LIMIT SUPPLYING AS A SIGNAL OF QUALITY

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Abstract

This paper analyzes the role of seller-induced excess demand as a signal of high quality of a new product provided by a monopoly firm. In discrete settings where neither price nor advertising can signal the quality, we show that creating shortages can successfully signal high quality and results in separating equilibrium. It is also shown that least-cost separating equilibria are accompanied with high price. This explains why high quality producers may prefer to use limit supplying (queuing, limited edition, capacity etc.) with high price to signal its quality and induce buying frenzy.

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1 Introduction

Signaling the existing quality of an experienced good is of vital importance to firms. Previous literature studied using price or dissipative advertisement expenditure or the combination of them to signal quality. The relationship between capacity choice or excess demand induced by limiting outputs by the sellers, and quality, has been rarely considered by the literature. Exceptions include DeGraba (1995), Allen and Faulhaber (1991), Bose (1996), von Ungern-Sternberg (1991) among a few others.

In DeGraba (1995), excess demand is purposely induced by a monopoly firm to promote buying frenzy. However in his model the generic quality of the product is known to everyone but it is assumed that consumers learn their types over time. Hence selling fewer units than the number of consumers induces all consumers to purchase while uninformed although they prefer to purchase after become informed, because anyone waiting to purchase until becoming informed finds no units available. This induced “buying frenzy” allows the monopolist to price higher and obtain a higher profit.

Allen and Faulhaber (1991) argue that if Nelson effect dominates Schmalensee effect, then the good firm uses low price to signal its high (expected) quality of products, hence it need not clear the market. The intuition is that if below market clearing price signals the quality, then limiting supply can reduce the signaling cost. Here the signal is conveyed by price but not excess demand.

In a related paper, Wilson (1980) identifies an equilibrium where there is an excess supply accompanied with high price. This is because higher market price induces high quality car owners (who have higher reservation value) to bring their cars to the market hence creating excess supply. Despite the fact that there is an excess supply, buyers are not willing to lower the price because high quality cars
will quit the market if otherwise, leaving market flooded by lemons only.

Bose (1996) and von Ungern-Sternberg (1991) consider the rationing problem in restaurants but neither discuss signaling role of rationing. Bose (1996) claims that restaurants using capacity and henceforth queuing to screen less profitable customers since serious customers who will spend more care less about waiting time. von Ungern-Sternberg’s argument is somewhat similar to peak load pricing: since restaurants can not charge the time customers spend in dining so excess demand is associated with profit maximizing price.

However, in industries other than restaurant, limiting supply is a commonly used business strategy. It is also documented in the popular media:

“Analyst speak for that indefinable aura that convinces a consumer to pay a lot of money for something he, or more likely she, could buy much more cheaply elsewhere... The destroyer of brand integrity is “brand dilution”, which is the perverse reward for popularity. If too many people have a supposedly exclusive Fendi handbag or Hermès scarf, in the customer’s view, no longer worth its vertiginous price.” —The Economist, March 6-12, 2004.

Christian Dior sued the supermarkets for carrying its products because wide availability of their products hurt the firm. The same Economist article sees the risk of oversupply through licensing primarily due to quality concerns:

“If a licensee sells the product at a discount, or lowers its quality, or sells it in the wrong place, or bundles it together with low-quality products, the “brand integrity” will be harmed, perhaps permanently. The best-known example is Pierre Cardin, whose licensing operations proliferated so much that by the 1980s he had lent his name up to 800 products, including toilet-seat covers.”
On the other hand, in many industries, firms rarely use advertising to signal their quality but rather create excess demand through some marketing practice such as ‘limited edition’ etc. For instance, the high quality handbag producer Louis Vuitton does not advertise against a cheap handbag imitation producer but frequently produces limited edition bags. Apart from limited editions, firms also limit the supply of a new product in its debut. In a recent example, the automaker Jaguar puts only 50 copies of a special Neiman Marcus edition of the redesigned 2010 Jaguar 5.0-liter V8 making 470 hp XJ at a price of $105,000 for order in October 2009 and it only took four hours and four minutes to finish all bookings. The dealers then will still take orders and compile a waiting list, and the car will be on sale nationwide in US after being revealed in July 2010 (Autoweek, October 20, 2009). Ferrari promises that it will not produce more than 4300 vehicles despite more than a two-year waiting list for its cars (Financial Times, April 11, 2002). In another Economist article (October 2-8, 2004), it is reported that the owner of the Italian brand Armani who has been steadily profitable for over 30 years, Mr. Giorgio Armani, said:

“they should be made more exclusive by restricting sales”.

In the current paper, we argue that limiting supply initially, or creating excess demand, can be a less costly tool to signal quality. It is possible to have an equilibrium, in which consumers rationally expect the firm to induce different amounts of excess demand for different product quality types. To best illustrate the idea, we confine analysis to a two period discrete model of a monopoly firm providing a new product of either high or low quality. The firm knows the quality but the consumers do not have this knowledge at the time the product is introduced. Further, the firm cannot vary the quality1. The two period model captures the introductory

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1As explained in Milgrom and Roberts (1986), this may correspond to the situation where the firm’s R&D effort has generated the product of a given quality that the firm must then decide how to introduce. As usual, we assume that the product of high-quality is more costly to produce.
phase of the product under the following conditions on information transmission. The product’s life cycle can be decomposed into two phases: the introductory phase and the mature phase. Signaling occurs during the introductory phase. We do not allow word-of-mouth learning or independent sources of quality revelation such as consumer reports. However all consumers know the product quality in the mature phase either through a separating equilibrium where consumers can rationally predict the true quality, or in a pooling equilibrium where all consumers buy in the introductory period and know the true quality from their experience. Hence the firm will choose its complete-information monopoly price and will not induce excess demand in the mature phase.

The rest of the paper is organized as follows. Section 2 introduces the model in the simple homogenous consumer case. Section 3 presents results on heterogeneous consumer case. Section 4 concludes the paper.

2 Quality Signaling Problem

Consider a two-period market for a new indivisible experience good supplied by a monopoly firm. The good can be of either high ($s_1$) or low quality (type $s_0$). The quality is not observable to the consumers nor is adjustable by the firm across time. The marginal cost of the firm is constant for either quality type. Denote by $c_t$ the marginal cost for quality type $t$. Assume as usual $c_0 < c_1$.

There is a continuum of consumers with each demanding for at most one unit of the good. Consumers may or may be homogenous with regard to their tastes or willingness to pay for quality. Let $\theta$ denote the value of quality for a consumer, so that if he consumes the good of quality $s_k$ at price $p$, then he obtains utility

$$\theta s_k - p.$$ (1)
Let $\delta \in (0,1]$ be the common discount factor. Since $s_1 > s_0$ and $c_0 < c_1$, the low-quality type has incentive to mislead the consumers into believing that it is of the high-quality type, provided that it is not too costly to do so. In addition, consumers are price-taking because there is a continuum of them.

There is no communication between the consumers. Therefore, those who purchase the good in the first period learn the quality and can make their second period purchase decisions based on that information. The consumers who do not purchase will rely on their updated beliefs using observable signals when making their second period purchase decisions.

3 Quality Signaling with Homogenous Consumers

In this section, we consider the simpler case in which consumers are homogenous, in the sense that they have the same taste for quality, represented by parameter $\theta$. In this case, the least-cost separating equilibrium with price and advertising as the signal of high quality is less preferable than the pooling equilibrium to the high-quality firm. It can be shown that using price alone or price-advertising combination can not signal quality in the case where low-quality is also socially desirable under complete information. However, a suitable price together with a certain amount of shortage can more profitably signal the high quality.

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2 Since technology is not adjustable, we can identify the type of the firm with the quality type of the good it produces.

3 Strangers to a tourism town often find themselves unable to judge the quality of local restaurants even if surrounded by advertisements.
3.1 Signal High quality by Price and Advertising

Suppose that the high-quality firm uses price $p_1$ and dissipative advertising with expenditure $A$ to signal its quality. As usual, the low-quality firm chooses its complete information monopoly price $\theta s_0$ but does not advertise in any separating equilibrium. Hence, the following incentive-compatibility constraints characterize all separating equilibria: 

\begin{equation}
 p_1 - A - c_1 \geq \theta s_0 - c_1 \tag{2}
\end{equation}

and

\begin{equation}
 p_1 - A - c_0 \leq \theta s_0 - c_0. \tag{3}
\end{equation}

Notice that (2) and (3) imply $(p_1 - A - c_0) = \theta s_0 - c_0$ or equivalently $p_1 - A = \theta s_0$. Thus, the net price of high-quality firm is the same as the low-quality firm’s price. This would make the separating equilibrium less attractive to the high-quality firm than the pooling equilibrium, in which both types of the firm charge the same price based on the consumers’ prior beliefs:

\begin{equation}
 p_1^\mu = \mu \theta s_1 + (1 - \mu) \theta s_0 > \theta s_0.
\end{equation}

By (1), this period 1 price results in the high-quality firm’s total profit equal to

\begin{equation}
 p_1^\mu - c_1 + \delta(\theta s_1 - c_1). \tag{4}
\end{equation}

We now turn to the case where the monopoly firm is allowed to use limiting supply to signal its product quality.

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4Let $(p_1, A)$ satisfy (1) and (2), and let $\mu(h|p_1', A')$ denote the probability with which each consumer believes that the good of high quality conditional on observing price $p_1$ and advertisement measure by expenditure $A'$. Then, $(p_1, A)$ and $(\theta s_0, 0)$ can be supported as the firm’s equilibrium choices by letting $\mu(h|p_1', A') = 1$ for all pairs $(p_1', A') \geq (p_1, A)$ and $\mu(h|p_1', A') = 0$ otherwise. Notice that such a belief system in turn is consistent with Bayes rule and the firm’s equilibrium strategy.
3.2 Signaling Quality by Limiting Supply

Suppose that, instead of supplying the entire market, the high-quality firm limits its supply so that only a fraction $\alpha \in (0, 1)$ of the population of consumers can get the good. That is, suppose that the high-quality firm induces a shortage equal the amount of $(1 - \alpha)$. Consumers can observe this shortage by proxies such as the time required to order in advance or queuing. They can then update their prior beliefs about the quality type using the observation. It follows that the high-quality firm may reveal its type to the consumers by properly choosing the amount of shortage, leaving it not desirable for low-quality type to mimic.

In a separating equilibrium, the low-quality type does not induce any shortage because it cannot mislead the consumers unless it induces the same amount of shortage. This means that all separating equilibria are equally profitable for the low-quality firm. However, this is not true for the high-quality firm. We are interested in the most profitable separating equilibria for the reason that they satisfy refinements such as the the Cho-Kreps intuitive criterion. Such equilibria are known as least-cost separating equilibria. A price-shortage pair $(p^*, \alpha^*)$ is a least-cost separating equilibrium if and only if it solves

$$\max_{\alpha, p_1 \leq \theta s_1} \alpha (p_1 - c_1) + \delta (\theta s_1 - c_1) \tag{5}$$

subject to

$$\alpha(p_1 - c_1) \geq \theta s_0 - c_1, \tag{6}$$

and

$$\alpha(p_1 - c_0) + \delta \max\{\theta s_0 - c_0, (1 - \alpha)(\theta s_1 - c_0)\} \leq (1 + \delta)(\theta s_0 - c_0). \tag{7}$$

Condition (6) is equivalent to the incentive constraint

$$\alpha(p_1 - c_1) + \delta(\theta s_1 - c_1) \geq \theta s_0 - c_1 + \delta(\theta s_1 - c_1),$$
under which the high-quality firm does not have any incentive to mimic the low-quality type’s period choice \((\theta s_0, 0)\). Similarly, (7) is equivalent to the incentive constraint, which makes it not desirable for the low-quality firm to mimic the high-quality firm’s period 1 choice of \((p, \alpha)\) and then either supplies to the entire population of consumers at price \(\theta s_0\) or supplies only to \((1 - \alpha)\) fraction of them by continuing to mimic the high-quality firm’s choice of charging price \(\theta s_1\) in period 2. When mimicking in period 1, the low-quality firm can mislead consumers to whom it did not supply in period 1 by mimicking the high-quality type’s period 2 choice.

**Lemma 1** Let \((p_1, \alpha)\) be a price-shortage pair. If

\[
\alpha(p_1 - c_0) + \delta \max\{\theta s_0 - c_0, (1 - \alpha)(\theta s_1 - c_0)\} < (1 + \delta)(\theta s_0 - c_0)
\]

Then, \((p_1, \alpha)\) cannot solve problem (5); hence, it cannot be the high-quality firm’s choice in the least-cost separating equilibrium.

**Proof.** Suppose first \(\theta s_0 - c_0 \geq (1 - \alpha)(\theta s_1 - c_0)\). In this case, if \(p_1 < \theta s_1\), then, by slightly increasing the price from \(p_1\) to \(p_1'\), then \((p_1', \alpha)\) also satisfies (8). Since \(\alpha(p_1' - c_0) > \alpha(p_1 - c_0)\), \((p_1', \alpha)\) satisfies (6) whenever \((p_1, \alpha)\) does. Notice also that (8) implies (7). It follows that \((p_1, \alpha)\) cannot solve problem (5). If \(p_1 = \theta s_1\), then \(\alpha < 1\) because \(\theta s_1 - c_0 > \theta s_0 - c_0\). Thus, by slightly increasing \(\alpha\) to \(\alpha'\), we can guarantee that \((p_1, \alpha')\) satisfy (8).\(^5\) Thus, since \(\alpha'(p_1 - c_1) > \alpha(p_1 - c_1)\), \((p_1, \alpha)\) cannot solve problem (5).

Suppose now \(\theta s_0 - c_0 < (1 - \alpha)(\theta s_1 - c_0)\). In this case, \(\alpha \neq 1\). Thus, as before, we can increase the maximum value of problem (5) by keeping price \(p_1\) and slightly increasing \(\alpha\) without violating (6) and (7). \(\blacksquare\)

Lemma 1 shows that (7) must be binding at any solution to problem (5). We

\(^5\)Notice \(\theta s_0 - c_0 \geq (1 - \alpha)(\theta s_1 - c_0)\) implies \(\theta s_0 - c_0 > (1 - \alpha')(\theta s_1 - c_0)\) for all \(\alpha' > \alpha\).
now apply this lemma to show there is a unique solution for problem (5), which implies positive shortage.

We assume the following conditions on the parameters:

**A1:**
\[ \theta s_k > c_k \text{ where } k = 0, 1 \text{ and } \delta > \max \left\{ \frac{\theta s_1 - c_0}{c_1 - c_0}, \frac{\theta s_0 - c_0}{\theta(s_1 - s_0)} \right\}. \]

The following proposition characterizes the unique least-cost separating equilibrium under assumption **A1**.

**Proposition 1** Assume **A1**. Then, (5) has a unique solution \((p_1^*, \alpha^*)\) given by
\[ \alpha^* = \frac{\theta(s_1 - s_0)}{\theta s_1 - c_0} \]
and
\[ p_1^* = c_0 + \frac{(\theta s_0 - c_0)(\theta s_1 - c_0)}{\theta(s_1 - s_0)}. \]

**Proof.** Let \((p_1, \alpha)\) be a solution for problem (8). By Lemma 1,
\[ \alpha(p_1 - c_0) + \delta \max\{\theta s_0 - c_0, (1 - \alpha)(\theta s_1 - c_0)\} = (1 + \delta)(\theta s_0 - c_0). \quad (9) \]
We break the rest of the proof into two cases.

- **Case 1:** \(\alpha \geq \theta(s_1 - s_0)/(\theta s_1 - c_0)\).

  In this case, we have \((\theta s_0 - c_0) \geq (1 - \alpha)(\theta s_1 - c_0)\). Thus, by (9), \(\alpha(p_1 - c_0) = (\theta s_0 - c_0)\), which implies \(p_1 = c_0 + (\theta s_0 - c_0)/\alpha\). It follows that \(\alpha(p_1 - c_1) = \theta s_0 - c_0 - \alpha(c_1 - c_0)\) is decreasing in \(\alpha\).

- **Case 2:** \(\alpha \leq \theta(s_1 - s_0)/(\theta s_1 - c_0)\).
In this case, we have \((\theta s_0 - c_0) \leq (1 - \alpha)(\theta s_1 - c_0)\). Thus, by (9), \(\alpha(p_1 - c_0) + \delta(1 - \alpha)(\theta s_1 - c_0) = (1 + \delta)(\theta s_0 - c_0)\) which implies

\[
p_1 = c_0 - \delta(\theta s_1 - c_0) \frac{1 - \alpha}{\alpha} + \frac{\theta s_0 - c_0}{\alpha}.
\]

It follows that \(\alpha(p_1 - c_1) = \theta s_0 - c_0 - \delta(\theta s_1 - c_0) + \delta \alpha(\theta s_1 - c_0) - \alpha(c_1 - c_0)\). Since \(\delta(\theta s_1 - c_0) > (c_1 - c_0)\), we conclude that \(\alpha(p_1 - c_1)\) is increasing in \(\alpha\).

In summary, we have shown that under (7), \(\alpha(p_1 - c_1)\) is decreasing in \(\alpha\) when \(\alpha \geq \theta(s_1 - s_0)/(\theta s_1 - c_0)\) and it is increasing when \(\alpha \leq \theta(s_1 - s_0)/(\theta s_1 - c_0)\). This concludes that in any least-cost separating equilibrium it must be

\[
\alpha = \frac{\theta(s_1 - s_0)}{\theta s_1 - c_0}
\]

(10)

and

\[
p_1 = c_0 + \frac{\theta s_0 - c_0}{\alpha} = c_0 + \frac{(\theta s_0 - c_0)(\theta s_1 - c_0)}{\theta(s_1 - s_0)}.
\]

(11)

This establishes the uniqueness.

To show the existence, notice first that (10) and (11) together with the assumptions imply \(0 < \alpha^* < 1\) and \(c_1 < p_1^* < \theta s_1\). By the preceding analysis, \((p_1^*, \alpha^*)\) maximizes \(\alpha(p_1 - c_1)\) subject to (7). Thus, to complete the rest of the proof, it suffices to shows that \((p_1^*, \alpha^*)\) satisfies (6). Notice

\[
\alpha^*(p_1^* - c_1) \geq (\theta s_0 - c_1) \iff \frac{\theta(s_1 - s_0)}{\theta s_1 - c_0} \leq 1.
\]

Since \(\theta s_0 > c_0\), the above condition is automatically satisfied.

By Proposition 1, the high-quality firm’s total profit in two periods is:

\[
\alpha^*(p_1^* - c_1) + \delta(\theta s_1 - c_1) = \theta s_0 - c_0 + \delta(\theta s_1 - c_1) - \frac{\theta(s_1 - s_0)}{\theta s_1 - c_0}(c_1 - c_0).
\]

(12)

From (4) and (12), it follows that the high-quality firm is better off signaling its quality type via the price-shortage pair if and only if \(p_1^* - c_1 < \alpha^*(p_1^* - c_1)\), which in turn is equivalent to

\[
\mu < \frac{(\theta s_0 - c_0)(c_1 - c_0)}{\theta(s_1 - s_0)(\theta s_1 - c_0)}.
\]

(13)
We summarize this result in the following proposition whose proof will be omitted.

**Proposition 2** Assume A1. Then, the price-shortage pair is a more profitable signal than the price-advertisement pair if and only if \( \mu \) satisfies (13).

The reason why price alone or price-advertising combination fails to signal high-quality is that the high quality firm’s choice can be always profitably mimicked by the low-quality type, and there is no gain in the future by doing so. The result is discussed in Tirole (1988) in which first period price is uninformative if the low-quality monopolist can earn profit under full information. However, it is shown in this paper that limiting supply (inducing shortage) is more costly to the low-quality firm since the opportunity cost in terms of the lost profit is higher due to cost differential \( c_0 < c_1 \).

4 **Signalling with Heterogeneous Consumers**

In this section we consider a familiar generalization of the model in section 3 that allows for heterogeneous consumers. The unit-demand differentiated consumer formulation of the problem follows Wolinsky (1983), Chan and Leland (1982), Cooper and Ross (1984, 1985), and Farrell (1980), and the notations follow Tirole (1988). Specifically, there are two types of consumers in terms of their tastes for quality. The value of the good with quality level \( s_k \) is \( \theta_1 s_k \) for type 1 consumers and \( \theta_0 s_k \) for type 0 consumers. The proportion of type 1 consumers is denoted by \( q_1 \). This is because the informed consumers exert positive externalities on the uninformed ones and in the second period all consumers either know the true quality (hard information) or can perfectly predict the true quality in a separating equilibrium. In this case, price and advertisement are still substitutes for the high-quality firm. But, we show that the high-quality firm is better off signaling its quality type than
not signaling at all. However, under certain range of values for the parameters of the model, the high-quality firm can be more profitable to signal its quality type by limiting its supply than by price.

We make the following further assumptions:

**A2:**

\[
q_1 \equiv \frac{\theta_0 s_0 - c_0}{\theta_1 s_1 - c_0} \leq q_1 \leq \frac{\theta_0 s_0 - c_0}{\theta_1 s_0 - c_0} \equiv q_1^*.
\]

Assumption **A2** implies that it is profitable for the low-quality firm to sell its good to both types of consumers under full information. In addition, it also makes the quality signaling problem non-trivial, because it implies that the low-quality firm is better off charging the high-quality firm’s complete information monopoly price, if in so doing it is perceived as the high-quality firm.

Notice that **A2** implies

\[
q_1 > \frac{\theta_0 s_0 - c_1}{\theta_1 s_1 - c_1}.
\]

This means that when information is complete, it is more profitable for the high-quality firm to sell its good type 1 consumers.

### 4.1 Signaling Quality by Price

When the high-quality firm uses price to signal its quality type, the incentive compatibility constraints become

\[
q_1 \left[ (p_1 - c_1) + \delta (\theta_1 s_1 - c_1) \right] \geq \theta_0 s_0 - c_1 + \delta q_1 (\theta_1 s_1 - c_1),
\]

(14)

and

\[
q_1 (p_1 - c_0) + \delta (\theta_0 s_0 - c_0) \leq (1 + \delta) (\theta_0 s_0 - c_0).
\]

(15)

Observe that (14) and (15) are consistent under assumption **A2**, in the sense that there are price-shortage pairs that simultaneously satisfy them. Observe also that
in least-cost separating equilibrium, the high-quality firm’s price solves

$$\max_{p_1 \leq \theta_1 s_1} q_1 [(p_1 - c_1) + \delta (\theta_1 s_1 - c_1)]$$

subject to (14) and (15).

Simple analysis shows that the high-quality firm’s price in the least-cost separating equilibrium must be:

$$p_1^* = c_0 + \frac{\theta_0 s_0 - c_0}{q_1}. \quad (17)$$

For later references, we summarize this result in the following proposition whose proof will be omitted.

**Proposition 3** Assume A1-A2. Then, there exists a unique least-cost separating equilibrium, in which the high-quality firm signals its quality type by period 1 price in (17).

**Example 2:** Let $c_1 = \frac{1}{2}, c_0 = 0, \theta_1 = 2, \theta_0 = 1, s_1 = 1, s_0 = \frac{1}{2}, \delta = 1$. Then, A1 and A2 are satisfied, with $\left[\frac{1}{4}, \frac{1}{2}\right]$ as the interval of feasible proportions $q_1$. The least-cost separating signalling price is monotonically decreasing from $p_1^* = 2$ when $q_1 = \frac{1}{4}$ and $p_1^* = 1$ when $q_1 = \frac{1}{2}$ (see Tirole, 1988, pp. 121, exercise 2.7 for the original example).

### 4.2 Signaling High Quality by Limiting Supply

As before, a consumer will not learn the quality of the good if he does not buy or cannot buy in period 1. Thus, when the high-quality firm signals its quality type by limiting supply, the incentive compatibility constraints become

$$\alpha q_1 (p_1 - c_1) + \delta q_1 (\theta_1 s_1 - c_1) \geq \theta_0 s_0 - c_1 + \delta q_1 (\theta_1 s_1 - c_1) \quad (18)$$
and
\[ \alpha q_1 (p_1 - c_0) + \delta \max \{\theta_0 s_0 - c_0, (1 - \alpha) q_1 (\theta_1 s_1 - c_0)\} \leq (1 + \delta) (\theta_0 s_0 - c_0). \] (19)
In least-cost separating equilibrium, the high-quality firm’s period choice \((p_1, \alpha)\) solves
\[ \max_{p_1 \leq \theta_1 s_1} \alpha q_1 [(p_1 - c_1) + \delta (\theta_1 s_1 - c_1)] \] (20)
subject to (18) and (19).
Furthermore, the incentive compatibility constraint (19) is binding in least-cost separating equilibrium. We summarize this result in the following lemma. Its proof is similar to the proof of Lemma 1. For this reason, we omit the proof.

**Lemma 2** Let \((p_1, \alpha)\) be the high-quality firm’s choice in least-cost separating equilibrium. Then, (19) must be binding.

In what follows we characterize the least-separating equilibrium separately for two disjoint ranges of parameter values. The first range is determined by the following assumptions:

**A3:**
\[ q_1 \geq 2(\theta_0 s_0 - c_0) \frac{\theta_1 s_1 - c_0}{\theta_1 s_1 - c_0} \equiv \tilde{\alpha}_1 \quad \text{and} \quad \delta > \frac{c_1 - c_0}{\theta_1 s_1 - c_0}. \]

The following proposition establishes a unique least-cost separating equilibrium under assumptions **A1, A2** and **A3**.

**Proposition 4** Assume **A1-A3**. Then, there is a unique least-cost separating equilibrium in which the high-quality firm chooses
\[ \alpha^* = 1 - \frac{\theta_0 s_0 - c_0}{q_1 (\theta_1 s_1 - c_0)}, \] (21)
and
\[ p_1^* = c_0 + \frac{(\theta_1 s_1 - c_0) (\theta_0 s_0 - c_0)}{q_1 (\theta_1 s_1 - c_0) - (\theta_0 s_0 - c_0)}. \] (22)
Proof. The proof is similar to that of Proposition 1 hence it is provided in the appendix.

Example 2 (continued): Consider the same parameter values for $\theta_1, \theta_0, s_1, s_0, c_1, c_0, \delta$ as before. When $q_1 = \frac{1}{2}$, a high quality monopolist’s first period profit in a signalling equilibrium by limit supplying is $\frac{3}{8}$ where it is $\frac{1}{2}$ if signals by price only, thus there is a cost saving of $\frac{1}{8}$.

Depending on consumers’ prior belief, the least-cost separating equilibrium in Proposition 3 may be less profitable than the pooling equilibrium. Thus, to guarantee that the high-quality firm has incentive to signal quality by limiting supply, we need the following condition on the prior belief.

\[ \mu \geq (1 - \alpha^*)(c_1 - c_0) - s_0(\theta_1 - \theta_0) \]

\[ \frac{1}{\theta_1(s_1 - s_0)}. \]

We now consider the other range of parameter values guaranteeing the existence of a least-cost separating equilibrium. This range is determined by

**A4:**

\[ \frac{\theta_0 s_0 - c_0}{\theta_1 s_1 - c_0} < q_1 \leq \frac{2(\theta_0 s_0 - c_0)}{\theta_1 s_1 - c_0}. \]

In the case, we show that the high-quality firm must set the highest price in least-cost separating equilibrium.

**Proposition 5** Assume **A1** and **A4.** Then, there is a unique least-cost separating equilibrium in which the high-quality firm’s choice is given by

\[ \alpha^* = \frac{\theta_0 s_0 - c_0}{q_1(\theta_1 s_1 - c_0)}, \] (23)

and

\[ p_1^* = \theta_1 s_1. \] (24)
Proof. By Lemma 2, (19) must be binding in least-cost separating equilibrium:

\[ \alpha q_1(p_1 - c_0) + \delta \max\{\theta_0 s_0 - c_0, (1 - \alpha)q_1(\theta_1 s_1 - c_0)\} = (1 + \delta)(\theta_0 s_0 - c_0). \] (25)

Notice that the price in (22) is not feasible under A4. Notice also that A4 together with the binding incentive compatibility constraint on the low-quality firm implies that \( \theta_0 s_0 - c_0 < (1 - \alpha)q_1(\theta_1 s_1 - c_0) \) cannot be satisfied. Thus, in the rest of the proof, we focus on price-shortage pairs \((p_1, \alpha)\) satisfying \( \theta_0 s_0 - c_0 \geq (1 - \alpha)q_1(\theta_1 s_1 - c_0) \) and (25). Equivalently,

\[ \alpha \geq 1 - \frac{\theta_0 s_0 - c_0}{q_1(\theta_1 s_1 - c_0)} \text{ and } p_1 = c_0 + \frac{(\theta_0 s_0 - c_0)}{\alpha q_1} \leq \theta_1 s_1. \] (26)

The constraint on price \( p_1 \) in (26) in turn implies

\[ \alpha \geq \frac{\theta_0 s_0 - c_0}{q_1(\theta_1 s_1 - c_0)}. \] (27)

With A4, the lower bound on \( \alpha \) in (27) is no less than the lower bound in (26). Furthermore, the high-quality firm’s period 1 profit is decreasing in \( \alpha \) over \((p_1, \alpha)\) satisfying (26) and (27). It follows that in least-separating equilibrium, the high-quality firm’s period 1 choice must be given by (23) and (24). This establishes the uniqueness of least-cost separating equilibrium. The existence is established by the fact the the price-shortage pair in (23) and (24) solves problem (20) associated with least-cost separating equilibrium. ■

In what follows, we characterize the existence and rankings of these separating equilibria as well as pooling equilibria when consumers differ. The following assumption guarantees the existence of separating equilibria using price-shortage pairs with price strictly below the highest possible price in complete information case.

A5:

\[ \theta_1(s_1 - s_0) > \theta_1 s_0 - c_0. \]
Assumption A5 guarantees that $\tilde{q}_1 < \overline{q}_1$ hence not all separating equilibria characterized by Proposition 4 are dominated. In particular, profit comparisons yield the following two corollaries.

**Corollary 1.** Assume A5 holds. then separating equilibria characterized by Proposition 4 in which the monopoly firm signal through a price-shortage pair with highest possible price are always most profitable separating equilibria for $q_1 \in [\hat{q}_1, \overline{q}_1]$ which is a non-empty set.

**Corollary 2.** Assume A2 holds. then separating equilibria exist. Let

$$\tilde{q}_1 = \frac{(\theta_0 s_0 - c_0)(\theta_1 s_1 - c_1)}{(\theta_1 s_1 - c_0)(c_1 - c_0)}.$$

Then if

$$c_1 - c_0 \leq \theta_1 s_1 - c_1 \leq 2(c_1 - c_0)$$

holds, then equilibria characterized by (17) using price alone to signal is the most profitable separating equilibria for $q_1 \in [\tilde{q}_1, \overline{q}_1]$; equilibria characterized by Proposition 5 in which firm signals through a price-shortage pair with highest possible price is the most profitable separating equilibria for $q_1 \in [\hat{q}_1, \overline{q}_1]$.

Likewise, we give the condition when pooling equilibria are Pareto superior to a high-quality monopolist. More formally,

$$\tilde{\alpha}' q_1 (\overline{p}_1 - c_1) \leq q_1 [\mu \theta_1 s_1 + (1 - \mu) \theta_1 s_0 - c_1],$$

from which we obtain

$$\mu \geq \frac{(\theta_0 s_0 - c_0)(\theta_1 s_1 - c_1) - q_1 (\theta_1 s_0 - c_1)}{q_1 (\theta_1 s_1 - \theta_1 s_0)(\theta_1 s_1 - c_0)} \equiv \tilde{\mu}'.$$

### 5 Conclusion

In this paper we have considered the possibility for a monopoly firm of a new product to signal quality by inducing excess demand. We have established results for a
simple two period model which illustrates the strategic role of limit supplying. In both homogenous and heterogeneous consumer assumptions we show that seller induced excess demand can signal high quality under general conditions. With some additional conditions, seller-induced excess demand is a more profitable signal of high quality than using price alone. In addition, signaling high quality by inducing excess demand is always accompanied with high price that exploits all consumer surplus for individual consumers who purchase in the introductory phase. Our results provide a rationale for “limited editions”, capacity constraints, or queuing, together with a high price in the introductory phase of a high-quality product provided by a monopoly firm.

References


6 Appendix

Proof of Proposition 4.

Proof. Consider the binding condition (25) in least-cost separating equilibrium. Let \((p_1, \alpha)\) satisfy (25). We break the rest of the proof into two cases.

- **Case 1:** \(\alpha \geq 1 - \frac{\theta_0s_0 - c_0}{q_1(\theta_1s_1 - c_0)}\).

  In this case, \(\theta_0s_0 - c_0 \geq (1 - \alpha)q_1(\theta_1s_1 - c_0)\). Thus, by (25), \(\alpha q_1(p_1 - c_0) = (\theta_0s_0 - c_0)\), which implies \(p_1 = c_0 + (\theta_0s_0 - c_0)/\alpha q_1\). It follows that high-quality firm’s first period profit \(\alpha q_1(p_1 - c_1) = \theta_0s_0 - c_0 - \alpha q_1(c_1 - c_0)\) is decreasing in \(\alpha\).

- **Case 2:** \(\alpha \leq 1 - \frac{\theta_0s_0 - c_0}{q_1(\theta_1s_1 - c_0)}\).

  In this case, \(\theta_0s_0 - c_0 \leq (1 - \alpha)(\theta_1s_1 - c_0)\). Thus, by (25), \(\alpha(p_1 - c_0) + \delta(1 - \alpha)q_1(\theta_1s_1 - c_0) = (1 + \delta)(\theta_0s_0 - c_0)\) which implies that the high-quality firm’s first period profit is \(\alpha q_1(p_1 - c_1) = (1 + \delta)(\theta_0s_0 - c_0) - \delta q_1(\theta_1s_1 - c_0) + \alpha q_1(\theta_1s_1 - c_0) - \alpha q_1(c_1 - c_0)\). Since \(\delta > (c_1 - c_0)/(\theta_1s_1 - c_0)\) and \(\theta_1s_1 - c_0 > c_1 - c_0\), it follows that \(\tilde{\alpha}q_1(\tilde{p}_1 - c_1)\) is increasing in \(\tilde{\alpha}\).

In summary, the analysis in case 1 and case 2 together concludes that in any least-cost separating equilibrium it must be

\[
\alpha = 1 - \frac{\theta_0s_0 - c_0}{q_1(\theta_1s_1 - c_0)}, \tag{A-1}
\]

and

\[
p_1 = c_0 + \frac{(\theta_1s_1 - c_0)(\theta_0s_0 - c_0)}{q_1(\theta_1s_1 - c_0) - (\theta_0s_0 - c_0)}. \tag{A-2}
\]

This establishes the uniqueness. To show the existence, notice first that A2 and (A-1) imply \(0 < \alpha < 1\). Notice also \(c_1 < p_1\) is automatically satisfied. By A3, \(q_1(\theta_1s_1 - c_0) > 2(\theta_0s_0 - c_0)\) which together with (A-2) implies \(p_1 < \theta_1s_1\). Since \((p_1, \alpha)\) in (A-1) and (A-2) maximizes the high-quality firm’s period 1 profit subject
to (19), to complete the rest of the proof it suffices to show that \((p_1, \alpha)\) also satisfies (18). Notice

\[
\alpha q_1(p_1 - c_1) \geq \theta_0 s_0 - c_1 \iff \frac{q_1(\theta_1 s_1 - c_0) - (\theta_0 s_0 - s_0)}{\theta_1 s_1 - c_0} \leq 1,
\]

which always holds. ■