FDI, Outsourcing and Wage Inequality: 
A North-South Model of Skill Accumulation

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ABSTRACT
This study develops a North-South model with skill accumulation to investigate the effects of international specialization on innovation, skill accumulation, wage inequality and the pattern of production. Southerners can choose to become skilled or unskilled and skill formation depends on private and public educational investments and skill externality. We find that if the skill formation elasticity of public educational investments is sufficiently high, increasing incentives of outsourcing will raise the extent of outsourcing, the adjusted R&D difficulty and international wage inequality for the skilled while the extent of FDI and relative wage rate in the South will be lower. Increasing incentives for FDI will raise the extent of FDI and relative wage rate in the South and reduce the extent of outsourcing, adjusted R&D difficulty and the international wage inequality for the skilled.

Keywords: FDI; Outsourcing; Skill accumulation; Wage inequality.

JEL Classification: F12; F23; O31.

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1. INTRODUCTION

Foreign direct investment (FDI) and international outsourcing have become quite common around the globe nowadays. Antras and Helpman (2004) once have described this phenomenon by giving two examples. “Intel Corporation provides an example of the FDI strategy,” they wrote, “it assembles most of its microchips in wholly owned subsidiaries in China, Costa Rica, Malaysia, and the Philippines. On the other hand, Nike provides an example of the arm’s length import strategy: it subcontracts most of its manufacturing to independent producers in Thailand, Indonesia, Cambodia, and Vietnam” (p. 553).

This phenomenon has made economists demonstrate considerable interest over recent years in the causes and effects of international specialization, both empirically and theoretically. The literature related to international production was dominated by two major research lines. The first research line focused on the examination of the impact of FDI and imitation in developing countries, based on a product cycle model originated by Vernon (1966) and extended by Grossman and Helpman (1991a, 1991b). Studies along this research line include Helpman (1993), Lai (2001) and Glass and Saggi (2002). By constructing a two country model with an innovating region (the North) and an imitating region (the South), these studies analyzed the influence of intellectual property rights protection on innovation, FDI and wage inequality.

The second research line investigated the effect of increased extent of international outsourcing of production on wage inequality. As Glass ad Saggi (2001) pointed out that while empirical studies in this research line were prevailing, there were only few theoretical studies explaining the linkage between these two.\(^1\) The early theoretical study of the effects of outsourcing can be found in Feenstra and Hanson

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\(^1\) See Feenstra and Hanson (1996b) for the empirical study of the effects of outsourcing on wage inequality.
(1996a) who assumed that final goods were produced from a continuum of intermediate goods that used different proportions of skilled and unskilled workers. Later, Glass and Saggi (2001) developed a North-South product cycle model to examine the effects of increased outsourcing of production on innovation, wages and pattern of production. While outsourcing in Feenstra and Hanson (1996a) was driven by factor endowment, outsourcing in Glass and Saggi (2001) was driven by differences in technology.

Thus, it is clear that the prior studies have tended to study FDI and international outsourcing activities separately, thereby ignoring the facts that firms can choose to adapt their production in developing countries between FDI and outsourcing. The early study of the trade-off between FDI and outsourcing can be found in Grossman and Helpman (1993) who assumed a final-good producer can manufacture goods by itself or by making specific investment governed by imperfect contracts. Recently, Antras and Helpman (2004) developed a model where firms can choose to engage in FDI, domestic outsourcing or international outsourcing. By assuming that there exists within-sectoral heterogeneity of firms, they showed that low-productivity firms outsource in the North while high-productivity firms outsource in the South.

There are two features to distinguish this paper from previous studies. The first feature is that we develop a simple North (developed countries)-South (developing countries) product cycle model with Northern production, FDI and international outsourcing. Following Glass and Saggi (2001), we assume that FDI and outsourcing are driven by differences in technology. Production is decomposed into advanced production and basic production. The advanced production only can be conducted in the North while the basic production can occur either in the North or in the South. Northern firms can choose to complete the whole production at home or only conduct advanced production and use basic goods produced in the South. When a Northern
firm makes basic goods made abroad, it engages in FDI. Instead, it can choose to outsource basic goods in the South. Outsourcing is plagued with contractual difficulties since contracts are incomplete. Hence, we assume that outsourcing is risky and is subject to a probability of contract defaulting.

The second feature of this study is that the choice of skill (human capital) accumulation is endogenized. All Northern agents are assumed to be skilled while Southern agents can choose to become skilled or unskilled. In order to become skilled workers, Southern agents need to sacrifice working time to go to schools for skill training. Unskilled Southerners work in the production sector while skilled Southerners work in the FDI sector. Northern firms engaging in FDI need to hire Southern skilled workers to manage business in the South. The heterogeneity of Southerners allows us to study effects on the relative wage for skilled and unskilled Southerners and international wage inequality for skilled workers.

We assume that skill (human capital) formation depends on private and public educational investments and skill externality. Skill formation is critical in determining the pattern of production (extents of Northern production, FDI and outsourcing) because it affects the allocation of Southern labor resources used for FDI and production and this will in turn affect the Northern production and innovation. When the skill formation elasticity of public educational investment is sufficiently large, the wage rate for skilled Southerners and the fraction of Southern workers being skilled will be positively correlated. Increasing incentives for Southern production (such as a larger proportion of basic production conducted in the South or a lower risk of outsourcing) will increase the demand of unskilled Southern workers and reduce the fraction of Southerners being skilled and the extent of FDI. An increase in the skill

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2 Skill accumulation was endogenized in Parello (2008) to study the impact of intellectual property rights protection in a product cycle model with FDI and imitation.
formation elasticity of private educational investment and decreases in the Northern labor supply and labor intensity for R&D will cause similar effects. Furthermore, a lower risk of outsourcing will raise the extent of outsourcing and R&D difficulty while the extent of FDI will become lower. On the other hand, the increasing incentives for FDI (such as a lower labor intensity for FDI) will increase the fraction of Southerners being skilled and the extent of FDI and lower the R&D difficulty and the extent of outsourcing.

Our analysis also shows that international and Southern wage inequalities are not affected by the changes in Northern labor and the elasticities of skill formation with respect to private and public educational investments. A decrease in the labor intensity for FDI which increases the demand of Southern skilled workers will raise the relative wage rate in the South and lower the international wage inequality for skilled workers. On the other hand, changes inducing more demand of Southern unskilled workers and reduce the relative wage rate in the South and raise the international wage inequality for skilled workers, except that international wage inequality for skilled workers remains the same in response to an increase in the share of basic production in the South.

The remainder of this paper is organized as follows. In the next section we describe the setting of the model and show the existence of the balanced-growth-path (BGP) equilibrium. In Section 3, we examine how labor supply, proportion of basic production conducted in the South, FDI opportunities, Southern economic environment and skill accumulation affect wage inequality, innovation and the pattern of production. Section 4 investigates the robustness of the setting of skill formation. The final section concludes. Proofs of propositions 1 and 2 are given in Appendix A.

2. THE MODEL
We start with a description of a quality-ladder model of growth in the spirit of Grossman and Helpman (1991) and Dinopoulos and Segerstrom (1999). Consumers live either the developed North country or the developing South country. In each country $i = \{N, S\}$, the economy is composed by $L_i(t)$ identical households at time $t$. Given the birth rate $\psi$ and the death rate $\delta$ in both countries, the growth rate of population $g$ is equal to $(\psi - \delta)$ and $L_i(t + T) = L_i(t)e^{gT}$. Assuming that the lifespan of each individual is $T$, the population dynamics imply that that $\psi L_i(t) = \delta L_i(t + T)$. This indicates that the number of birth at time $t$ equals the number of death at time $t+T$. Thus, we can express that $\psi = \frac{g e^{gT}}{e^{gT-1}}$ and $\delta = \frac{g}{e^{gT-1}}$. Agents in the North are all skilled workers, while agents in the South can be differentiated between skilled and unskilled workers.

Northern firms engage in R&D activity and produce quality frontier of products through innovation. Following Glass and Saggi (2001), we assume that production can be broken into two stages: a basic stage and an advanced stage. Once successful in innovating a higher quality level of a product, a Northern firm can attempt to adapt its basic production technology in the South with lower costs by outsourcing or FDI. Unskilled Southern workers are hired to conduct basic production in the South. If a firm engages in FDI, it also needs to hire skilled Southerners to manage and monitor business in the South. Alternatively, a Northern firm can choose to license a Southern firm to conduct basic production by signing contract. Although outsourcing does not incur the cost of recruiting skilled Southerners, the Northern firm needs to face the risk of contract defaulting.

2.1. Consumers

Consumers choose from a continuum of products $z \in [0, 1]$ available at different quality levels ($j$). The quality increment between the product of quality $j$ from the one of
quality $j-1$ is constant and equals $\lambda > 1$. Thus, for each product with the quality level $j$ provides quality $\lambda^j$. All products begin at time $t=1$ with the quality level $j=0$ and the base quality $\lambda^0 = 1$.

The representative household in the country $i$ faces the lifetime utility:

$$U_i(0) = \int_0^\infty L_i(0)e^{-(\rho-g)t} \log u_i(t) \, dt; \quad L_i(0) > 0; \, \rho > g,$$

where $\rho$ denotes the subjective discount factor. The instantaneous utility is:

$$\log u_i(t) = \int_0^1 \log [\sum_j \lambda^j q_{ij}(z,t)] \, dz,$$

where $q_{ij}(z,t)$ is the household consumption in the country $i$ for the quality level $j$ of product $z$ at time $t$.

The total expenditure for all products with different quality levels under the price $p_{ij}(z,t)$ is:

$$E_i(t) = \int_0^1 [\sum_j p_{ij}(z,t)q_{ij}(z,t)] \, dz.$$

Consumers will maximize the lifetime utility subject to the aggregate intertemporal budget constraint:

$$W_i(t) + A_i(t) = \int_t^\infty E_i(\tau) e^{\gamma \tau} e^{-[R(\tau)-R(t)]} \, d\tau,$$

where $W_i(t)$ denotes the discount wage income of the households from country $i$, and $A_i(t)$ represents the present value of financial assets at time $t$. The cumulative interest rate up to time $t$ is represented by $R(t) = \int_0^t r(\tau) \, d\tau$, where $r(\tau)$ is instantaneous interest rate at time $\tau$.

The optimization problem can be solved by backward induction in three steps. In the final step, consumers allocate expenditure at each instant for each product across available quality levels. Due to the Dixit-Stiglitz specification in Eq. (2), consumers will choose the quality which gives the lowest adjusted price $\frac{p_{ij}(z,t)}{\lambda^j}$. That is, consumers
are willing to pay $\lambda$ for a one quality level improvement in a product. In the equilibrium, only the highest quality level available will sell.

In the second step, consumers allocate expenditure across products at each instant. Because the elasticity of substitution between any two products is constant at unity, expenditures across products are the same. Then the overall unit-elastic demand function for each top-quality product is:

$$q(z,t) = \frac{E(t)}{p(z,t)},$$  \hspace{1cm} (5)

where $E(t) = E_N(t)L_N(t) + E_S(t)L_S(t)$ is the global expenditure at time $t$.

In the final step, consumers allocate lifetime wealth across time by maximizing Eq. (1) subject to the intertemporal budget constraint (Eq. (4)). This yields the optimal expenditure path for the representative agent in each country:

$$\frac{E_i(t)}{E_i(t)} = r(t) - \rho.$$  \hspace{1cm} (6)

To ensure the existence of equilibrium, we assume that $r(t) = \rho$.

### 2.2. Skill accumulation

All Northerners are skilled workers. They provide all of their time on work to earn the wage rate $w^N$. Agents in the South can choose to be unskilled and earn the wage rate $w^S_L$, which is normalized to 1, or to spend an exogenously time period $D$ in schools for skill training and receive the skilled wage rate $w^S_H$ after they complete education.

Let $\phi$ denote the proportion of population who remain unskilled in the South. At time $t$, $\phi L_S(t)T$ individuals are unskilled; $(1 - \phi)L_S(t)(T - D)$ individuals complete the education process and devote themselves to the labor market. Thus, the total population consists of three parts which can be described in the following equation:
\[
\phi L_S(t) + (1 - \phi)L_S(t) \frac{D}{T} + (1 - \phi)L_S \frac{T - D}{T} = L_S(t).
\]  

(7)

The first term on the left hand side of Eq. (7) represents the unskilled labor supply at time \(t\). The second term is labor receiving the education presently, and the third term is the supply of skilled labor.

It is commonly agreed in the literature of human capital that private educational investments (such as time spent in schools), public educational investments (such as government spending in education) and skill externality are important determinants to the accumulation of human capital.\(^3\) Therefore, we consider a skill (human capital) formation depending on these three important determinants. In this paper, we focus our analysis on the BGP equilibrium. We assume that the total public educational investment in period \(t\) is \(G(t) = g_G L_S(t)\), where \(g_G > 0\). This assumption allows the total educational investment to grow at the BGP growth rate which equals the population growth rate. Then the subsidy each Southern skilled worker can receive is \(g_S = \frac{G(t)}{(1 - \phi)L_S(t)}\). Individuals choose to be educated if:

\[
\int_t^{t + T} e^{-[R(\tau) - R(t)]} w_t^S d\tau \leq \int_{t + D}^{t + T} e^{-[R(\tau) - R(t)]} w_t^S h(D) g_S \left( \frac{L_S^S(t)}{L_S(t)} \right)^\eta d\tau ,
\]

(8)

where the function \(h(D)\) is the skill production function of the time spent in schools with \(h'(D) < 0\) and \(h''(D) > 0\). The parameter \(\gamma \in (0,1)\) represents the elasticity of skill accumulation with respect to the public educational investment.\(^4\) Let \(L_S^S(t)\) denote the effective skilled Southerners. The externality of skill is represented by the proportion of effective skilled Southerners to the total Southern population, \(\left( \frac{L_S^S(t)}{L_S(t)} \right)^\eta\).

The parameter \(\eta \in (0,1)\) measures the degree of skill externality and we assume that

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\(^3\) For example, Kaganovich and Zilcha (1999) considered a human capital accumulation function depending on both private and public investments in education. Galor and Tsiddon (1997) considered a human capital accumulation function depending on the human capital externality (global externality).

\(^4\) The Cobb-Douglas formation of the human capital accumulation function has been widely used in the literature; see, for example, Glomm and Ravikumar (1992) and Chen (2005, 2006).
Note that Eq. (8) holds with equality in an equilibrium where skilled and unskilled workers coexist in the South. Then the optimal time spent in schools ($\bar{D}$) is determined by the following equation:

$$\rho h(\bar{D}) = (1 - e^{-\rho(T-\bar{D})})h'(\bar{D}). \quad (9)$$

Eq. (9) indicates that $\bar{D}$ only depends on the skill production function $h(\bar{D})$ and is not affected by the wage rates and skill externality.

From Eq. (8), we can derive the relative wage rate (skill premium) within the South as:

$$\frac{w_H^S}{w_L^S} = w_H^S = \frac{\sigma(\bar{D})}{[h(\bar{D})B(\bar{D})^{\eta}g_c^{\gamma}(1 - \phi)^{\eta-\gamma}]^{\frac{1}{1-\eta}}}, \quad (10)$$

where $\sigma(\bar{D}) = \frac{1-e^{-\rho T}}{e^{-\rho \bar{D}}-e^{-\rho T}} > 1$ and $B(\bar{D}) = \frac{T-D}{T} < 1$. Eq. (10) indicates that the larger the size of externality is, the larger the skill premium is.

At equilibrium, the supply of unskilled labor ($L^U_S$) is:

$$L^U_S(t) = \phi L_S(t). \quad (11)$$

And the supply of effective skilled labor is:

$$L^S_S(t) = [h(\bar{D})B(\bar{D})^{\eta}g_c^{\gamma}(1 - \phi)^{1-\gamma}]^{\frac{1}{1-\eta}}L_S(t). \quad (12)$$

2.3. Producers

Innovation occurs only in the North. A Northern firm in industry $z$ undertaking innovation intensity $t_R(z,t)$ for a time interval $dt$ will succeed in one quality level improvement in the final product with probability $t_R(z,t)dt$. To achieve this, it requires $a_R t_R(z,t)X(t)dt$ units of labor with the cost $w^N a_R t_R(z,t)X(t)dt$, where $X(t)$ denotes the difficulty of conducting R&D. Based on the temporary effects on

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5 As we can see from Eq. (10), the relative wage of skilled and unskilled workers in the South is independent of the proportion of southerners being unskilled (skilled) if $\eta = \gamma$. 


growth (TEG) approach by Segerstrom (1998), we assume that the R&D difficulty grows with the innovation intensity: \( \frac{X(t)}{X(t)} = \xi t_R(t) \) with \( 0 < \xi < \frac{1}{k} \). 6 This assumption captures the concept that

Production can be broken into a basic stage and an advanced stage. Of the one unit of labor used to produce one unit of the final product, \( \alpha \) units of labor is needed in the basic stage and \( (1 - \alpha) \) units of labor is used in the advanced stage. We assume that only Northern firms can conduct the advanced-stage production while the basic-stage production can be executed either in the North or in the South. This assumption of production implies that these two stages of production can occur in different countries.

A Northern firm can choose to adapt its basic production in the South by FDI or by outsourcing. To adapt its basic production in the South by FDI, the firm needs to recruit Southern skilled workers to manage or monitor its production process. Undertaking FDI intensity \( t_F(z, t) \) for a time interval \( dt \) needs \( a_F t_F(z, t)X(t)dt \) units of labor with the cost \( w^S a_F t_F(z, t)X(t)dt \) and leads to the successful probability of \( t_F(z, t)dt \). On the other hand, to adapt its basic production in the South by outsourcing, a Northern firm needs to hire Northern workers to file paper work of contracts. 7 Undertaking outsourcing intensity \( t_O(z, t) \) for a time interval \( dt \) needs \( a_O t_O(z, t)X(t)dt \) units of labor with the cost \( w^N a_O t_O(z, t)X(t)dt \) and leads to the successful probability of \( t_O(z, t)dt \).

Let \( v_N \) denote the expected value a firm gains from successful innovation. To avoid the infinite intensity of innovation, a Northern firm will choose its research intensity such that the expected gains from innovation do not exceed cost, with

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6 By using this setting of the R&D difficulty growth, we can remove the scale effect where innovation increases with the labor supply.

7 The same setting of outsourcing was used by Glass and Saggi (2001).
equality when innovation happens with positive intensity:

\[ v_N \leq w^N a_R X, \quad t_R > 0 \iff v_n = w^N a_R X. \tag{13} \]

Let \((v_F - v_N)\) represent capital gains from adapting basic production in the South by FDI. A Northern firm will choose its FDI intensity such that the expected gains from FDI do not exceed cost, with equality when FDI happens with positive intensity:

\[ v_F - v_N \leq w^N a_F X, \quad \tau_F > 0 \iff v_F - v_n = w^N a_F X. \tag{14} \]

Let \((v_O - v_N)\) represent capital gains from outsourcing basic production. A Northern firm will choose its outsourcing intensity such that the expected gains from international outsourcing do not exceed cost, with equality when outsourcing happens with positive intensity:

\[ v_O - v_N \leq w^O a_O X, \quad \tau_O > 0 \iff v_O - v_n = w^O a_O X. \tag{15} \]

Eqs. (13)-(15) imply that along the BGP equilibrium:

\[
\frac{\dot{X}(t)}{X(t)} = \frac{\dot{v}_N(t)}{v_N(t)} = \frac{\dot{v}_F(t)}{v_F(t)} = \frac{\dot{v}_O(t)}{v_O(t)} = \xi \tau_R(t). \tag{16}
\]

We assume that the firms face Bertrand competition and old technologies which designs have been improved are available internationally. Thus, Southern firms are able to produce final goods by using old technologies. However, Northern firms produce with state-of-the-art technologies will charge prices that just prevent its closest rivals from earning positive profits.

Therefore, producing firm will charge the price \(p = \lambda\) and make the sale \(q = E/\lambda\).\(^8\) The instantaneous profits for firms complete its final production in the North are:

\[ \pi_N = E \left( 1 - \frac{w^N}{\lambda} \right). \tag{17} \]

\(^8\) To guarantee positive profits (\(\pi_N\) and \(\pi\)), we need \(1 < w^N < \lambda\). As we will see later, this condition will be guaranteed by Eq. (33).
Firms that adapt basic production (either by FDI or by outsourcing) in the South can save costs by hiring unskilled Southern workers to produce basic goods and their marginal costs are \( c = \alpha + (1 - \alpha)w^N \). The instantaneous profits are:
\[
\pi = E \left( 1 - \frac{c}{\lambda} \right).
\] (18)

Using Eq. (16), the reward for a Northern firm which successfully innovates can be expressed as:
\[
\nu_N = \frac{\pi_N + \tau_F[v_N - w^N_H a_F X] + \tau_O(v_O - w^N a_O X)}{\rho + (1 - \xi)\tau_R + \tau_F + \tau_O}.\] (19)

A Northern firm which adapts its basic production in the South can choose to do so by FDI or by outsourcing. The reward for a firm which successfully adapts its basic production in the South by FDI is:
\[
\nu_F = \frac{\pi_F}{\rho + (1 - \xi)\tau_R}.\] (20)

On the other hand, if the Northern firm chooses to outsource its production in the South, it faces a risk of contract defaulting.\(^9\) We assume that once the contract is defaulted, the Northern firm will shut down and lose all of its value. Using \( \varepsilon \) to denote the probability that Southern firms default the contract, the reward for a firm which successfully adapts its basic production in the South by outsourcing is:
\[
\nu_O = \frac{\pi_O}{\rho + (1 - \xi)\tau_R + \varepsilon}.\] (21)

2.3. The BGP equilibrium and factor markets

We focus on the analysis along the BGP equilibrium. Along the BGP equilibrium, the growth rate of R&D difficulty should equal the growth rate of population. This allows

\(^9\) We can also assume that there is a risk for a Northern firm which chooses to adapt its basic production in the South by FDI. Since FDI utilizes Southern skilled workers to manage production process in the South, it is reasonable to assume that the risk of FDI is smaller than the risk of outsourcing. By assuming that the risk of FDI is smaller than the risk of outsourcing, the results will be the same qualitatively as those derived from our model. Note that our model setting induces that \( w^N_H a_F X > w^N a_F X \). This implies that the costs of paper work of outsourcing are less than the management costs of FDI.
us to derive the long-run innovation rate as:

$$\nu_R = \frac{g}{\xi}.$$  \hspace{1cm} (22)

Let $n_N$ denote the fraction of products completely produced in the North (the extent of Northern production). Similarly, let $n_F$ and $n_O$ respectively represent the fractions of products with basic production conducted in the South by FDI (the extent of FDI) and by outsourcing (the extent of outsourcing). The sum of these three product measures should be one:

$$n_N + n_F + n_O = 1. \hspace{1cm} (23)$$

Along the BGP equilibrium, the flow into basic production by FDI or outsourcing should equal the flow out of it:

$$\nu_F n_N = \nu_R n_F, \hspace{1cm} (24)$$

$$\nu_O n_N = (\nu_R + \epsilon)n_O. \hspace{1cm} (25)$$

We define two stationery variables as the adjusted level of R&D difficulty ($x = X/L_S$) and the adjusted global expenditure ($\hat{E} = E/L_S$). Since Northern labor can be used for R&D, outsourcing and production, the labor-market clearing condition for the North implies that:

$$a_R \nu_R x + a_O \nu_O x n_N + [n_N + (1 - \alpha)(n_F + n_O)] \frac{\hat{E}}{\lambda} = \frac{L_N}{L_S}. \hspace{1cm} (26)$$

The Southern skilled workers are used for FDI while its unskilled workers are used for production. The labor-market clearing conditions for the South indicate that:

$$a_F \nu_F x n_N = [h(D)B(D)g_G(1 - \phi)^{1-\gamma}]^{\frac{1}{1-\eta}}, \hspace{1cm} (27)$$

$$\alpha(n_F + n_O) \frac{\hat{E}}{\lambda} = \phi. \hspace{1cm} (28)$$

Substituting Eqs. (13)-(15) and (17) into Eq. (19), Eqs. (13), (14) and (18) into Eq. (20) and Eqs. (13), (15) and (18) into Eq. (21), we have:

$$\hat{E} \left(1 - \frac{w^N}{\lambda}\right) = [\rho + (1 - \xi)\nu_R]w^N a_R x, \hspace{1cm} (29)$$
where $\mu = [\rho + (1 - \xi)\tau_R]a_o + \epsilon(a_o + a_R)$.

The economy is composed by Eqs. (9), (10) and (22)-(31) with twelve variables $\{w^N, w^S_H, \bar{D}, \phi, x, \tilde{E}, n_N, n_F, n_O, \tau_R, \tau_F, \tau_O\}$. From Eq. (29), we have:

$$\tilde{E} = \frac{x\lambda[\lambda \mu + [\rho + (1 - \xi)\tau_R]a_R\lambda]}{\mu(\lambda - 1)}.$$  \hspace{1cm} (32)

Plugging Eq. (32) into Eq. (31), we can derive the Northern wage rate as:

$$w^N = \frac{\lambda \mu + [\rho + (1 - \xi)\tau_R]a_R\lambda}{\mu + [\rho + (1 - \xi)\tau_R]a_R\lambda} > 1,$$  \hspace{1cm} (33)

where $\tau_R$ is given by Eq. (22). From Eq. (30), and the wage rate for the Southern skilled workers as:

$$w^S_H = \frac{\mu}{[\rho + (1 - \xi)\tau_R]a_F}\cdot w^N.$$  \hspace{1cm} (34)

Eqs. (33) and (34) indicate that the international wage inequality for skilled workers are:

$$w_H = \frac{w^N}{w^S_H} = \frac{[\rho + (1 - \xi)\tau_R]a_F}{\mu}.$$  \hspace{1cm} (35)

Since the wage rate of Southern unskilled workers is normalized to one, the wage inequality within the South equals $w^S_H$. Combining Eqs. (10) and (34), we can calculate the proportion of Southern population who remain unskilled:

$$\phi = 1 - \left[\frac{1}{h(\bar{D})B(\bar{D})g_G\gamma\left(w^S_H\right)^{\eta-\gamma}}\right].$$  \hspace{1cm} (36)

Using Eqs. (24), (25), (27) and (28), we can express $\tau_o n_N$ by:

$$\tau_o n_N = (\tau_R + \epsilon)\left\{\frac{\lambda \phi}{a_F\tilde{E}}\cdot \left[\frac{h(D)B(D)g_G\gamma(1 - \phi)^{1-\gamma}}{\tau_R a_F x}\right]^{\frac{1}{\eta-\gamma}}\right\}.$$  \hspace{1cm} (37)

Substituting Eqs. (23) and (37) into Eq. (26), we can calculate:
\[ x = \frac{\alpha(\lambda - 1)\left\{ \frac{L_N}{L_S} + \frac{a_o(t_R + \varepsilon)}{t_R a_F} [h(D)B(D)gG^\gamma (1 - \phi)^{1-\gamma}]^{\frac{1}{1-\eta}} + \phi \theta \right\}}{\lambda \mu + [\rho + (\lambda - \xi)t_R]a a_R} > 0, \quad (38) \]

where \( \theta = 1 - \frac{\alpha_o(t_R + \varepsilon)(\lambda - 1)}{\lambda \mu + [\rho + (\lambda - \xi)t_R]a a_R} = \frac{a_o[t_R(1-\lambda \xi + \rho + \lambda \varepsilon)] + a_R[(\rho + (1-\xi)t_R]a + \lambda \varepsilon]}{\lambda \mu + [\rho + (1-\xi)t_R]a a_R} > 0. \)

Furthermore, substituting Eq. (23) into Eq. (28), we have:

\[ n_N = 1 - \frac{\lambda \phi}{a E}. \quad (39) \]

Plugging Eq. (27) into Eq. (24) generates:

\[ t_F = \frac{[h(D)B(D)gG^\gamma (1 - \phi)^{1-\gamma}]^{\frac{1}{1-\eta}}}{a_F x n_N}. \quad (40) \]

Finally, using Eqs. (24), (23) and (25), we can determine \( n_F = \frac{\rho n_N}{t_R} \), \( n_O = 1 - n_N - n_F \) and \( t_O = \frac{(t_R + \varepsilon)n_O}{n_N} \), respectively. Thus, we have completely solved the model and are ready to demonstrate the existence of the BGP equilibrium:

**Proposition 1.** There exists a unique BGP equilibrium in the North-South model with FDI, outsourcing and TEG setting of the R&D difficulty index under a certain condition. \(^{10}\) Along the BGP equilibrium, \( \{w_N, w_R^N, \tilde{D}, x, \tilde{E}_x, t_R, t_F, t_O\} \) are positive constants and \( \{\phi, n_N, n_F, n_O\} \) are constants between zero and one.

### 3. FDI AND OUTSOURCING

In this section, we examine effects of labor supply, basic production, FDI and outsourcing opportunities and Southern economic environment on innovation, wage inequality and pattern of production. Due to the complexity of the model, the theoretical analysis of these impacts may not be feasible. Thus, our analysis in this

\(^{10}\) In Appendix 1, we show that given \( a_f^\phi, a_f^\beta \) and \( a_f^\gamma \) defined by Eqs. (A1.4), (A1.6) and (A1.8), the existence of a unique BGP equilibrium depends on \( \eta \) and \( \gamma \). When \( \eta > \gamma \), there exists a unique BGP equilibrium if \( a_F \in \left( a_F^\gamma, \min \{a_F^\phi, a_F^\beta\}\right) \). When \( \eta < \gamma \), a unique BGP equilibrium exists if \( a_F \in \left( \max\{a_F^\phi, a_F^\beta\}, a_F^\gamma\right) \).
section is mostly based on numerical results. Before conducting numerical experiments, we first need to calibrate parameter values used in the model.

For the benchmark model, the per capita real GDP growth rate is set to $g = 2\%$. The discount factor $\rho = 0.03$ is chosen to generate a 3\% real interest rate. Following Glass and Saggi (2001), we set the one-stage quality improvement to $\lambda = 2$, the share of basic production produced in the South to $\alpha = 0.5$ and the labor intensities for R&D and outsourcing to $a_R = 2$ and $a_O = 1$. The labor intensity for FDI is set to $a_F = 1.6$ to match the extent of Northern production ($n_N$) roughly equal to 50\%.\(^{11}\) The Southern population ($L_S$) is normalized to one. We assign the Northern population to $L_N = 3$ and the risk of contract defaulting to $\varepsilon = 0.01$ so that the extents of FDI ($n_F$) and outsourcing ($n_O$) are roughly equal. We set the parameter of the growth rate of R&D difficulty at $\xi = 0.4$ to generate the wage ratio of skilled to unskilled Southerners equal to 1.5.

Based on the studies of Acemoglu and Angrist (2001) and Krueger and Lindahl (2001), the elasticity of human capital accumulation with respect to human capital eternality ($\eta$) is small. Thus, we choose the value used by de la Croix and Doepke (2003) and set $\eta = 0.1$. Parameter $\gamma$ represents the human capital accumulation elasticity for education expenditure. Note that the accumulation of human capital is highly correlated with income. The results of the empirical study by Johnson and Stafford (1973) showed that income elasticity for education expenditure was 0.198, whilst the figure used by Fernandez and Rogerson (1997), based on the estimates of Card and Kreuger (1992), was 0.2. Since the figures provided by these studies are virtually identical, we also set $\gamma$ as being equal to 0.2.

We assume that human capital production function follows $h(D) = AD^\beta$, where $A$ represents the productivity of skill production and $\beta$ represents the elasticity of

\(^{11}\) We follow Glass and Saggi (2001) to match the extent of Northern production to 50\%.
human capital accumulation with respect to the time spent in schools. Following Dinopoulos and Segerstrom (1999), we assume that each agent has a working life of 40 years which is normalized to one in the model \( T = 1 \), and an “unskilled” high-school graduate becomes skilled worker by spending \( D = 4 \) years in college. Then we calibrate \( \beta = 0.11 \) to match the value that skilled workers spend about 10% of working life on skill accumulation and 90% of working life on work. The parameter \( A \) is set to 1.165 so that less than 20% Southern workers are skilled. The public educational investment is set to the 3% of adjusted global expenditure and this gives us \( g_G = 0.2 \).

Under the parameterization in the benchmark model, 19.35% of Southern workers will spend 9.79% of time for education and become skilled workers. The Northern wage rate is 1.6 and the wage rate for Southern skilled workers is 1.5. Thus, the international wage inequality for skilled workers is 1.07. The resulting adjusted global expenditure is 6.37 and the adjusted R&D difficulty is 6.38. The intensities of R&D, FDI and outsourcing are 0.05, 0.02 and 0.3, respectively. The extents of Northern production, FDI and outsourcing are 49.40%, 24.32% and 26.28%, respectively. The benchmark values are presented in Table 1. Besides, Table 1 also summarizes the impacts of following events on the key macroeconomic variables in our model.

3.1. Labor supply

We begin our study by investigating the impact of a decrease in the Northern labor supply. The numerical results of the effects of 1% decrease in the Northern labor supply are given in Table 1. A decrease in the Northern labor supply implies that less labor can be devoted to innovation and production in the North. Thus, it leads to a

\[ \text{The parameterization used by Dinopoulos and Segerstrom (1999) guarantees that the fraction of the labor force becomes skilled is less than 25%. Since skilled Southern workers only work for the FDI sector, we set the fraction of the Southern labor force being skilled is less than 20%}. \]
lower adjusted level of R&D difficulty, a smaller extent of Northern production and higher extent of FDI.

Because Northern labor supply does not directly affect the incentives of innovations, FDI and outsourcing, the Northern and Southern wages rates ($w^N$ and $w^S_H$) are unaffected. Thus, the international wage inequality for skilled workers ($w_H$) does not change. With no change in the incentive of skill accumulation, the fraction of Southern labor being skilled remains the same. We summarize the results in the following proposition.

**Proposition 2.** A decrease in the Northern labor supply will lead to a decrease in the adjusted level of R&D difficulty. The extent of Northern production will decrease while the extent of FDI and the FDI intensity will increase. However, it does not affect the Northern and Southern wage rates and the fraction of Southern workers being skilled.

Although a lower Northern labor supply will shift production from the North to the South and this will raise the extent of outsourcing, it will also reduce the extent of outsourcing because outsourcing requires Northern labor for the paper work of contract. Our numerical results show that the former will dominate the latter and the extent of outsourcing will increase. The outsourcing and FDI intensities will increase to restore the steady-state conditions (Eqs. (24) and (25)).

### 3.2. Basic production

Along with the development of the South, technology adaption becomes easier and a greater share of basic production can be produced in the South. An increase in $\alpha$ reduces the demand of Northern labor for production. Therefore, the Northern wage rate becomes lower. Since in the North, more labor is available for innovation, the
adjusted level of R&D difficulty will increase. However, an increase in the share of basic production raises the demand of unskilled labor in the South and this will in turn decrease the relative wage rate in the South, leaving the international wage inequality for skilled workers unchanged. The decrease in the Southern wage rate for skilled workers reduces the fraction of Southern workers being skilled. The following proposition summarizes our findings of wage rates.\textsuperscript{13}

**Proposition 3.** The higher the share of basic production that can be produced by FDI or outsourcing, the lower the Northern wage rate and the Southern wage rate for skilled workers, the smaller proportion of Southern workers being skilled, leaving the international wage inequality for skilled workers unchanged.

The decrease in the Northern wage rate lowers the cost of Northern production. Thus, it raises the extent of Northern production and lowers the extents of FDI and outsourcing. Moreover, the decrease in the supply of Southern skilled workers lowers the extent of FDI while the increase in the supply of Southern unskilled workers raises the extents of FDI and outsourcing. Our numerical results show that over all, the extent of FDI will decrease while the extent of outsourcing will increase. Consequently, both the FDI intensity and the outsourcing intensity will decrease to satisfy the steady-state conditions.

### 3.3. Labor intensity for R&D

A decrease in the labor intensity for R&D ($a_R$) lowers the cost of R&D activity and raises the incentives for R&D activity. This raises the adjusted level of R&D difficulty. Since more Northern labor is available for Northern production, the extent of Northern production will increase. The increase in the demand of Northern labor raises the

\textsuperscript{13} It is easy to prove Propositions 3-5 by taking partial differentials of Eqs. (33)-(36) with respect to $\alpha$, $a_F$, $a_O$ and $\varepsilon$. Proofs are available upon request.
Northern wage rate. Thus, the demand of Southern unskilled workers increases. This lowers the relative wage rate in the South and increases the international wage inequality for skilled workers. Then the fraction of Southern workers being skilled will decrease and this will in turn reduce the extent of FDI. Furthermore, the increase in the supply of Southern unskilled workers will raise the extent of outsourcing and reduce the extent of Northern production. Our numerical results show that over all, the extent of Northern production will be lower. To maintain the steady-state conditions, the FDI intensity will decrease while the outsourcing intensity will increase.

3.4. Labor intensity for FDI

In order to attract Northern firms to adapt their basic production in the South, the Southern government can make efforts to improve Southern economic environment. This can be represented by a decrease in the labor intensity for FDI ($a_F$). A decrease in $a_F$ lowers the cost of FDI, increases the incentives of FDI and lowers the incentives of outsourcing. This raises the extent of FDI and reduces the extent of outsourcing. The demand of Southern skilled workers becomes higher and this in turn will increase the relative wage rate in the South. The increase in the Southern relative wage rate lowers the international wage inequality for skilled workers because $a_F$ does not affect the Northern wage rate directly. The higher relative wage rate in the South will raise the fraction of Southern workers being skilled. Proposition 4 summarizes our findings of wage rates.

**Proposition 4.** A fall in the labor intensity for FDI will not affect the Northern wage rate. However, it will increase the Southern wage rate for skilled workers and the fraction of population being unskilled. Thus, the international wage inequality for skilled workers will become lower.
A decrease in the Southern unskilled workers will reduce the extent of outsourcing even more. With a large drop in the extent of outsourcing, the extent of Northern production increases. This implies that less Northern labor can be used for R&D activity and the adjusted level of R&D difficulty will decrease. The FDI intensity will increase while outsourcing intensity will decrease to satisfy the steady-state conditions.

3.5. Outsourcing opportunities

Improvement in Southern economic environment can be captured by a decrease in the labor intensity of outsourcing \( a_o \) or the risk for outsourcing \( \varepsilon \). A decrease in \( a_o \) (\( \varepsilon \)) lowers the cost of outsourcing and raises the incentives for firms to conduct basic production in the South. This reduces the Northern wage rate and the extent of Northern production. With more Northern labor available for R&D, the adjusted level of R&D difficulty increases. The relative wage rate in the South will decrease due to the increase in the demand of Southern unskilled workers for production. The international wage inequality for skilled workers will also become higher. The lower wage rate for Southern skilled workers will cause a lower fraction of Southern population being skilled. Results related to the wage rates are summarized in the following proposition.

**Proposition 5.** The better the Southern economic environment, the higher the Northern wage rate, the larger the Southern wage rate for skilled workers, the higher the international wage inequality for skilled workers and the greater proportion of Southern workers being unskilled.

With a smaller proportion of Southern workers being skilled, the extent of FDI decreases. Since both the extents of Northern production and FDI will decrease, the
extent of outsourcing will increase. Consequently, the FDI intensity will decrease while the outsourcing intensity will increase. Results shown in Table 1 also show that one percentage increase in $a_o$ cause larger percentage changes of key variables than the one percentage increase in $\epsilon$.

Our findings of the impacts of $\alpha$, $a_R$ and $a_o$ on the pattern of production are very different from those found in Glass and Saggi (2001). In their study, an increase in $\alpha$ ($a_R$) or a decrease in $a_o$ will lower the extent of Northern production and raise the extent of outsourcing. However, we show that with the consideration of FDI and skill accumulation in the South, changes in $\alpha$, $a_R$ and $a_o$ will generate different effects on patterns of production.

3.6. Skill accumulation

Finally, we study the impact of skill accumulation in the South by allowing the skill formation elasticity of the time spent in schools ($\beta$) to increase. Because an increase in $\beta$ does not affect wage rates directly, the Northern wage rate, the Southern wage rate and the international wage inequality for skilled workers remain the same.

An increase $\beta$ will increase time spent in schools and will cause two effects on the fraction of Southern workers being skilled. An increase in time spent in schools will lower it while the shorter time spent by skilled workers on work will raise it in order to keep the lifetime income unchanged so that the equilibrium with the coexistence of skilled and unskilled Southern workers can be restored. Our numerical results show that the former dominates the latter, so the fraction of Southern workers being skilled will decrease. Thus, the extent of FDI will decrease and the extent of outsourcing will increase. Because in the South, more unskilled labor is available for production, the extent of Northern production will decrease. As with more Northern

14 Note that a 1% increase in $\beta$ will increase time spent in schools by 0.9%. 

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labor can be used for R&D activity, the adjusted level of R&D difficulty will increase. The FDI intensity will decrease while the outsourcing intensity will increase to maintain the steady-state conditions.

4. SKILL FORMATION

One important feature of this paper is that we endogenize skill accumulation in a North-South model. In this section, we examine the robustness of setting of skill formation by allowing the value of $\gamma$ to vary. Table 2 presents the results of $\gamma = 0.6$ and its associated equilibrium values are given in the 2nd row. Since $w^N$, $w^S_H$ and $w_H$ determined by Eqs. (33)-(35) are not affected by $\gamma$, results shown in the 2nd, 3rd and 4th columns in Table 1 remain valid. Comparing the results of $\gamma = 0.6$ if Table 2 with those in Table 1, we find that they are different in two places: (1) the extent of outsourcing decreases as the share of basic production decreases; and (2) the extent of Northern production increases as the labor intensity of R&D decreases. The reason is that as $\gamma$ becomes larger, it causes a smaller percentage change in the fraction of Southern workers being skilled. Since the fraction of Southern skilled workers will affect the extent of FDI directly, this will in turn induce a smaller percentage of the extent of FDI. This can be easily seen from Eq. (36). By using Eq. (36), we can derive:

$$\frac{\Delta(1 - \phi)}{1 - \phi} = \frac{1}{\eta - \gamma} \left\{ - \frac{\Delta h(\bar{D})}{h(\bar{D})} + \eta \frac{\Delta B(\bar{D})}{B(\bar{D})} + (1 - \eta) \left[ \frac{\Delta \sigma(\bar{D})}{\sigma(\bar{D})} - \frac{\Delta w^S_H}{w^S_H} \right] \right\},$$

where $\Delta X$ denotes the change of variable $X$. Note that in Tables 1 and 2, time spent in school $(\bar{D})$ is not affected, except for the case of changes in $\beta$. If $\bar{D}$ remains unchanged, there will be no changes in $h(\bar{D})$, $B(\bar{D})$ and $\sigma(\bar{D})$. Thus, with the same percentage change in $w^S_H$, the larger $|\eta - \gamma|$ is, the smaller percentage change of $(1 - \phi)$ will be.

When $\alpha$ increases, the lower Northern wage rate reduces the extent of outsourcing while the increase in the supply of Southern unskilled workers raises the
extent of outsourcing. The former will dominate the latter if \( \gamma \) is high and the extent of outsourcing will decrease. When \( a_R \) decreases, it increases the extent of Northern production directly. However, it also causes a small increase in \( \phi \) which in turn will induce a small decrease in the extent of Northern production. Thus, the extent of Northern production will increase if \( \gamma \) is high.

In our benchmark, we set \( \gamma > \eta \) based on empirical studies. However, there are other settings of skill formation (calibration) in the literature of human capital formation. For example, de la Croix and Doepke (2004) set the parameter \( \gamma \) larger than 0.7 when conducting numerical analysis for a growth model with endogenous fertility. Parallo (2008) provided other example by assuming that skill accumulation does not depend on public educational investments (i.e. \( \gamma = 0 \)) in a North-South model of intellectual property rights.

We then turn to examine the case of \( \gamma < \eta \) by setting \( \gamma = 0.05 \). Results are also presented in Table 2 where the equilibrium values of key variables are given in the 10\(^{th}\) row. Comparing results in Table 1 and 2, we find the impacts of \( L_N \) on key variables under different values of \( \gamma \) are qualitatively the same. However, the effects of \( \alpha (a_R, a_F, a_O, \varepsilon, \beta) \) on key variables are very different. The 12\(^{th}\) row in Table 2 shows that an increase in \( \alpha \) will reduce the fraction of Southern workers being unskilled. Thus, the extent of FDI decreases with an increase in the extent of outsourcing. Similarly, due to the effects of public educational investments on the skill formation, changes in \( a_R (a_F, a_O, \varepsilon, \beta) \) cause \( \phi \) to react towards a different directions from those obtained in Table 1. This will further induce the extent of FDI to react towards the opposite directions from those obtained in Table 1. Their impacts on the adjusted R&D difficulty and the extents of Northern production and outsourcing are not clear because changes in \( x, n_N \) and \( n_O \) depend on the changes of demands of Northern and Southern labor.
Next, we consider the case where public educational investments do not affect skill accumulation by setting $\gamma = 0$. Numerical results have shown in Table 2 with the equilibrium values of key variables given in the 18th row. We find that these results are quite similar to those obtained when $\gamma = 0.05$ and one can think this case as a special case of $\gamma < \eta$.

Results of $\phi$ in Tables 1 and 2 indicate that skill accumulation function is a very important determinant to the pattern of production. Eq. (8) implies that in order to maintain the equilibrium where skilled and unskilled workers coexist in the South, $w^S_H$ and $(1 - \phi)$ will be negatively correlated if $\gamma < \eta$. We use changes in $\alpha$ as an example. As $\alpha$ increases, $w^S_H$ becomes lower and the equality of Eq. (8) does not hold since the right-hand side of Eq. (8) becomes lower. To restore the equilibrium with the coexistence of unskilled and skilled Southern workers, the fraction of Southern workers being skilled needs to increase so that the externality of skill becomes higher and Eq. (8) will hold. This will further affect the extents of Northern production, FDI and outsourcing through the labor market clearing conditions.

However, the setting of $\gamma < \eta$ generates some unreasonable results. Under this setting, a decrease in the incentive of skill accumulation will cause an increase in the fraction of Southern workers being skilled. Besides, the impacts on the pattern of production are counter-intuitive since an increase in the incentive of FDI (a lower $a_F$) will reduce the extent of FDI and an increase in the incentive of outsourcing (a lower $a_O$ or $\epsilon$) will reduce the extent of outsourcing. Notice that $w^S_H$ and $(1 - \phi)$ becomes positively correlated if $\gamma > \eta$. Thus, changes in $a_R (a_F, a_O, \epsilon, \beta)$ will cause $\phi$ and $n_F$ to move toward opposite directions in Tables 1 and 2.

5. CONCLUSION

In this paper, we develop a North-South model with skill accumulation to investigate
the effects of international specialization on innovation, skill accumulation, wage inequality and pattern of production. This study demonstrates the important role of skill formation in determining pattern of production. The wage rate for Southern skilled workers and the fraction of Southern workers being skilled will be positively correlated if the elasticity of skill formation with respect to the public educational investment is larger than the elasticity with respect to skill externality and vice versa. This will further affect pattern of production through labor market clearing conditions. Our numerical results indicate that a sufficiently large elasticity of skill formation with respect to the public educational investment will generate more reasonable results since under this setting, the extent of FDI (outsourcing) will increase in response to an increase in the incentives of FDI (outsourcing).

Regarding wage inequalities, we find that changes in Northern labor, the elasticities of skill formation with respect to time spent in schools and the public educational investment do not affect international and Southern wage inequalities. However, if the demand of Southern unskilled workers increases, the relative wage rate in the South will decrease and the international wage inequality for skilled workers will increase (except changes in $\alpha$) and vice versa.

Our paper concludes with the suggestion that this study could easily be extended and applied to a variety of issues which would seem to be ripe for future study, pointing out two possible directions. First, considering imitation activity in the South would allow us to investigate the effects of intellectual property rights protection on FDI and outsourcing. Second, the outsourcing contracts can be endogenized. By designing elaborate contracts, Northern firms can avoid the loss of profits caused by a bad shock of Southern economic environment. It would be interesting to examine how Southern economic environment affects the contracts designing.
REFERENCES


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Table 1: Numerical Results

<table>
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<tr>
<th>Benchmark</th>
<th>$w^N$</th>
<th>$w^S_H$</th>
<th>$w_H$</th>
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<th>$x$</th>
<th>$n_N$</th>
<th>$n_F$</th>
<th>$n_O$</th>
<th>$\lambda_F$</th>
<th>$\lambda_O$</th>
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<td>1.5000</td>
<td>1.0667</td>
<td>0.8065</td>
<td>6.6410</td>
<td>0.4940</td>
<td>0.2432</td>
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<td>1.0101</td>
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<td>2.2083</td>
<td>9.5970</td>
<td>-13.0353</td>
<td>7.2291</td>
<td>-14.9142</td>
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Note: Numbers reported in rows 3-9 are percentage changes of key variables from their benchmark values (presented in row 2) due to each exogenous shift.
Table 2: Human Capital Formation

<table>
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<tr>
<th>γ = 0.6</th>
<th>( \phi )</th>
<th>( x )</th>
<th>( n_N )</th>
<th>( n_E )</th>
<th>( n_G )</th>
<th>( t_E )</th>
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<table>
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<td>0.8256</td>
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<td>0.5618</td>
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<td>-3.0951</td>
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<tr>
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<td>0.2290</td>
<td>-4.1553</td>
<td>-16.7393</td>
<td>29.2931</td>
<td>-13.1295</td>
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<td>4.3481</td>
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<td>( \varepsilon ) down by 1%</td>
<td>-1.9434</td>
<td>0.1104</td>
<td>1.7637</td>
<td>7.4502</td>
<td>-12.8450</td>
<td>5.8800</td>
<td>-14.4983</td>
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<tr>
<td>( \beta ) up by 1%</td>
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<td>-0.0692</td>
<td>1.3208</td>
<td>5.3063</td>
<td>-9.2940</td>
<td>3.9335</td>
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<th>( \phi )</th>
<th>( x )</th>
<th>( n_N )</th>
<th>( n_E )</th>
<th>( n_G )</th>
<th>( t_E )</th>
<th>( t_G )</th>
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<td>-0.8193</td>
<td>0.8253</td>
<td>0.8253</td>
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<td>1.6581</td>
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<tr>
<td>( \alpha ) up by 1%</td>
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<td>1.2022</td>
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<td>-3.4968</td>
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<td>( a_R ) down by 1%</td>
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<td>0.4884</td>
<td>0.3151</td>
<td>0.5669</td>
<td>-1.2822</td>
<td>0.2510</td>
<td>-1.5923</td>
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<td>0.1158</td>
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<td>14.0157</td>
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<tr>
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<td>1.9868</td>
<td>8.9717</td>
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Note: Numbers reported in rows 3-9, 11-17 and 19-25 are percentage changes of key variables from their benchmark values (presented in row 2 and 10) due to each exogenous shift.
APPENDIX A

A.1. Proof of Proposition 1

In this appendix, we prove the existence of the BGP equilibrium. Since the left-hand side of Eq. (9) is increasing in $\bar{D}$ and the right-hand side of Eq. (9) is decreasing in $\bar{D}$, there exists a unique solution of $\bar{D}$. Solutions of $t_R, w^N, w_H^S, \phi$ and $x$ are given by Eqs. (22), (33), (34), (36) and (38). Substituting the solution of $x$ in Eq. (38) into Eq. (32), we can derive $\hat{E}$. Solutions of $n_N$ and $t_F$ are given by Eqs. (39) and (40). Then from Eq. (24), we can derive:

$$n_F = \frac{t_F n_N}{t_R}.$$  \hspace{1cm} (A1.1)

Eqs. (23) and (25) imply that

$$n_O = 1 - n_N - n_F,$$  \hspace{1cm} (A1.2)

$$t_O = \frac{(t_R + \varepsilon) n_O}{n_N}.$$  \hspace{1cm} (A1.3)

Then, we need to restrict solutions of $\{\phi, n_N, n_F, n_O\}$ to be between zero and one. We first consider the case for $\eta > \gamma$. Eq. (36) indicates that $\phi$ is less than one. Combining Eqs. (34) and (36), we can show that $\phi$ is positive if

$$a_F < a_F^0 = \frac{\mu w^N (h(\bar{D}) B(\bar{D})^\eta g^\gamma)}{\sigma(\bar{D}) [\rho + (1 - \xi) t_R]} \frac{1}{1-\eta},$$  \hspace{1cm} (A1.4)

where $w^N$ is given by Eq. (33).

Eq. (39) shows that $n_N$ is less than 1. Furthermore, it indicates that $n_N$ is positive if

$$\frac{\lambda \phi}{a_F \hat{E}} < 1.$$  \hspace{1cm} (A1.5)

Substituting Eqs. (32), (34), (36) and (38) into Eq. (A1.5), then we have

$$a_F > a_F^1,$$  \hspace{1cm} (A1.6)
where \( a_F^1 = \left\{ \frac{1 - \Gamma(\theta + \frac{\mu N}{L_S})}{\frac{1 - \Gamma}{\mu N} h(D) B(D) g_C^\gamma (1 - \phi)^{1 - \eta}} \right\}^{\frac{\eta}{1 - \eta}} \) and

\[
\Gamma = \frac{a(\lambda + [\rho + (1 - \xi)B]a_R)}{\lambda + [\rho + (1 - \xi)B]a_R}.\]

Eq. (A1.1) implies that \( n_F > 0 \). Combining Eqs. (40) and (A1.1), we have \( n_N + n_F < 1 \) if

\[
\frac{\lambda \phi}{a_F^1} > \frac{h(D) B(D) g_C^\gamma (1 - \phi)^{1 - \eta}}{a_F^2},
\]

Substituting Eqs. (32), (34), (36) and (38) into Eq. (A1.7), then Eq. (A1.7) holds if

\[
a_F < a_F^2,
\]

where \( a_F^2 = \left\{ \frac{\lambda - 1}{\lambda + [\rho + (1 - \xi)B]a_R} \right\}^{\frac{\eta - \gamma}{1 - \eta}} \). When Eq. (A1.8) holds, we have \( n_F < 1 \). Furthermore, from Eq. (A1.2), it also implies that \( n_O > 0 \). Since both \( n_N \) and \( n_F \) are positive, Eq. (A1.2) indicates that \( n_O < 1 \). Thus, the unique BGP equilibrium exists if \( a_F \in (a_F^1, \min \{a_F^0, a_F^2\}) \).

Similarly, one can show that if \( \eta < \gamma \), BGP equilibrium exists if \( a_F \in (\max\{a_F^0, a_F^2\}, a_F^1) \). Note that \( a_F^1 < \min \{a_F^0, a_F^2\} \) if \( \frac{L_N}{L_S} \) is large enough and vice versa.

Q.E.D.

A.2. Proof of Proposition 2

Note that Eqs. (33), (34) and (36) show that \( w^N, w_H^S \) and \( \phi \) do not depend on \( L_N \). These imply that \( w_H^S \) is also independent of \( L_N \). Using Eq. (38) to differentiate \( x \) with respect to \( L_N \), we have:
\[
\frac{\partial x}{\partial L_N} = \frac{\alpha(\lambda - 1)}{L_S\{\lambda \mu + [\rho + (\lambda - \xi)\tau_R]aa_R}\lambda} > 0. \quad (A2.1)
\]

Substituting Eqs. (32) and (36) into Eq. (39), we can compute the differentiation of \( n_N \) with respect to \( L_N \):
\[
\frac{\partial n_N}{\partial L_N} = \frac{\phi}{ax} \frac{\partial x}{\partial L_N} > 0. \quad (A2.2)
\]

Differentiating \( t_F \) with respect to \( L_N \) by using Eq. (40) gives us:
\[
\frac{\partial t_F}{\partial L_N} = -\frac{[h(D)B(D)g_{\chi}(1 - \phi)^{1-\gamma}]^{1/\eta}}{a_F(xn_N)^2} \left( n_N \frac{\partial x}{\partial L_N} + x \frac{\partial n_N}{\partial L_N} \right) < 0. \quad (A2.3)
\]

Eq. (A1.1) implies that:
\[
\frac{\partial n_F}{\partial L_N} = \frac{1}{t_R} \left[ n_N \frac{\partial t_F}{\partial L_N} + t_F \frac{\partial n_F}{\partial L_N} \right]. \quad (A2.4)
\]

Substituting Eqs. (A2.3) and (40) into Eq. (A2.4) generates:
\[
\frac{\partial n_F}{\partial L_N} = \frac{t_F}{t_R} \left[ \frac{1}{x} \frac{\partial x}{\partial L_N} + \left( 1 - \frac{1}{n_N} \right) \frac{\partial n_N}{\partial L_N} \right]. \quad (A2.5)
\]

Eq. (A2.5) indicates that \( \frac{\partial n_F}{\partial L_N} < 0 \) since \( n_N \in (0,1) \).

Eqs. (A2.1)-(A2.3) and (A2.5) indicate that an increase in \( L_N \) will raise \( x \) and \( n_N \), reduce \( t_F \) and \( n_F \) and leave \( w^N, w^S_H, w_H \) and \( \phi \) unchanged.

\[ Q.E.D. \]