Interest Rate Rules and the Credit Channel of Monetary Policy Transmission

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Abstract

This paper develops a general equilibrium model with the banking system and the reserves market, where the central bank affects the federal funds rate through open market operations. We show that when the coefficient that measures the response of the federal funds rate to the inflation rate falls below a threshold value, which is very small, the economy will have two steady states, where the low-equilibrium federal funds rate one is a saddle and the high-equilibrium federal funds rate one is a sink. Otherwise, the economy has a unique steady state which exhibits local determinacy.

Keywords: Nominal Interest Rate Rules, Indeterminacy, The Credit Channel of Monetary Policy Transmission.

JEL Classification: E51, E58.

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1 Introduction

This paper develops a general equilibrium model with the banking system, the reserves market, and central bank’s open market operations to revisit the controversial issue in the literature of monetary economics regarding the macroeconomic stabilizing property of the nominal interest rate rules. It is motivated by the notice that central bankers whose monetary policies are described as the nominal interest rate rules conduct open market operations to adjust the supply of reserves in the reserves market, with an aim to achieve their targets for the overnight loan rate (the federal funds rate in the United States). Nevertheless, as a conventional practice in the literature on the nominal interest rate rules, the authors all use the nominal interest rate on government bonds as a proxy for the federal funds rate. Such a kind of manipulation seems plausible since the nominal interest rates on alternative financial instruments tend to move together over time. Technically, it is because the theoretical models do not incorporate the banking system nor the reserves market, and hence is incapable of dealing with issues regarding the federal funds rate.

The ability that the central bank affects the federal funds rate through open market operations is known as “the liquidity effect” of monetary policy. Changes in the federal funds rate in turn influence commercial banks’ demand for excess reserves and hence the supply of loans to borrowers, which eventually affect aggregate demand in the economy. Such a channel of transmission for monetary policy, known as “the credit channel of monetary transmission,” is absent in traditional models without the banking system and the reserves market. The literature on the credit channel of monetary policy transmission has demonstrated the important role of bank lending in explaining the length and the depth of business fluctuations [Bernanke 1983; Bernanke and Blinder 1988; Bernanke and Gertler 1995]. The development of this literature is in the light of the asymmetric treatment on bank assets and bank liabilities in traditional models. Specifically, as Bernanke and Blinder (1988, p.435) points out, “Money, the bank liability, is given a special role in the determination of aggregate demand. In contrast, bank loans are lumped together with other debt

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instruments in a “bond market,” which is then conveniently suppressed by Walras’ Law.” In that paper, Bernanke and Blinder develop a variant of the IS/LM model which allows roles for both money and credit (bank loans). Both this paper and Bernanke and Gertler (1995) demonstrate the enhancement mechanism of the credit channel. Due to the existence of the credit channel, monetary policy still matters even in a liquidity trap. Bernanke and Blinder (1992), Kashyap et al. (1993), and Kashyap and Stein (1995) then provide empirical evidence that monetary transmission works through bank loans as well as bank deposits.2

Since the seminal work of Taylor (1993), a vast literature on the nominal interest rate rules has been developed. It is well-known that the Taylor rule (Taylor, 1993) is the kind of the nominal interest rate rule that sets the federal funds rate as a function of the inflation gap (the deviation of the inflation rate from its target level) and the output gap (the deviation of output from its natural rate level), with the coefficient on the inflation gap exceeding unity. Taylor (1993, 1999) emphasize that a rule of this type has the benefit of stabilizing inflation. Monetary policy operation procedure of this kind is then known as the “Taylor principle” (Woodford, 2001). It is well-known that many authors suggest that to avoid real indeterminacy the central bank should adhere to the Taylor principle. Nevertheless, still many others demonstrate that steering under the Taylor principle may introduce real indeterminacy in an otherwise determinacy economy.3

As we emphasized, one common model feature of the existing works is the use of the nominal interest rate of government bonds as a proxy for the federal funds rate. This paper departs from this conventional manipulation by incorporating into the model the banking system and the reserves market, in so doing the equilibrium federal funds rate can be determined in the market for reserves. By means of the framework, the main purpose of this paper is the re-examine the macroeconomic stabilizing property of the nominal interest rate rules. To facilitate comparison, we start the analysis with a simple model without the banking system. The result obtained is very standard. Specifically, active rules (rules that satisfy the Taylor

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2 Other authors who analyze and support this credit view include Fuerst (1992), Li (2000), Einars-son and Marquis (2002), Li and Chang (2004), Gillman and Kejak (2004), Auray and Fève (2005), Chang et al. (2007), and Claus (2007), among others.

3 See Benhabib and Farmer (1999), McCallum (2003), and Woodford (2003) for a literature review.
principle) ensure equilibrium uniqueness, whereas passive rules (rules that violate the Taylor principle) generate equilibrium indeterminacy.

We then extend the basic model by incorporating the banking system and the reserves market. Meanwhile, we characterize open market operations of the central bank. Our numerical result shows that under active rules, there is always a unique steady state which exhibits saddle-path stability. Passive rules can also ensure a unique steady state and local determinacy as long as the coefficient that measures the response of the federal funds rate to the inflation rate does not fall below a threshold value. If the coefficient that measures the response of the federal funds rate to the inflation rate falls below the threshold value, then the economy will have two steady states, where the low-equilibrium federal funds rate one exhibit saddle-path stability and the high-equilibrium federal funds rate one is subject to endogenous business fluctuations. We demonstrate that, under plausible parameter values, the threshold value for the coefficient on the inflation rate is very small. This indicates that the indeterminacy region under the nominal interest rate rules is very small. Furthermore, we find that even when two steady states exist, the high-equilibrium federal funds rate one which exhibits local indeterminacy has a federal funds rate that is too high to be empirically plausible. Thus, our results lead to the conclusion that, within a cash-in-advance economy without investment and under plausible values of the federal funds rate, the nominal interest rate rules are always stabilizing, no matter whether it is active or passive.

The remainder of this paper is organized as follows. Section 2 presents a basic model without the banking system as a benchmark. Section 3 describes a general equilibrium model with the banking system and the reserves market and analyzes the existence and number of the economy’s steady state, together with the local stability properties. Section 4 concludes.

2 Basic Model without Banking System

The model is essentially a simplified version of Meng (2002) in that we assume inelastic labor supply and a log utility to make things simple – as Meng (2002) demonstrates, endogenous labor supply along with the CRRA utility function complicate the macroeconomic stabilizing property of the interest rate rules. It can also be inter-
interpreted as modifying the flexible-price model in Benhabib et al. (2001) by considering a production economy with a cash-in-advance constraint. We assume that there are no fundamental uncertainties present in the economy.

There is a continuum of identical competitive firms in the economy, with the total number normalized to one. The specification of the firms’ production technology follows that used in the sticky-price model in Benhabib et al. (2001):

\[ y_t = h_t^\alpha, \quad 0 < \alpha < 1, \]  

(1)

where \( y_t \) is output and \( h_t \) is labor input.

Let \( w_t \) denote the real wage rate. Under the assumption that labor market is perfectly competitive, the first-order condition for the firms’ profit maximization problem gives

\[ w_t = \frac{\alpha y_t}{h_t}. \]  

(2)

Equation (2) then implies that firms earn positive economic profits which equal

\[ \Pi_{ft} = (1 - \alpha) y_t. \]  

(3)

The economy is also populated by a unit measure of identical infinitely-lived households, each endowed with one unit of time and supplies its time inelastically to the production of output. The representative household maximizes a stream of discounted utilities over sequences of consumption:

\[ U = \int_0^\infty \ln c_t e^{-\rho t} dt, \]  

(4)

where \( c_t \) is the individual household’s consumption, \( \rho \in (0, 1) \) denotes the subjective discount rate.

Assume that all consumption purchases must be financed by the household’s real balances \( m_t \), which equals the nominal money supply \( M_t \) divided by the price level \( P_t \). The representative household thus faces the following cash-in-advance constraint:

\[ c_t \leq m_t. \]  

(5)
The household also holds nominal government bonds \( B_t \) that pay the nominal interest rate \( R_t > 0 \). As owners of all firms, households receive profits in the form of dividends. The budget constraint faced by the representative household thus is given by

\[
\dot{m}_t + \dot{b}_t = (R_t - \pi_t)b_t - \pi_t m_t + w_t - c_t + \Pi_{ft}, \quad m_0, b_0 > 0 \quad \text{given,}
\]

where \( b_t (\equiv B_t/P_t) \) denotes real government bonds, and \( \pi_t \) is the inflation rate.

Let us denote the real financial wealth as \( a_t \equiv m_t + b_t \). We can then express the representative household’s budget constraint (6) as the following form that appears in Meng (2002) and Benhabib et al. (2001), among many others:

\[
\dot{a}_t = (R_t - \pi_t)a_t - R_t m_t + w_t - c_t + \Pi_{ft}.
\]

The representative household treats \( R_t, \pi_t, w_t, \) and \( \Pi_{ft} \) as given and maximizes (4) subject to (5) and (7) by choosing a sequence \( \{c_t, m_t, a_t\}_{t=0}^{\infty} \). Let us denote \( \lambda_t \) as the shadow value of real financial wealth and \( \eta_t \) as the Lagrange multiplier for the CIA constraint (5). As is common in the literature, we assume that the CIA constraint (5) is strictly binding in equilibrium, thus \( \eta_t > 0 \) for all \( t \). The first-order conditions for the representative household with respect to the indicated variables and the associated transversality conditions (TVC) are

\[
c_t \quad : \quad c_t^{-1} = \lambda_t + \eta_t, \quad \text{ (8)}
\]

\[
m_t \quad : \quad R_t = \frac{\eta_t}{\lambda_t}, \quad \text{ (9)}
\]

\[
a_t \quad : \quad \dot{\lambda}_t = (\rho - R_t + \pi_t)\lambda_t, \quad \text{ (10)}
\]

\[
\text{TVC} \quad : \quad \lim_{t \to \infty} e^{-\rho t} \lambda_t a_t = 0. \quad \text{ (11)}
\]

Equation (8) states that the marginal benefit of consumption equals its marginal cost, which is the marginal utility of having an additional real dollar. Equation (9) equates the marginal costs of holding money balances, which is the nominal interest rate, to the marginal benefits from holding money balances. Equations (8) and (9) together imply that \( R_t = \frac{1}{c_t \lambda_t} - 1 \). Equation (10) governs the evolution of the shadow value of real wealth.
The government issues money and bonds to finance its expenditure on interest payments. The flow budget constraint of the government is thus given by:

\[
\dot{B}_t + \dot{M}_t = R_t B_t,
\]

which can be expressed as

\[
\dot{a}_t = (R_t - \pi_t) a_t - R_t m_t.
\]

In accordance with the exiting literature, we assume that the central bank follows an interest rate feedback rule as follows:

\[
R_t = \psi(\pi_t),
\]

where the function \( \psi(\cdot) \) is positive, increasing, and differentiable. Let \( \pi^* \) denotes the steady-state inflation rate. In line with Leeper (1991), Meng (2002), and Benhabib et al. (2001), we refer to monetary policy as passive at \( \pi^* \) if \( \psi'(\pi^*) < 1 \) and as active at \( \pi^* \) if \( \psi'(\pi^*) > 1 \).

From (14), we obtain an expression of \( \pi_t \) as \( \pi_t = \psi^{-1}(R_t) \), where \( \psi^{-1}(\cdot) \) is the inverse function of \( \psi(\cdot) \). Therefore, \( \psi^{-1} = \frac{1}{\psi'} > 0 \). We then substitute this expression of \( \pi_t \) into (10) to obtain

\[
\dot{\lambda}_t = [\rho - R_t + \psi^{-1}(R_t)] \lambda_t.
\]

By using (2) and (3) to substitute out \( w_t \) and \( \Pi_{ft} \) in (7) and combining the subsequent equation with (13) and imposing the labor market equilibrium condition: \( h_t = 1 \), we obtain the economy’s consolidated budget constraint as follows:

\[
c_t = y_t = 1.
\]

Using the above condition, it then follows from \( R_t = \frac{1}{c_t \lambda t} - 1 \) that the nominal interest rate has a one-to-one relation to the shadow value of real financial wealth:

\[
R_t = \frac{1}{\lambda_t} - 1.
\]
Equations (15) and (17) together give the differential equation of the nominal interest rate $R_t$ that govern the dynamics of this economy:

$$\dot{R}_t = (1 + R_t)[R_t - \psi^{-1}(R_t) - \rho].$$

(18)

By linearizing (18) around the steady state $R^*$, we have

$$\dot{R}_t = \Delta(R_t - R^*),$$

(19)

where $\Delta = \frac{(1+R^*)[\psi'(R^*)-1]}{\psi'(R^*)} > 0$, as $\psi'(R^*) > 1$. Since $R_t$ is a jump variable, we conclude that the steady state is a saddle under active rules and is a sink under passive rules. This is a standard result in the literature.

3 Model with Banking System and the Reserves Market

The previous section shows that in a simple cash-in-advance model with inelastic labor supply, the use of the nominal interest rate of government bonds as the proxy of the federal funds rate (so that the nominal interest rate rules are described by reaction functions of the government bonds rate to the inflation rate) lead to a clear-cut conclusion that active rules can maintain saddle-path stability while passive rules give rise to sunspots fluctuations. In this section, we will demonstrate that the result will be quite different when we introduce into the model the banking system, the reserves market (the federal funds market), the borrowing-lending activities between firms and commercial banks, the central bank’s open market purchases or sales with commercial banks, and, most importantly, we let the federal funds rate to feedback to the inflation rate. To facilitate comparison, the model builds on the one in the previous section and borrows quite a lot from Agénor (1997) for the description of the borrowing-lending activities between firms and commercial banks.

3.1 Producers

Following Agénor (1997), we assume for simplicity that firms have no access to capital markets. Since they cannot raise external funds by issuing corporation bonds and/or equity, the only way they can finance their working capital is borrowing from commercial banks. Working capital needs consist solely of labor costs and must be
financed prior to the sale of output. Total production costs faced by firms equal the wage bill plus the interest payments made on bank loans.

The representative firm’s production technology is the same as that in the previous section (equation (1)): \( y_t = h_t^\alpha \), \( 0 < \alpha < 1 \). Given this production technology, the representative firm’s objective is to choose a sequence \( \{ h_t, l_t^d \}_{t=0}^\infty \) so as to maximize its real (net) profits

\[
\Pi_{ft} = y_t - w_t h_t - r_t l_t^d,
\]

subject to the financial constraint:

\[
w_t h_t \leq l_t^d, \tag{21}
\]

where \( l_t^d \equiv L_t^d / P_t \) is the real amount of loans obtained from commercial banks with \( L_t^d \) denoting nominal amount of loans, and \( r_t \) is the real lending rate charged by commercial banks.

Assume that the firm’s financial constraint (21) is continuously binding since, given that borrowing is costly, there is no reason for the firm to borrow excess funds from commercial banks. As a result, we can re-write the representative firm’s profit function as follows:

\[
\Pi_{ft} = h_t^\alpha - (1 + r_t) w_t h_t. \tag{22}
\]

The representative firm’s first-order condition with respect to \( h_t \) leads to

\[
(1 + r_t) w_t = \frac{\alpha y_t}{h_t}, \tag{23}
\]

which shows that labor demand is inversely related to the effective cost of labor, \( (1 + r_t) w_t \). Comparison of (23) with (2) reveals that when firms borrow from commercial banks to finance their wage bills, the existence of interest burden lowers the maximum wage rate they are willing to offer to workers.

By combining (21) and (23), we derive the firm’s demand for credit as

\[
l_t^d = \frac{\alpha y_t}{1 + r_t}, \tag{24}
\]

which is increasing in output, \( y_t \), and is decreasing in the loan rate, \( r_t \).
Substituting (23) into (22), we obtain that the economic profits the representative firm earns in each period equals

$$\Pi_{ft} = (1 - \alpha)y_t > 0.$$  \hfill (25)

Firms transfer their net income, $q_{ft}$, to their owners, households:

$$q_{ft} = y_t - (1 + r_t)t_l^d - \pi_t l_t^d,$$  \hfill (26)

where the term $\pi_t l_t^d$ accounts for the inflation tax on loans.

### 3.2 Households

Without loss of generality, we assume for simplicity that households have no access to investing in government bonds. As we will describe later, the assumption that both commercial banks and the central bank hold government bonds is enough to characterize the central bank’s open market operations when implementing the interest rate rules. Actually, if households also hold government bonds, an additional no-arbitrage condition between holding government bonds and deposits will be introduced through the households’ first-order conditions of utility maximization. Hence, in this section, we assume that in addition to cash balances $M_t$, households also hold deposits $D_t$ that pay the nominal deposits rate $R_{dt} > 0$. Let $m_t \equiv M_t/P_t$ denotes the real cash balances and $d_t \equiv D_t/P_t$ the real deposits. Then, we assume that the liquidity constraint faced by the representative household is of the form:

$$c_t \leq m_t + \theta d_t,$$  \hfill (27)

which states that all cash holdings and a fraction $\theta \in [0, 1)$ of deposits are used for financing the household’s consumption purchases. The specification that only a fraction $\theta$ of deposits are used for financing consumption purchases can be interpreted as describing the fact that, due to the transactions costs of withdrawing deposits, households will be less willing to use their deposits for purchasing consumption goods. It can also be interpreted as accounting for the transactions costs households bear when they withdraw deposits.

The flow budget constraint of the representative household is given by
\[ \dot{m}_t + d_t = w_t - c_t + r_{dt}d_t - \pi_t m_t + q_{ft} + q_{bd}, \]  

(28)

where \( r_{dt} (= R_{dt} - \pi_t) \) is the real deposits rate, and \( q_{ft} \) and \( q_{bd} \) respectively represent real income received from firms and commercial banks.

The representative household’s objective is to choose a sequence \( \{c_t, d_t, m_t\}_{t=0}^{\infty} \) so as to maximize its life-time utility (4), subject to the liquidity constraint (27) and the budget constraint (28), taking as given \( m_0, d_0, \) and the time paths of \( w_t, r_{dt}, \pi_t, \) \( q_{ft} \) and \( q_{bd} \). As before, we let \( \lambda_t \) denote the shadow value of real financial wealth and \( \eta_t \) the Lagrange multiplier for the CIA constraint (27); moreover, we assume that the CIA constraint (27) is strictly binding in equilibrium, thus \( \eta_t > 0 \) for all \( t \). The first-order conditions for the representative household with respect to the indicated variables and the associated transversality conditions (TVC) are

\[
\begin{align*}
 c_t & : \quad c_t^{-1} = \lambda_t + \eta_t, \\
 d_t & : \quad \dot{\lambda}_t/\lambda_t = \rho - r_{dt} - \theta \frac{\eta_t}{\lambda_t}, \quad (30) \\
 m_t & : \quad \dot{\lambda}_t/\lambda_t = \rho + \pi_t - \frac{\eta_t}{\lambda_t}, \quad (31) \\
 \text{TVC}_1 & : \quad \lim_{t \to \infty} e^{-\rho t} \lambda_t m_t = 0, \quad (32) \\
 \text{TVC}_2 & : \quad \lim_{t \to \infty} e^{-\rho t} \lambda_t d_t = 0. \quad (33)
\end{align*}
\]

Equation (29) is the same as (8) in the previous section which states that the marginal benefit of consumption equals its marginal cost. From (29)-(31) we obtain the nominal rate of deposits as:

\[ R_{dt} = r_{dt} + \pi_t = (1 - \theta) \left[ \frac{1}{c_t \lambda_t} - 1 \right]. \]  

(34)

Actually, if households also hold government bonds, \( \left[ \frac{1}{c_t \lambda_t} - 1 \right] \) in (34) will be the equilibrium nominal interest rate of government bonds and (34) will represent the no-arbitrage condition between holding deposits and government bonds. It is clear that this nominal interest rate of government bonds is the same as that derived in the previous section. In addition, (34) depicts that the existence of the transactions costs of withdrawing deposits (which is captured by \( \theta \)) drives a wedge between the rate of return on deposits and that on government bonds.
3.3 Commercial banks

Assets of commercial banks consist of credit extended to firms, $l_t$, required reserves, $RR_t$, excess reserves, $ER_t$, and the real stock of government bonds, $b_{pt} \equiv B_{pt}/P_t$, which pay the nominal interest rate $R_t > 0$. Assume that commercial banks have no access to money and capital markets. Thus, bank liabilities consist solely of deposits held by households, $d_t$. We further follow Agénor (1997) in assuming for simplicity that banks have no net worth. Commercial banks’ balance sheet can then be written as

$$RR_t + ER_t + l_t + b_{pt} = d_t.$$  \hfill (35)

Required reserves held at the central bank do not pay interest and are determined by

$$RR_t = vd_t, \quad v \in (0,1).$$  \hfill (36)

where $v$ is the reserve requirement ratio. Excess reserves also do not pay interest. The reason for commercial banks to hold excess reserves is that these reserves are insurance against deposit outflows. The opportunity cost of holding excess reserves is the interest rate that could have been earned on lending these reserves out in the reserves market, which is the federal funds rate, $R_{ff}$. Thus, an increase in the federal funds rate represents an increase in the opportunity cost of holding excess reserves. Holding everything else constant, this will induce the commercial banks to reduce their demand for excess reserves. We characterize this behavior of commercial banks by specifying that the excess reserve ratio, $e_t \equiv \frac{ER_t}{d_t}$, is a decreasing function of the federal funds rate:

$$e_t = e(R_{ff}), \quad e' < 0.$$  \hfill (37)

The specification of (37) is also seen in Taylor (2001), Carpentera and Demirap (2004) and Mishkin (2007).

From (35)-(37), we obtain the supply of credit as

$$l_t^s = (1 - v - e_t)d_t - b_{pt}.$$  \hfill (38)
Assume that banks have no operating costs. The net-profits of the representative bank are

\[ \Pi_t = r_{lt}l^b_t + r_t b_{pt} - r_{dt}d_t, \quad (39) \]

where \( r_t = R_t - \pi_t \) is the real interest rate on government bonds.

For commercial banks’ simultaneous holdings of bank loans and government bonds to be a rational behavior, a no-arbitrage condition between loans and government bonds should be imposed, which requires that these two assets yield the same real (net) rate of return: \( r_{lt} = r_t \). It then follows from the zero-net-profit condition of the representative bank that

\[ r_{dt}d_t = r_{lt}(l^b_t + b_{pt}), \quad (40) \]

By using (38) to substitute out \((l^b_t + b_{pt})\) in (40), we then obtain the following relationship between the rates of deposits and lending:

\[ r_{dt} = r_{lt} [1 - v - c(R_{fft})]. \quad (41) \]

The above equation clearly shows that the lending rate \((r_{lt})\) is \textit{ceteris paribus} positively related to the reserve requirement ratio \((v)\). In particular, other things being equal, an increase in the reserve requirement ratio by the central bank reduces the available funds that commercial bank can lend to firms. This tends to induce commercial banks to raise the lending rate on each dollar of the loans.

Finally, since banks do not accumulate assets, net income transferred to households will be

\[ q_{bt} = (1 + r_{lt})l^b_t + r_t b_{pt} - r_{dt}d_t - \pi_t (RR_t + ER_t), \quad (42) \]

where the term \( \pi_t (RR_t + ER_t) \) measures the inflation tax paid on reserves.

### 3.4 The government and the central bank

The central bank follows the same interest rate feedback rule as that in the previous section. However, since we no longer use the government bonds rate as the proxy for the federal funds rate, the expression of the interest rate feedback rule is now
As in the previous section, the function $\psi(\cdot)$ is positive, increasing, and differentiable, and we refer to monetary policy as passive at $\pi^*$ if $\psi'(\pi^*) < 1$ and as active at $\pi^*$ if $\psi'(\pi^*) > 1$.

Assume that the central bank lends only to the government. The central bank’s balance sheet is thus given by

$$b_{gt} = m_t + RR_t + ER_t,$$  \hspace{1cm} (44)

where $b_{gt} = B_{gt}/P_t$ is the real stock of government bonds held by the central bank, with $B_{gt}$ denoting the nominal stock.

The government’s expenditure consists of interest payments to its bond holders, commercial banks and the central bank. Hence, the total interest payments are $R_tB_t$, where $B_t = B_{pt} + B_{gt}$. It receives transfers of the central bank’s revenue $\Omega_t = R_tB_{gt}$. The government’s deficits are then financed by the issuance of government bonds. In nominal terms, the flow budget constraint of the government is written as

$$\dot{B}_t = R_tB_t - \Omega_t = R_{bt}B_{pt}.$$  \hspace{1cm} (45)

In real terms, (45) is expressed as

$$\dot{b}_t = r_tb_{pt} - \pi_tb_{gt} = r_tb_t - (r_t + \pi_t)b_{gt}.$$  \hspace{1cm} (46)

Note from (44) that $b_{gt} = m_t + RR_t + ER_t$. Hence, $\pi_tb_{gt}$ in the first equality of (46) measures the inflation taxes on cash balances and reserves.

### 3.5 Open market operation and reserve market equilibrium

An open market purchase of government bonds from commercial banks increases the stock of government bonds held by the central bank and decreases the stock of government bonds held by commercial banks by the same amount. Therefore, it does not affect the outstanding stock of government bonds. However, an open market purchase injects reserves into the banking system, thereby increases the available
funds that can be used for making new loans. This is the so-called lending channel of monetary policy transmission.

Since our focus is on how the central bank implements the interest rate rules through open market operations, rather than on how the extension of credit by the central bank to the government causes inflation, we assume for simplicity and without loss of generality that at every issuance of bonds the government allocates a fixed proportion $\phi \in (0, 1)$ of the government bonds to the central bank and the remaining proportion $1 - \phi \in (0, 1)$ to commercial banks. The realized stock of government bonds held by the central bank, $b_{gt}$, will be $\phi b_t$ plus the amount due to open market purchases. This indicates that the amount of open market purchases and hence the amount of reserves injected into the reserves market is $b_{gt} - \phi b_t$.

The quantity of reserves demanded equals the sum of required reserves and excess reserves. We assume that the federal funds rate is below the discount rate, which is true at almost every date for every country. Thus, commercial banks will not borrow from the discount window and hence the supply of reserves will equal the nonborrowed reserves, that is the amount of reserves supplied by the central bank’s open market operations: $b_{gt} - \phi b_t$. Reserves market equilibrium requires that the quantity of reserves demanded equals the quantity of reserves supplied, which is written as

$$RR_t + ER_t = b_{gt} - \phi b_t.$$ 

According to (47), the demand of reserves is a downward-sloping curve in a diagram with the federal funds rate on the vertical axis. On the other hand, the supply of reserves is a vertical line in the diagram. Such a viewpoint is also made by Taylor (2001) and Mishkin (2007).

The central bank’s balance sheet (44) and the reserves market equilibrium condition (47) together imply that

$$m_t = \phi b_t,$$

which states that at each instant in time, currency in circulation, $m_t$, equals the real credit allocated by the central bank to the government, $\phi b_t$. 

14
3.6 Credit market equilibrium and the resource constraint

Credit market equilibrium requires that firms’ demand for credit equals credit extended to firms by commercial banks. By using (24) and (38), we can write down this condition as follows:

\[
\frac{\text{\textit{\alpha}}_{\textit{yt}}}{1 + r_{\textit{lt}}} = (1 - v - c_{\textit{t}})d_{\textit{t}} - b_{\textit{pt}} = l_{\textit{t}},
\]

(49)

The above credit market equilibrium condition determines the equilibrium lending rate.

By combining the representative commercial bank’s balance sheet (35) with the central bank’s balance sheet (44) and given that \( l_{\textit{lt}} = l_{\textit{st}} = l_{\textit{t}} \) holds when credit market is in equilibrium, we obtain

\[
m_{\textit{t}} + d_{\textit{t}} = l_{\textit{t}} + b_{\textit{t}},
\]

(50)

which indicates that the money supply, \( m_{\textit{t}} + d_{\textit{t}} \), equals the quantity of loans plus the stock of government bonds. Notice that the quantity of loans is the credit extended to firms by commercial banks, and the stock of government bonds equals the credit extended to the government by commercial banks and the central bank.

By taking time derivation on both sides of (50) and using the government’s budget constraint (46) to substitute out \( \dot{b}_{\textit{t}} \) in the subsequent equation, with (44), we then obtain that changes in the money supply equal

\[
\dot{m}_{\textit{t}} + \dot{d}_{\textit{t}} = \dot{l}_{\textit{t}} + r_{\textit{bt}}b_{\textit{pt}} - \pi_{\textit{t}}(m_{\textit{t}} + RR_{\textit{t}} + ER_{\textit{t}}),
\]

(51)

By substituting \( q_{\textit{ft}} \) in (26) and \( q_{\textit{bt}} \) in (42) into the household’s budget (28), and given that \( l_{\textit{lt}} = l_{\textit{st}} = l_{\textit{t}} \) and \( h_{\textit{t}} = 1 \) hold when both credit market and labor market are in equilibrium, we then have another expression of changes in the money supply as follows:

\[
\dot{m}_{\textit{t}} + \dot{d}_{\textit{t}} = 1 - c_{\textit{t}} + l_{\textit{t}} - \pi_{\textit{t}}l_{\textit{t}} + r_{\textit{bt}}b_{\textit{pt}} - \pi_{\textit{t}}(m_{\textit{t}} + RR_{\textit{t}} + ER_{\textit{t}}).
\]

(52)

Equations (51) and (52) together give the economy’s resource constraint as follows:
\[ \dot{l}_t = 1 - c_t + l_t - \pi_t l_t. \] (53)

### 3.7 Analysis of Local Dynamics

This subsection analyzes the existence and uniqueness of the model’s steady state(s), together with the associated local dynamics. From the model’s equilibrium conditions, we derive the following dynamical system that governs the dynamics of the model:

\begin{align*}
\dot{l}_t &= 1 - [\phi + \theta(1 - \phi)]b_t + [1 - \theta - \psi^{-1}(R_{fft})] \lambda_t, \quad (54) \\
\dot{b}_t &= r_{lt}b_t - r_{lt} + \psi^{-1}(R_{fft}) b_{gt}, \quad (55) \\
\dot{\lambda}_t &= \left[ \rho + \psi^{-1}(R_{fft}) + 1 \right] \lambda_t - \frac{1}{[\phi + \theta(1 - \phi)]b_t + \theta \lambda_t}, \quad (56)
\end{align*}

where \[ \psi^{-1} = \frac{1}{\psi} > 0, \text{ and} \]

\begin{align*}
R_{fft} &= R_{fft}(l_t, b_t, \lambda_t), \quad (57) \\
\dot{b}_{gt} &= b_g(l_t, b_t, \lambda_t), \quad (58) \\
\dot{r}_{lt} &= r_l(l_t), \quad (59)
\end{align*}

with \[ R_{fft,l} \equiv \frac{\partial R_{fft}}{\partial l_t} > 0, \quad R_{fft,b} \equiv \frac{\partial R_{fft}}{\partial b_t} < 0, \quad R_{fft,\lambda} \equiv \frac{\partial R_{fft}}{\partial \lambda_t} < 0, \quad b_{g,l} \equiv \frac{\partial b_g}{\partial l_t} > 0, \quad b_{g,\lambda} \equiv \frac{\partial b_g}{\partial \lambda_t} > 0, \quad \text{ and} \quad r_{l,t} \equiv \frac{\partial r_l}{\partial l_t} < 0. \]

Given the above dynamical system (54)-(56), the steady state is characterized by positive real numbers \((l^*, b^*, \lambda^*)\) that satisfy \[ \dot{l}_t = \dot{b}_t = \dot{\lambda}_t = 0. \] In the second part of the appendix, we demonstrate that the model’s steady-state conditions can be reduced to the following two equations which describe the stationary relationships between \(R_{fft}^*\) and \(l^*\):

\[ l^* = \frac{\alpha \left[ 1 - v - e(R_{fft}^*) \right]}{(1 - \theta)\rho - \theta \psi^{-1}(R_{fft}^*) + 1 - v - e(R_{fft}^*)}, \quad (60) \]

\(^4\)See the appendix for the detailed derivation.

\(^5\)For the details of the partial derivatives of \(R_{fft}, b_{gt}, \text{ and } r_{lt}\) with respect to \(l_t, b_t, \text{ and } \lambda_t\), please see the appendix.
\[
\left\{ \frac{\alpha}{l^*} - 1 - \left[ \frac{\alpha}{l^*} - 1 + \psi^{-1}(R_{ff}^*) \right] \left\{ [v + e(R_{ff}^*)] (1 - \phi) + \phi \right\} \left\{ 1 + [1 - \theta - \psi^{-1}(R_{ff}^*)] l^* \right\} \right. \\
= \left[ \phi + \theta(1 - \phi) \right] \left[ \frac{\alpha}{l^*} - 1 + \psi^{-1}(R_{ff}^*) \right] [v + e(R_{ff}^*)] l^* ,
\]

(61)

where an asterisk denotes the steady state value of a variable. A quick idea to prove the existence and uniqueness of the model’s steady state(s) is to plot (60) and (61) in the \( R_{ff} - l^* \) space. Let us denominate the locus describing the relationship in (60) as \( SS_1 \) and the locus describing the relationship in (61) as \( SS_2 \). By taking total differentiation on both sides of (60) and (61), we obtain

\[
\frac{dl^*}{dR_{ff}} \bigg|_{SS_1} = \frac{(\theta/\psi' - e'\pi^*) (l^*)^2}{\alpha(1 - v - e^*)} > 0, \tag{62}
\]

\[
\frac{dl^*}{dR_{ff}} \bigg|_{SS_2} = -\frac{R_t^* \left[ e'd^* + \frac{(v+e^*)(l^*)^2}{\psi'} \right] + \frac{(v+e^*)(1-\phi) + \psi b^* + (v+e)l^*}{\psi'} \left[ 1 + \frac{(\theta+\pi^*-1)l^*}{\psi(1-\phi)b^*} \right] + \frac{ab^*}{(l^*)2}}{R_t^* (v + e^*)} > 0, \tag{63}
\]

where \( e^* = e(R_{ff}^*) \). Equations (62) and (63) state that the \( SS_1 \) locus is upward-sloping in the \( R_{ff} - l^* \) space and the slope of the \( SS_2 \) locus can be positive or negative. Due to the complicated form of (63), we cannot analytically obtain the conditions that determine the slope of the \( SS_2 \) locus. We will resort to numerical method to deal with this issue later on.

In terms of the steady-state’s local stability properties, we compute the Jacobian matrix \( J \) of the dynamical system (54)-(56) evaluated at the steady state. By linearizing the dynamical system (54)-(56) around the steady state, we have

\[
\begin{bmatrix}
\dot{l}_t \\
\dot{b}_t \\
\dot{\lambda}_t \\
\end{bmatrix} = J \begin{bmatrix}
l_t - l^* \\
b_t - b^* \\
\lambda_t - \lambda^* \\
\end{bmatrix}, \tag{64}
\]

where the arguments of the Jacobian, \( J_{ij} \), \( i = 1, 2, 3 \), \( j = 1, 2, 3 \), are given by:

\[
J_{11} \equiv \frac{\partial l_t}{\partial l_t} = 1 - \theta - \pi^* - \frac{l^* R_{ff,l}}{\psi'}, \nonumber
\]

\[
J_{12} \equiv \frac{\partial l_t}{\partial b_t} = -\left[ \phi + \theta(1 - \phi) \right] - \frac{l^* R_{ff,b}}{\psi'}, \nonumber
\]

17
\[
\begin{align*}
J_{13} & \equiv \frac{\partial \hat{b}_t}{\partial \lambda_t} = -\frac{l^* R_{ff,\lambda}}{\psi'}, \\
J_{21} & \equiv \frac{\partial \check{b}_t}{\partial \lambda_t} = b^* r_{t,t} - R^*_t b_{g,t} - \left[ r_{t,t} + \frac{R_{ff,t}}{\psi'} \right] b^*_g, \\
J_{22} & \equiv \frac{\partial \check{b}_t}{\partial b_t} = r^*_t - R^*_t b_{g,b} - \frac{R_{ff,b}}{\psi'} b^*_g, \\
J_{23} & \equiv \frac{\partial \check{b}_t}{\partial \lambda_t} = -R^*_t b_{g,\lambda} - \frac{R_{ff,\lambda}}{\psi'} b^*_g, \\
J_{31} & \equiv \frac{\partial \check{\lambda}_t}{\partial l_t} = \frac{\lambda^* R_{ff,l}}{\psi'} + \frac{\theta}{(c^*)^2}, \\
J_{32} & \equiv \frac{\partial \check{\lambda}_t}{\partial b_t} = \frac{\lambda^* R_{ff,b}}{\psi'} + \frac{\phi + \theta(1 - \phi)}{(c^*)^2}, \\
J_{33} & \equiv \frac{\partial \check{\lambda}_t}{\partial \lambda_t} = \rho + \pi^* + 1 + \frac{\lambda^* R_{ff,\lambda}}{\psi'},
\end{align*}
\]

where all the partial derivatives are evaluated at the steady state.

The stability of a steady state is determined by comparing the eigenvalues of \( J \) that have negative real parts to the number of initial conditions in the dynamical system (54)-(56), which is one because \( l_t \) and \( \lambda_t \) are both jump variables, and \( b_t \) is pre-determined. As a result, the steady state displays saddlepath stability and equilibrium uniqueness when two eigenvalues have positive real parts and one eigenvalue has negative real part. If more than one eigenvalues have negative real parts, then the steady state is locally indeterminate (a sink) and can be exploited to generate endogenous business fluctuations driven by agents’ self-fulfilling expectations or sunspots. If all eigenvalues have positive real parts, then the steady state is a source.

Due to the complexity of the arguments of the Jacobian matrix \( J \), the proof of the number of eigenvalues with positive real parts cannot be done analytically. Thus, we turn to numerical analysis. For this purpose, we first need to specify explicit functional forms for the excess reserve ratio function \( e(\cdot) \) and the interest rate feedback rule \( \psi(\cdot) \).

Following Taylor (2001), we specify a linear excess reserve ratio function as follows:

\[
e_t = e_0 - e_1 R_{ff,t}, \tag{65}
\]

where \( e_0 \) is a constant intercept term and \( e_1 > 0 \) measures the slope of the excess reserve ratio function.
We then follow McCallum and Nelson (1999) and Kurozumi (2006), among others, in specifying the following interest rate feedback function:

$$\psi(\pi_t) = \psi_0 + \psi_1 \pi_t,$$

(66)

where $\psi_0$ is a constant intercept term; $\psi_1 > 1$ and $\psi_1 < 1$ respectively represent the cases of active and passive rules.

Our benchmark parameterization is as follows. The time unit is assumed to be a quarter. The labor share $\alpha$ and the rate of time preference $\rho$ are set at standard values used in the literature: $\alpha = 0.7$ and $\rho = 0.0045$, where the latter is chosen to imply an annual 1.8% discount rate [see, for example, Benhabib et al. (2001), Dupor (2001)].

In order to obtain values for the intercept and the slope of the excess reserve ratio function, i.e. $e_0$ and $e_1$ in (65), we estimate (65) using monthly data of the effective federal funds rate and aggregate reserves and deposits of depository institutions provided by the Board of Governors of the Federal Reserve System, over the period 1980 : 1 – 2009 : 12. The estimation result is presented as follows (standard errors in parentheses):

$$e_t = 0.015482 - 0.00178R_{f,t} \quad \sigma_e = 0.02 \quad R^2 = 0.099$$

(67)

It is clear that both the intercept and the coefficient on the federal funds rate have very low estimated standard errors and are very significantly different from zero. The estimation result is consistent with Taylor’s (2001, p. 23) view that “...transactions costs and high penalties for overnight overdrafts suggest that $\alpha$ (which is $e_1$ in (65)) should be less than infinity and possibly quite small.” In addition, the fact that the coefficient on the federal funds rate is significantly different from zero support Taylor’s (2001, p. 23) view that the coefficient is greater than zero.

The reserve requirement ratio is set at its average value in the same sample period: $v = 0.01482$. The response of the federal funds rate to the inflation rate, $\psi_1$, is set at 1.5 for active rules, so that at the steady state the interest rate rule has the slope suggested by Taylor (1993) [Benhabib et al. (2001) and Dupor (2001), among others].
For passive rules, we follow Dupor (2001) in adopting a value $\psi_1 = 0.99$. We then set the intercept of the interest rate feedback function $\psi_0 = 0.015$, the fraction of deposits that are used for financing the household’s consumption purchases $\theta = 0.9$, and the proportion of government bonds that are allocated to the central bank at every issuance of government bonds $\phi = 0.5$. The benchmark parameterization implies that the federal funds rate is 6% per year, which equals the average effective federal funds rate in the period 1980 : 1 – 2009 : 12.

We are now in a position to analyze the existence and uniqueness of the model’s steady state(s) and the associated local dynamics. To examine the existence and number of the economy’s steady state in a transparent manner, we use (60) to substitute out $l^*$ in (61) and express the subsequent equation as $f(R_{ff}^*) = 0$. Therefore, the equilibrium $R_{ff}^*$ will be located at the (possibly more than one) intersection(s) of $f(R_{ff}^*)$ and the horizontal axis. We allow the federal funds rate to vary between 0% and 25%. Note that this range covers all the possibilities of the federal funds rate since the time unit is assumed to be a quarter.

The top panels of Figure 1 illustrate the results of the benchmark cases: $\psi_1 = 1.5$ and $\psi_1 = 0.99$. In addition to the benchmark cases, in Figure 1 we also plot $f(R_{ff}^*)$ for some other values of $\psi_1$, for the purpose of examining how the result changes when the response of the federal funds rate to the inflation rate changes. Under each parameterization, we calculate the eigenvalues of the Jacobian matrix $J$ in (64). The result is presented in the first panel of Table 1. As Figure 1 depicts, in the benchmark case, there always exists a unique steady state under active rules within the range $0% < R_{ff}^* < 25%$. We then see from Table 1 that the active steady state is characterized by two positive roots and one negative root, which indicates that the active steady state is a saddle. We then turn to the cases of passive rules. Figure 1 clearly shows that when $\psi_1 = 0.99$, there exists a unique steady state. As the interest rate rules become more and more passive ($\psi_1$ gets smaller), the number of the model’s steady state turns from one to two. The threshold value of $\psi_1$ for generating two steady states is at about 0.356, which represents a very passive rule. We then see from Table 1 that, under passive rules, when there exists a unique steady state, the steady state is a saddle; when there are two steady states, the low-equilibrium federal funds rate one is a saddle and the high-equilibrium federal funds rate one is a sink (the
stable manifold has dimension two). We noticed that the monthly data of the effective federal funds rate per year never exceed 20% during the period 1980 : 1 – 2009 : 12 (in fact, since when the data is available at the Board of Governors of the Federal Reserve System).\(^6\) This indicates that any \(R_{ff}^*\) that exceeds 5% is not empirically plausible. However, as Table 1 illustrates, in all cases the indeterminate steady state has a \(R_{ff}^*\) that is far beyond 5%. This implies that under empirically plausible values of the equilibrium federal funds rate neither active nor passive rules can generate local indeterminacy.

Recall that in the basic model without banking system where the nominal interest rate of government bonds is used as the proxy for the federal funds rate (section 2), a clear-cut result which is consistent with the existing literature’s finding was obtained: active rules always guarantee equilibrium uniqueness and passive rules always lead to local indeterminacy. In this section, we abandon this conventional practice in the literature and let the federal funds rate feeds back to the inflation rate. Our numerical results demonstrate that active rules remain stabilizing. However, in contrast to the traditional finding, passive rules do not necessarily generate local indeterminacy; the indeterminacy region is actually very small and the indeterminate steady state has an equilibrium federal funds rate that is too high to be empirically plausible.

To assess the robustness of the benchmark parameterization results, in what follows we consider variations in the values of the fraction of deposits that are used for financing the household’s consumption purchases, \(\theta\), and the proportion of government bonds that are allocated to the central bank at every issuance of government bonds, \(\phi\).

Firstly, in Figure 2 and the second panel of Table 1, we consider a reduction in \(\theta\) from 0.9 to 0.5, which represents a higher transactions costs in financing consumption purchases using deposits. It is apparent that the threshold value of \(\psi_1\) that generates two steady states and local indeterminacy decreases from 0.356 to 0.209. Other things remain the same. Specifically, there is a unique active steady state which is locally determinate. Under passive rules, local determinacy is ensured if a unique steady state exists; if two steady states emerge, the one that is associated with a lower equilibrium federal funds rate is locally determinate, whereas the one that is

\(^6\)The biggest annual federal funds rate occurs in 1980 : 01 which is 19.08%. 

21
associated with a higher equilibrium federal funds rate generates endogenous business fluctuations. Nonetheless, the steady states that exhibit local indeterminacy have $R^*_ff$ that are too high to be empirically plausible. Figure 3 and the third panel of Table 1 illustrate the extreme case where only cash is used for financing consumption purchases. We can see that the threshold value of $\psi_1$ that generates two steady states and local indeterminacy further decreases to 0.09.

To examine the role of $\phi$, we first consider in Figure 4 and the fourth panel of Table 1 an increase in $\theta$ from 0.5 to 0.9. We then consider in Figure 5 and the fifth panel of Table 1 a reduction in $\phi$ from 0.5 to 0.1. It is obvious that changes in the value of $\phi$ insignificantly affect the results, and the threshold value of $\psi_1$ that generates two steady states and local indeterminacy remains to be at about 0.356. This indicates that $\phi$ does not play a crucial role in the macroeconomic stabilizing property of the nominal interest rate rules.

4 Conclusion

This paper formally characterizes the banking system and the reserves market in a general equilibrium model. With the framework, we are capable of describing how the central banker’s open market operations in the reserves market affect the federal funds rate when it adopts a regime of the nominal interest rate rules. The reasons why pursuing this analysis is essential are that, firstly, bank loans are the main source of external finance of firms in most countries. Secondly, the central bank conduct open market operations mainly with commercial banks. How much of the reserves injected into the banking system through open market operations will be released to firms and/or consumers depends on the willingness of borrowing and lending between firms/consumers and commercial banks. Thirdly, a central banker whose monetary policy is described as the nominal interest rate rules adjusts the supply of reserves in the reserves market so as to achieve its target for the overnight loans rate. Thus, to fully evaluate the performance of the nominal interest rate rules, a model with the banking system and the reserves market is required.

Our analysis shows that the macroeconomic stabilizing property of the nominal interest rate rules changes quite a lot when moving from a standard model without the banking system to a model with the banking system and the reserves market,
where in the former case the government bonds rate is used as a proxy for the federal funds rate as a convention. To be specific, we find that in a model which neglects the banking system, active rules always maintain saddle-path stability and passive rules always lead to equilibrium indeterminacy. By contrast, when the banking system and the central banker’s open market operations in the reserves market are formally characterized, local indeterminacy arises only when the interest rate rules is very passive. Moreover, even in such cases, the indeterminate steady states have equilibrium federal funds rates are too high to be empirically plausible. Thus, our results indicate that under empirically plausible values of the equilibrium federal funds rate, neither active nor passive rules can generate local indeterminacy. Regarding possible extensions of our analysis, it would be worthwhile to investigate an investment economy. A remarkable insight drawn by Dupor (2001) is that, the macroeconomic stabilizing property of the nominal interest rate rules might be quite different if the interest rate affect real activities also through the investment decisions of the firms. We plan to pursue this research project in the near future.
Appendix: Derivations

Derivation of the dynamical system. We first summarize the macroeconomic equilibrium conditions as follows:

\[ c_t = m_t + \theta d_t, \]  
\[ \lambda_t = (\rho + \pi_t + 1)\lambda_t - \frac{1}{c_t}, \]  
\[ r_{dt} = (1 - \theta) \left( \frac{1}{c_t} - 1 \right) - \pi_t, \]  
\[ r_{dt} = r_t[1 - v - e(R_{fft})], \]  
\[ R_{fft} = \psi(\pi_t), \]  
\[ b_{gt} = m_t + [v + e(R_{fft})]d_t, \]  
\[ m_t = \phi b_t, \]  
\[ \dot{b}_t = r_t b_t - (r_t + \pi_t) b_{gt}, \]  
\[ l_t = 1 - c_t + l_t - \pi_t l_t, \]  
\[ r_{lt} = r_b. \]

It is obvious from the above equations that we will have a three-dimensional dynamical system in \((l_t, b_t, \lambda_t)\). Specifically, (69), (75), and (79) will constitute the set of differential equations of the dynamical system. For this purpose, we have to express all the other endogenous variables in (68)-(80) as functions of \((l_t, b_t, \lambda_t)\). Firstly, by making use of (74) and (78), we derive from (68) \(c_t\) as a function of \(l_t\) and \(b_t\):

\[ c_t = [\phi + \theta(1 - \phi)]b_t + \theta l_t. \]  

We then use (74) and (78) to obtain \(d_t\), which turns out to be also a function of \(l_t\) and \(b_t\):

\[ d_t = l_t + (1 - \phi)b_t. \]  

The derivation of other endogenous variables as functions of \((l_t, b_t, \lambda_t)\) are not so straightforward. In what follows we illustrate how the derivation is carried out.

From (72), we express \(\pi_t\) as a function of \(R_{fft}\):

\[ \pi_t = \psi^{-1}(R_{fft}), \quad \psi^{-1} = \frac{1}{\psi'} > 0. \]  

By using (81) and (83) to substitute out \(c_t\) and \(\pi_t\) in (70), we re-express the deposits rate as a function of \((l_t, b_t, \lambda_t, R_{fft})\):

\[ r_{dt} = (1 - \theta) \left[ \frac{1}{\{\phi + \theta(1 - \phi)\}b_t + \theta l_t} - 1 \right] - \psi^{-1}(R_{fft}). \]  

We then use (82) to substitute out \(d_t\) in (77). From the subsequent equation we derive the equilibrium lending rate as a function of \((l_t, b_t, \lambda_t, R_{fft}, b_{gt})\):
\( r_{lt} = \frac{\alpha}{[1 - v - e(R_{fft})][l_t + (1 - \phi)b_t] - b_t + b_{gt} - 1}. \)  

(85)

By substituting (84) and (85) into (71), we have the following relationship, which contains five variables \((l_t, b_t, \lambda_t, R_{fft}, b_{gt})\):

\[
(1 - \theta) \left[ \frac{1}{\{\phi + \theta(1 - \phi)\} b_t + \theta l_t} \right] - 1 = \psi^{-1}(R_{fft})
\]

\[
= \left\{ \frac{\alpha}{[1 - v - e(R_{fft})][l_t + (1 - \phi)b_t] - b_t + b_{gt} - 1} \right\} [1 - v - e(R_{fft})].
\]

(86)

We then use (86) and (76) to simultaneously solve \((R_{fft}, b_{gt})\) as functions of \((l_t, b_t, \lambda_t)\):

\[
R_{fft} = R_{ff}(l_t, b_t, \lambda_t),
\]

(87)

\[
b_{gt} = b_g(l_t, b_t, \lambda_t),
\]

(88)

where the partial derivatives are

\[
R_{ff,l} \equiv \frac{\partial R_{ff}}{\partial l_t} = \frac{1}{r_{lt} e^{1/\psi}} \left[ \frac{\theta(1 - \theta)}{c^2 \lambda_t} - \frac{\alpha(1 - v - e_t)}{l_t^2} \right] < 0,
\]

\[
R_{ff,b} \equiv \frac{\partial R_{ff}}{\partial b_t} = (1 - \theta)\frac{(\phi + \theta(1 - \phi))}{(r_{lt} e^{1/\psi}) c^2 \lambda_t} < 0
\]

\[
R_{ff,\lambda} \equiv \frac{\partial R_{ff}}{\partial \lambda_t} = \frac{1 - \theta}{(r_{lt} e^{1/\psi}) c^2 \lambda_t^2} < 0
\]

\[
b_{g,l} \equiv \frac{\partial b_{gt}}{\partial l_t} = d_t e^{R_{ff,l}} + v + e' > 0,
\]

\[
b_{g,b} \equiv \frac{\partial b_{gt}}{\partial b_t} = d_t e^{R_{ff,b}} + \phi + (v + e_t)(1 - \phi) > 0,
\]

\[
b_{g,\lambda} \equiv \frac{\partial b_{gt}}{\partial \lambda_t} = d_t e^{R_{ff,\lambda}} > 0.
\]

By making use of (87) and (88), we obtain from (85) the equilibrium lending rate as a function of \((l_t, b_t, \lambda_t)\):

\[
r_{lt} = r_l(l_t, b_t, \lambda_t),
\]

(89)

where \(r_{l,l} \equiv \frac{\partial r_{l}}{\partial l_t} = -\frac{\alpha}{l_t^2} < 0\), \(r_{l,b} \equiv \frac{\partial r_{l}}{\partial b_t} = 0\), and \(r_{l,\lambda} \equiv \frac{\partial r_{l}}{\partial \lambda_t} = 0\).

This completes the derivation.

**Derivation of the stationary relationships.** The main purpose of this part of the appendix is to illustrate the derivation of (60) and (61). At the steady state, \(\hat{l}_t = \hat{b}_t = \hat{\lambda}_t = 0\) hold. Let an asterisk denotes the steady state value of a variable. From (77), we have

\[
r_{l}^* = \frac{\alpha}{\hat{l}}^* - 1.
\]

(90)

From (56) with \(\hat{\lambda}_t = 0\), we obtain an expression of \(\frac{1}{e^{\lambda}}\) as follows:
\[ \frac{1}{c^* \lambda^*} = \rho + \psi^{-1}(R_{ff}^*) + 1. \]  

(91)

From (70) and (71), with (90), we have another expression of \( \frac{1}{c^* \lambda^*} \):

\[ \frac{1}{c^* \lambda^*} = \left( \frac{\alpha}{\bar{v}} - 1 \right) \left[ 1 - v - e(R_{ff}^*) \right] + \psi^{-1}(R_{ff}^*) + 1. \]  

(92)

Equation (60) is derived by equating the right-hand side of (91) to the right-hand side of (92):

\[ \frac{(1 - \theta) \rho - \theta \psi^{-1}(R_{ff}^*)}{1 - v - e(R_{ff}^*)} = \frac{\alpha}{l^*} - 1. \]  

(93)

Now let us move on to the derivation of (61). By using (90) to substitute out \( r_t^* \) in (55) and employing the condition that \( \dot{b}_t = 0 \), we have

\[ \left( \frac{\alpha}{l^*} - 1 \right) b^* = \left[ \frac{\alpha}{l^*} - 1 + \psi^{-1}(R_{ff}^*) - 1 \right] b^*_g. \]  

(94)

By using (82) to substitute out \( d^* \) in (76), we have

\[ b^*_g = [v + e(R_{ff}^*)] \left[ l^* + (1 - \phi) b^* \right] + \phi b^*. \]  

(95)

With \( \dot{l}_t = 0 \), we obtain from (54)

\[ b^* = \frac{1 + \left[ 1 - \theta - \psi^{-1}(R_{ff}^*) \right] l^*}{\phi + \theta(1 - \phi)}. \]  

(96)

Equation (61) is derived by putting together (96), (94), and (95):

\[ \left\{ \frac{\alpha}{l^*} - 1 - \left[ \frac{\alpha}{l^*} - 1 + \psi^{-1}(R_{ff}^*) \right] \right\} \left\{ [v + e(R_{ff}^*)] (1 - \phi) + \phi \right\} \left\{ 1 + \left[ 1 - \theta - \psi^{-1}(R_{ff}^*) \right] l^* \right\} \]

\[ = \left[ \phi + \theta(1 - \phi) \right] \left[ \frac{\alpha}{l^*} - 1 + \psi^{-1}(R_{ff}^*) \right] \left[ v + e(R_{ff}^*) \right] l^*. \]  

(97)
References


Table 1:

Benchmark case: $\phi = 0.5$, $\theta = 0.9$

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<th>$\psi_i$</th>
<th>1.5</th>
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<th>0.3</th>
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<th>0.1</th>
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<td>(roots)</td>
<td>($++$) ($++$) ($++$) ($++$) ($++$) ($++$) ($++$) ($++$)</td>
<td>0.24945</td>
<td>0.21256</td>
<td>0.1467</td>
<td>0.08084</td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>($++$) ($++$) ($++$) ($++$) ($++$) ($++$) ($++$) ($++$)</td>
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<td></td>
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<td></td>
</tr>
</tbody>
</table>

Reducing $\theta$: $\phi = 0.5$, $\theta = 0.5$

<table>
<thead>
<tr>
<th>$\psi_i$</th>
<th>1.5</th>
<th>0.99</th>
<th>0.5</th>
<th>0.3</th>
<th>0.209</th>
<th>0.2</th>
<th>0.1</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R'_y$</td>
<td>0.01713</td>
<td>0.01641</td>
<td>0.01571</td>
<td>0.0153</td>
<td>0.01528</td>
<td>0.01514</td>
<td>0.01927</td>
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</tr>
<tr>
<td>(roots)</td>
<td>($++$) ($++$) ($++$) ($++$) ($++$) ($++$) ($++$) ($++$)</td>
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<tr>
<td></td>
<td>0.24926</td>
<td>0.23917</td>
<td>0.12708</td>
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Only cash is used for purchasing: $\phi = 0.5$, $\theta = 0$

<table>
<thead>
<tr>
<th>$\psi_i$</th>
<th>1.5</th>
<th>0.99</th>
<th>0.5</th>
<th>0.3</th>
<th>0.2</th>
<th>0.1</th>
<th>0.09</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R'_y$</td>
<td>0.02139</td>
<td>0.01922</td>
<td>0.01713</td>
<td>0.01628</td>
<td>0.01585</td>
<td>0.01542</td>
<td>0.01538</td>
<td>0.02778</td>
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<td></td>
</tr>
</tbody>
</table>

Increasing $\phi$: $\phi = 0.9$, $\theta = 0.9$

<table>
<thead>
<tr>
<th>$\psi_i$</th>
<th>1.5</th>
<th>0.99</th>
<th>0.5</th>
<th>0.365</th>
<th>0.3</th>
<th>0.2</th>
<th>0.1</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R'_y$</td>
<td>0.01505</td>
<td>0.01503</td>
<td>0.01502</td>
<td>0.01501</td>
<td>0.01501</td>
<td>0.01501</td>
<td>0.01515</td>
<td>0.01511</td>
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<tr>
<td>(roots)</td>
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<tr>
<td></td>
<td>0.24946</td>
<td>0.21257</td>
<td>0.1467</td>
<td>0.08085</td>
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<td></td>
</tr>
</tbody>
</table>

Reducing $\phi$: $\phi = 0.1$, $\theta = 0.9$

<table>
<thead>
<tr>
<th>$\psi_i$</th>
<th>1.5</th>
<th>0.99</th>
<th>0.5</th>
<th>0.365</th>
<th>0.3</th>
<th>0.2</th>
<th>0.1</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R'_y$</td>
<td>0.01563</td>
<td>0.01542</td>
<td>0.01521</td>
<td>0.01515</td>
<td>0.01513</td>
<td>0.01508</td>
<td>0.01504</td>
<td>0.01626</td>
</tr>
<tr>
<td>(roots)</td>
<td>($++$) ($++$) ($++$) ($++$) ($++$) ($++$) ($++$) ($++$)</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>0.24945</td>
<td>0.21256</td>
<td>0.1467</td>
<td>0.08084</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: $\alpha = 0.7$, $\rho = 0.0045$, $e_o = 0.015482$, $e_i = 0.00178$, $v = 0.01482$, $\psi_o = 0.015$,
Figure 1: Existence of the Steady State: benchmark case

$\alpha = 0.7$, $\rho = 0.0045$, $e_o = 0.015482$, $e_i = 0.001781$, $\psi = 0.01482$

$\psi_o = 0.015$, $\phi = 0.5$, $\theta = 0.9$
Figure 2: Existence of the Steady State: reducing $\psi$

$\alpha = 0.7$, $\rho = 0.0045$, $e_o = 0.015482$, $e_i = 0.00178$, $v = 0.01482$, $\psi = 0.015$, $\phi = 0.5$, $\theta = 0.5$
Figure 3: Existence of the Steady State: only cash is used for purchasing

\[
\begin{align*}
\alpha &= 0.7, \quad \rho = 0.0045, \quad e_o = 0.015482, \quad e_i = 0.001781, \quad v = 0.01482, \\
\psi_o &= 0.015, \quad \phi = 0.5, \quad \theta = 0
\end{align*}
\]
Figure 4: Existence of the Steady State: increasing $\phi$

$\alpha = 0.7$, $\rho = 0.0045$, $e_0 = 0.015482$, $e_i = 0.00178$, $\nu = 0.01482$

$\psi_0 = 0.015$, $\phi = 0.9$, $\theta = 0.9$
Figure 5: Existence of the Steady State: reducing $\phi$

$\alpha = 0.7$, $\rho = 0.0045$, $e_o = 0.015482$, $e_i = 0.001781$, $v = 0.01482$

$\psi_o = 0.015$, $\phi = 0.1$, $\theta = 0.9$