) satisfying the inequality:

\[ \frac{\omega - (1 - q) - \epsilon}{\omega} [\ln (t + \kappa) + \gamma - \ln t - 1] \geq \frac{1 - q + \epsilon}{\omega} \]

ensures that after \( \kappa_B \) steps, the system will reach a state described by Case 1.B. Since in a state described by Case 1.B, we have that both \( \phi_o^{N,t} \) and \( \phi_p^{N,t} \) grow, in a finite number of steps the system will eventually reach a state such that:

\[ \phi_o^{N,t} > q - \epsilon \quad \text{and} \quad \phi_p^{N,t} \]

Condition (19) is equivalent to \( c_t > q - \epsilon \). Since \( \phi_p^{N,t+1} = \frac{t \phi_p^{N,t+1}}{t+1} \) and \( \phi_p^{N,t+1} = \frac{t \phi_p^{N,t+1}}{t+1} \), we have \( c_{t+1} = \frac{t \phi_p^{N,t+1}}{t+1} \). We can thus use the same argument as above to show that \( c_{t+1} > q - \epsilon \) will obtain after a finite number of steps. In particular, we can show that for \( c_t < q \), the minimal size of the step is given by \( c_{t+1} - c_t \geq \frac{1 - q}{t+1} \), and thus,

\[ c_{t+\kappa} - c_t \geq \sum_{i=1}^{\kappa} \frac{1 - q}{i+1} = \sum_{i=1}^{\kappa} \frac{1 - q}{i} - \sum_{i=1}^{t} \frac{1 - q}{i} \geq \geq (1 - q) [\ln (t + \kappa) + \gamma - \ln t - 1]. \]

Since the minimal value of \( c_t \) in the region of interest is given by \( (1 - q + \epsilon) \), choosing \( \kappa_C \) to be the smallest integer value of \( \kappa \) such that:

\[ (1 - q) [\ln (t + \kappa) + \gamma - \ln t - 1] \geq q - \epsilon - (1 - q + \epsilon) = 2 (q - \epsilon) - 1 \]

ensures that the system will reach a state characterized by (19) after at most \( \kappa_C \) steps.

While the behavior of \( \phi_o^{N,t} \) in a state characterized by (19) might be ambiguous (see the graph), we have that \( \phi_p^{N,t+1} > \phi_p^{N,t} \). Suppose that at time \( t \), the system is in a state described by Case 1.B and \( \phi_p^{N,t} = 0 \). We want to compute the maximal number of steps necessary for \( \phi_p^{N,t} \) to
exceed $1 - \epsilon$. For $\phi^p_{N,t} \in [0; 1 - \epsilon]$, the size of the step is given by:

$$\phi^p_{N,t+1} - \phi^p_{N,t} = \frac{1 - \phi^p_{N,t}}{t + 1} \geq \frac{\epsilon}{t + 1}.$$ 

It follows (as above) that after $\kappa$ steps,

$$\phi^p_{N,t+\kappa} - \phi^p_{N,t} \geq \sum_{i=1}^{\kappa} \frac{\epsilon}{t + i} = \sum_{i=1}^{\kappa} \frac{\epsilon}{t} - \sum_{i=1}^{\kappa} \frac{\epsilon}{i} \geq \epsilon [\ln(t + \kappa) + \gamma - \ln t - 1].$$

Hence, for each $\epsilon$, we can choose $\kappa = \kappa_A$ where $\kappa_A$ is the smallest integer value of $\kappa$ satisfying the inequality:

$$\epsilon [\ln(t + \kappa) + \gamma - \ln t - 1] > 1 - \epsilon,$$

ensuring that $\phi^p_{N,t+\kappa}$ exceeds $1 - \epsilon$ in a finite number of periods. Furthermore, since $\phi^p_{N,t}$ grows at values larger than $(1 - \epsilon)$, $\phi^p_{N,t}$ will remain above $(1 - \epsilon)$, once it has crossed the threshold.

Hence, $\phi^p_{N,t} \in (\phi^p_{N} - \epsilon; \phi^p_{N} + \epsilon)$ for all $t \geq \bar{t} + \kappa_A + \kappa_B + \kappa_C$.

Finally, consider the behavior of $\phi^o_{N,t}$ given that the system is in a state corresponding to Case 1.A or 1.B. Since $\phi^o_{N,t}$ increases below $\phi^o_{N,t}$ and decreases above this value, we can use the same argument as above to show that it will reach a state such that:

$$\phi^o_{N,t} \in \left(\frac{q - \epsilon}{\omega} - \frac{(1 - \omega)}{\omega} \phi^p_{N,t}; \frac{q + \epsilon}{\omega} - \frac{(1 - \omega)}{\omega} \phi^p_{N,t}\right)$$

in a finite number of steps, say $\kappa_D$, regardless from the starting point. Furthermore, we have that the maximal size of an upwards step in this region is given by:

$$\frac{1 - \phi^o_{N,t}}{t + 1} \leq \frac{1 - \frac{q - \epsilon}{\omega} + \frac{(1 - \omega)}{\omega} \phi^p_{N,t}}{t + 1} \leq \frac{1 - \frac{q - \epsilon}{\omega} + \frac{(1 - \omega)}{\omega}}{t + 1} = 1 + \frac{1 - \omega - q + \epsilon}{\omega}.$$ 

This upwards step will be less than $\epsilon$ if

$$\frac{1 - \epsilon + \frac{1 - \omega - q + \epsilon}{\omega}}{\epsilon} < t.$$ 

Similarly, the maximal size of a downward step is:

$$\phi^o_{N,t} - \phi^o_{N,t+1} = \phi^o_{N,t} - \frac{t \phi^o_{N,t}}{t + 1} = \frac{\phi^o_{N,t}}{\omega(t + 1)} - \frac{q + \epsilon}{\omega(t + 1)}(1 - \epsilon) = q + 2\epsilon - 1 + \omega(1 - \epsilon) \omega(t + 1).$$ 

This will be less than $\epsilon$ if

$$\frac{q + 2\epsilon - 1 + \omega(1 - \epsilon)}{\omega \epsilon} < t.$$
Hence, choosing
\[ \hat{t} > \max \left\{ \tilde{t} + \kappa_A + \kappa_B + \kappa_C + \kappa_D + \frac{1 - \omega + \omega^2}{2}; \quad \tilde{t} + \kappa_A + \kappa_B + \kappa_C + \kappa_D + \frac{q + 2 \omega - 1 - \omega (1 - \epsilon)}{\omega} - 1 \right\} \]
ensures that \( \phi_{N,t}^o \) remains in the interval
\[ \phi_{N,t}^o \in \left( \frac{q - \epsilon}{\omega} - \frac{(1 - \omega)}{\omega} \phi_{N,t}^p - \epsilon; \quad \frac{q + \epsilon}{\omega} - \frac{(1 - \omega)}{\omega} \phi_{N,t}^p + \epsilon \right). \]

Since
\[ \frac{q - \epsilon}{\omega} - \frac{(1 - \omega)}{\omega} \phi_{N,t}^p - \epsilon \geq \frac{q - \epsilon}{\omega} - \frac{(1 - \omega)}{\omega} - \epsilon = \frac{\omega - (1 - q)}{\omega} - \frac{(\omega + 1)}{\omega} \epsilon = \phi_{N}^o - \frac{(\omega + 1)}{\omega} \epsilon \]
and
\[ \frac{q + \epsilon}{\omega} - \frac{(1 - \omega)}{\omega} \phi_{N,t}^p + \epsilon \leq \frac{q + \epsilon}{\omega} - \frac{(1 - \omega)}{\omega} (1 - \epsilon) + \epsilon = \frac{\omega - (1 - q)}{\omega} + \frac{2 \epsilon}{\omega} = \phi_{N}^p + \frac{2 \epsilon}{\omega} \]
and (obviously) \( \frac{\omega + 1}{\omega} < \frac{2}{\omega} \), we can claim that
\[ \phi_{N,t}^o \in \left( \phi_{N}^o - \frac{2 \epsilon}{\omega}; \phi_{N}^o + \frac{2 \epsilon}{\omega} \right) \]
and
\[ \phi_{N,t}^p \in \left( \phi_{N}^p - \epsilon; \phi_{N}^p + \epsilon \right) \]
holds for all \( t \geq \hat{t} \). It follows that for all \( t \geq \hat{t} \), \( (\phi_{N,t}^o; \phi_{N,t}^p) \) is contained in a rectangle with sides
\( 2 \epsilon \) and \( \frac{2 \omega \epsilon}{\omega} \). Note that the circle that can be circumscribed around this rectangle has a radius of:
\[ 2 \sqrt{\frac{\epsilon^2 + \frac{\epsilon^2}{\omega^2}}{2}} = \frac{\epsilon}{\omega} \sqrt{1 + \frac{\epsilon^2}{\omega^2}} \leq \frac{2 \epsilon}{\omega} = \xi. \]

It follows that for all \( t \geq \hat{t} \), \( (\phi_{N,t}^o; \phi_{N,t}^p) \in B_\xi (\phi_{N}^o; \phi_{N}^p) \).

We have thus shown that on almost every path \( \sigma \), for every \( \xi > 0 \), there exists a \( \hat{t}_\xi (\sigma) \) such that \( (\phi_{N,t}^o (\sigma); \phi_{N,t}^p (\sigma)) \in B_\xi (\phi_{N}^o; \phi_{N}^p) \) for all \( t \geq \hat{t}_\xi (\sigma) \). Hence, the system converges almost surely towards the steady state \( (\phi_{N}^o; \phi_{N}^p) \).

**Proof of Proposition 4:**

We use the notation from the proof of Proposition 3 and similar reasoning to show that the dynamics of the system for \( \omega < 1 - q \) is given by the following table:
Given the conditions in (9), we have:

\[
\begin{align*}
    f_{T_0+T_1+1} (a_N; \bar{r}) &< f_{T_0+T_1+1} (a_O; \bar{r}) \\
    f_{T_0+T_1+1} (a_N; \bar{r}) &< f_{T_0+T_1+1} (a_O; \bar{r})
\end{align*}
\]

and therefore, \(a_{T_0+T_1+1}^{a_N} = a_N\) and \(a_{T_0+T_1+1}^{a_O} = a_O\). In order for these inequalities to be reversed, the new technology has to be chosen for a minimum of \(\hat{T} = \min \left\{ \frac{(1-q)T_0 + qT_1}{\omega}, \frac{qT_0 + (1-q)T_1}{\omega} \right\}\) periods. Fix \(\epsilon \in (0; 1-q)\) and \(\xi > 0\). By the Law of Large Numbers, for any such \(\epsilon\) and \(\xi\), we can find a sufficiently large \(\hat{T} (\epsilon; \xi)\) such that with probability of at least \(1 - \xi\), the observed frequency of success of \(a_N\) is in the interval \((q - \epsilon; q + \epsilon)\) and the observed frequency of success of \(a_O\) is in the interval \((1 - q - \epsilon; 1 - q + \epsilon)\) for all \(T \geq T_0 + T_1 + \hat{T} (\epsilon; \xi)\). Since decreasing
ω increases \( \bar{T} \), we have that: for fixed \( T_0, T_1 \) and \( q \), and for each \( \epsilon > 0, \xi > 0 \), there is a (sufficiently small) \( \hat{\omega} > 0 \) such that \( \bar{T} \geq \hat{T}(\epsilon; \xi) \).

Now, choose \( \omega \) so that \( \omega < \hat{\omega} \) and \( \epsilon + \omega < 1 - q \) and note that on a set of paths with measure of at least \( 1 - \xi \), after \( T_0 + T_1 + \bar{T} \) periods, the system will be in a state \( \phi^p_{N,T_0+T_1+T} = 0 \) and \( \phi^o_{N,T_0+T_1+T} = \frac{T}{T_0 + T_1 + \bar{T}} \).

Since the state \( \phi^p_{N,T_0+T_1+\bar{T}} = 0 \) and \( \phi^o_{N,T_0+T_1+\bar{T}} = \frac{T}{T_0 + T_1 + \bar{T}} \) corresponds to Case 2.C, we have that

\[
\phi^p_{N,T_0+T_1+\bar{T}+1} = \phi^p_{N,T_0+T_1+\bar{T}} = 0
\]

and

\[
\phi^o_{N,T_0+T_1+\bar{T}+1} = \frac{(T_0 + T_1 + \bar{T}) \phi^o_{N,T_0+T_1+T} + 1}{T_0 + T_1 + \bar{T} + 1} > \phi^o_{N,T_0+T_1+\bar{T}}.
\]

Using the same reasoning as in the proof of Proposition 3, we can show that for any \( \epsilon > 0 \), \( \phi^o_{N,t} > 1 - \epsilon \) will obtain in a finite number of steps. It follows that \( \lim_{t \to \infty} \phi^o_{N,t} = 1 \), while \( \lim_{t \to \infty} \phi^p_{N,t} = \phi^p_{N,T_0+T_1} = 0 \) on a set of paths with measure at least \( 1 - \xi \).

**Proof of Proposition 5:**

Note that pessimists will chose \( a_N \) in those periods, in which \( f_T(a_N; \bar{r}) - f_T(a_O; \bar{r}) \geq 0 \). Hence,

\[
T^* = \min_{T \in \mathbb{N}} \{ T \mid f_T(a_N; \bar{r}) - f_T(a_O; \bar{r}) \geq 0 \},
\]

or, equivalently,

\[
T^* = \min_{T \in \mathbb{N}} \{ T \mid T [f_T(a_N; \bar{r}) - f_T(a_O; \bar{r})] \geq 0 \}
\]

We thus analyze the properties of the stochastic process \( \nu_T = T [f_T(a_N; \bar{r}) - f_T(a_O; \bar{r})] \), which expresses the difference in the number of successes observed for \( a_N \) and the number of successes with \( a_O \) and we compute the expected time necessary for it to cross 0 from below.

By assumption, the initial state of the economy is characterized by \( \nu_T < 0 \). Note that if no one in the population holds \( a_N \), \( \nu_T \) decreases weakly with probability 1. In contrast, if everyone in the society holds \( a_N \), then period \( T^* \) has already been reached. Hence, we consider \( \nu_T \) in those periods, in which only optimists hold \( a_N \) and show that the expected number of periods for it to cross 0 from below is infinity. Note that in those periods \( T \), in which \( a_p^p = a_N, a_T^p = a_O \), we
have:
\[
\nu_{T+1} - \nu_T = \begin{cases} 
\omega - (1 - \omega) & \text{if } (a_T^P = a_O; r_T^P = \tilde{r}) \text{ and } (a_T^O = a_N; r_T^O = \tilde{r}) \\
-(1 - \omega) & \text{if } (a_T^P = a_O; r_T^P = \tilde{r}) \text{ and } (a_T^O = a_N; r_T^O = \tilde{r}) \\
\omega & \text{if } (a_T^P = a_O; r_T^P = \bar{r}) \text{ and } (a_T^O = a_N; r_T^O = \tilde{r}) \\
0 & \text{if } (a_T^P = a_O; r_T^P = \bar{r}) \text{ and } (a_T^O = a_N; r_T^O = \bar{r})
\end{cases}
\]

Since the joint distribution of outcomes for the technologies has not been specified, we assume that it is given by:
\[
\Pr\{(a_O; \tilde{r}); (a_N; \tilde{r})\} = P_1 \\
\Pr\{(a_O; \tilde{r}); (a_N; \bar{r})\} = P_2 \\
\Pr\{(a_O; \bar{r}); (a_N; \tilde{r})\} = P_3 \\
\Pr\{(a_O; \bar{r}); (a_N; \bar{r})\} = 1 - P_1 - P_2 - P_3
\]

where \(P_1, P_2, P_3 \in [0; 1]\) with \(\sum_{k=1}^{3} P_k \leq 1\). Furthermore, to be consistent with Assumption .1, \(P_1, P_2\) and \(P_3\) have to satisfy:
\[
\Pr\{(a_O; \tilde{r})\} = P_1 + P_2 = 1 - q \\
\Pr\{(a_N; \tilde{r})\} = P_1 + P_3 = q.
\]

It follows that the expected values of \(\nu_{T+1} - \nu_T\) is given by:
\[
E[\nu_{T+1} - \nu_T] = q\omega - (1 - q)(1 - \omega) = \omega - (1 - q) < 0.
\]

Since the outcomes are independent across time, \(\nu_T\) is a random walk with a negative expected increment. It follows that starting at \(\nu_{T_0} < 0\), the expected time before it crosses 0 from below is \(\infty\), i.e., \(E[T^*] = \infty\).]

Proof of Proposition 7:

Parts (i) and (ii) of this proposition is proven in the text. To see that the set of parameters satisfying the inequality in (iii) is indeed non-empty, set \(f(a_O; \tilde{r}) = q, \omega = \frac{1 - q}{2}\). It is easy to see that the inequality (12) will be satisfied for sufficiently large values of \(T\). Since we did not choose an extreme value of \(\omega\), the inequality will be preserved if we increased \(\omega\), but decreased \(f(a_O; \tilde{r})\). Hence, by continuity, the set of parameters for which (12) is satisfied has a non-empty interior.

Proof of Proposition 7:
Part (i) and (ii) of this proposition is proven in the text. To see that the set of parameters satisfying the inequality in (iii) is indeed non-empty, set $f(a_O; \bar{r}) = q$ and observe that the condition in the statement of the proposition reduces to:

$$\left( 1 - \frac{1}{T + 1} \right) > \frac{(1 - \omega)(2q - 1)}{(1 - q)q}.$$ 

We now show that there is a non-empty set of parameters, for which

$$\frac{(1 - \omega)(2q - 1)}{(1 - q)q} < 1,$$

or

$$q^2 + q - 1 - 2q\omega + \omega < 0.$$ 

To see this, note that since $2q > 1$, we have $\omega < 2q\omega$ and choosing a value of $q$ such that $q + q^2 - 1 < 0$, i.e., $q \in \left( \frac{1}{2}, \frac{\sqrt{5} - 1}{2} \right)$ implies the desired result. Hence, the statement in the proposition will be satisfied, whenever $T$ is sufficiently large. By continuity, decreasing $f(a_O; \bar{r})$ in a small range of $q$ will preserve the strict inequality. Hence, the set of parameters for which the inequality is satisfied indeed has a non-empty interior.

**Proof of Proposition 9:**

From Proposition 1, we know that pessimists prefer $a_O$ to $a_N$ given $D$ if and only if

$$f_D(a_O; \bar{r}) - f_D(a_N; \bar{r}) > 0.$$ 

Using the utility function in (14), we derive:

$$\hat{V}_p(a_O; D \cup \bar{D}) = (1 - \gamma_{T+\bar{T}}) \frac{Tf_D(a_O; \bar{r}) + \frac{T\bar{T}}{2}(1 - q)}{T + sT},$$

and

$$\hat{V}_p(a_N; D \cup \bar{D}) = (1 - \gamma_{T+\bar{T}}) \frac{Tf_D(a_N; \bar{r}) + \frac{T\bar{T}}{2}q}{T + sT}.$$ 

Hence, a pessimist will prefer to choose $a_N$ given $D \cup \bar{D}$ if and only if:

$$\frac{Tf_D(a_N; \bar{r}) + \frac{T\bar{T}}{2}q}{T + sT} > \frac{Tf_D(a_O; \bar{r}) + \frac{T\bar{T}}{2}(1 - q)}{T + sT},$$

or

$$\frac{T\bar{T}}{2}(2q - 1) > T[f_D(a_O; \bar{r}) - f_D(a_N; \bar{r})],$$

which proves part (i). To derive the condition in part (ii), which ensures that the pessimist will
prefer to choose \(a_N\) given \(D \cup \tilde{D}\) to choosing \(a_O\) given \(D\), we compare:

\[
\tilde{V}_p^p(a_N; D \cup \tilde{D}) = (1 - \gamma_{T+\tilde{T}}) \frac{Tf_D(a_N; \bar{r}) + \tilde{s}_{T}^T T_f^p q}{T + \tilde{s}_T} > V_p^p(a_O; D) = (1 - \gamma_T) \frac{Tf_D(a_O; \bar{r})}{T}
\]

Since \(\gamma_T = \frac{1}{T+1}\), this is equivalent to:

\[
\frac{T + \tilde{T}}{T + T + 1} \frac{Tf_D(a_N; \bar{r}) + \tilde{s}_{T}^T T_f^p q}{T + \tilde{s}_T} > \frac{T}{T + 1}f_D(a_O; \bar{r})
\]

Note that \(\frac{T + \tilde{T}}{T + T + 1} > \frac{T}{T + 1}\). Hence, the inequality will be always satisfied if

\[
\frac{Tf_D(a_N; \bar{r}) + \tilde{s}_{T}^T T_f^p q}{T + \tilde{s}_T} > f_D(a_O; \bar{r}).
\]

Simplifying, we obtain:

\[
\tilde{s}_{T}^T \left[ q - f_D(a_O; \bar{r}) \right] > T \left[ f_D(a_O; \bar{r}) - f_D(a_N; \bar{r}) \right],
\]

or

\[
\frac{\tilde{s}_{T}^T}{2} > \frac{T \left[ f_D(a_O; \bar{r}) - f_D(a_N; \bar{r}) \right]}{\left[ q - f_D(a_O; \bar{r}) \right]}.
\]

**Proof of Proposition 10:**

From Proposition 1, we know that optimists prefer to choose \(a_N\) given \(D\) if and only if

\[
f_D(a_N; \underline{\ell}) - f_D(a_O; \underline{\ell}) < 0.
\]

Furthermore,

\[
\tilde{V}_o^o(a_O; D \cup \tilde{D}) = (1 - \gamma_{T+\tilde{T}}) \frac{T \left[ 1 - f_D(a_O; \underline{\ell}) \right] + \tilde{s}_{T}^T \left[ 2 - q \right]}{T + \tilde{s}_T} + \gamma_{T+\tilde{T}}
\]

\[
\tilde{V}_o^o(a_N; D \cup \tilde{D}) = (1 - \gamma_{T+\tilde{T}}) \frac{T \left[ 1 - f_D(a_N; \underline{\ell}) \right] + \tilde{s}_{T}^T \left( 1 + q \right)}{T + \tilde{s}_T} + \gamma_{T+\tilde{T}}
\]

Hence, an optimist will prefer to choose \(a_N\) given \(D \cup \tilde{D}\) if and only if:

\[
\frac{\tilde{s}_{T}^T}{2} (2q - 1) > T \left[ f_D(a_N; \underline{\ell}) - f_D(a_O; \underline{\ell}) \right],
\]

which is always satisfied, since the l.h.s. is positive, whereas, the r.h.s. is always negative, whenever the optimist prefers \(a_N\) under \(D\). Finally, to derive the condition under which the optimist will prefer to choose \(a_N\) given \(D\) to choosing \(a_N\) given \(D \cup \tilde{D}\), we compare:

\[
\tilde{V}_o^o(a_N; D \cup \tilde{D}) = (1 - \gamma_{T+\tilde{T}}) \frac{T \left[ 1 - f_D(a_N; \underline{\ell}) \right] + \tilde{s}_{T}^T \left( 1 + q \right)}{T + \tilde{s}_T} + \gamma_{T+\tilde{T}} < V_o^o(a_N; D) = (1 - \gamma_T) \frac{T \left[ 1 - f_D(a_N; \underline{\ell}) \right]}{T} + \gamma_T
\]

45
Using the fact that $γ_T = \frac{1}{T+1}$, we note that this is equivalent to:

$$\frac{T + \tilde{T}}{T + T + 1} \left[ 1 - f_D(a; N, r) \right] + \frac{\tilde{s}T}{T + sT} (1 + q) + \frac{1}{T + T + 1} < \frac{T}{T + 1} \left[ 1 - f_D(a; N, r) \right] + \frac{1}{T + 1}$$

Note that if $f_D(a; N, r) = 0$, i.e. the new technology has been chosen never before, the expression simplifies to:

$$\frac{T + \tilde{T}}{T + T + 1} \frac{T + s \tilde{T} \left( 1 + q \right)}{T + sT} + \frac{1}{T + T + 1} < 1$$

which is always satisfied, since $\frac{T + s \tilde{T} \left( 1 + q \right)}{T + sT} < 1$, which is always satisfied, since $\frac{T + s \tilde{T} \left( 1 + q \right)}{T + sT} = 1$.

**References**


