Procurement Auctions with Pre-award Subcontracting

by

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Abstract

To be the lowest bidders in procurement auctions, contractors commonly solicit subcontract bids at the bid preparation stage. A remarkable feature of the subcontract competition is that “winning is not everything”; the lowest subcontractor gets a job conditional on his contractor’s successful bid. This paper makes the first attempt to establish a model for such pre-award subcontract competitions included in procurement auctions. It is found that subcontractors strategically provide larger discounts on their bids in response to increasing competition among contractors, which results in an endogenous downward shift in the distribution of bidders’ private information in the downstream auction as the number of rivals increases or the reservation price drops. The process has a striking impact on the analysis of the optimal reservation price and the empirical identification of the bidder’s cost distribution in procurement auctions.

1 Introduction

Subcontracting and outsourcing are common business practices in procurement markets. In a highway construction project, for instance, the winning bidder may subcontract road marking or signal work to specialty firms. In addition, the contractor may purchase raw materials or equipment from other sources, which can also be considered subcontracting in the broader sense. In construction projects which cover a wide range of work, it is impractical for the contractor to perform all the work. Therefore, for prime contractors, the bulk of the cost of a large construction project consists of subcontract payments.

To obtain qualified subcontracts at fair prices, prime contractors (PCs) commonly solicit irrevocable pre-bid price quotes from subcontractors (SCs) in the bid preparation stage. This practice also satisfies a PC’s need to estimate the project cost for tendering\(^1\). Some

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\(^1\)There are several legal issues regarding the irrevocability of subcontract bids. The California Supreme Court case of Drennan v. Star Paving Co.,\(^2\) suggested that a subcontract bid is irrevocable once it is relied
procurement buyers require contractors to submit a proposed subcontracting plan that must be approved by the contracting officer prior to bidding.\textsuperscript{2}

Such pre-award subcontract competitions are commonly seen not only in public procurement but also in private markets (Dyer and Kagel (1996), Degn and Miller (2003)). In this sense, PCs are not only bidders in the downstream procurement auction but also auctioneers in the upstream subcontract auction.

The aim of this research is to model subcontract auctions included in the procurement auction. A remarkable feature of such subcontract auctions is that winning is not everything; the awarded SC gets a job only if his PC wins in the downstream competition. Presumably, analyzing subcontract auctions requires a different framework from standard auctions where the lowest bidder always obtains a payoff.

The first question regarding upstream auctions, therefore, is whether the SC’s bidding strategy depends on the downstream competition. Since an SC obtains a payoff only if his PC wins in the downstream auction, the downstream competition should be a non-trivial concern of the SCs. In particular, if the PC faces intense competition in the downstream auction, lowering the subcontract bid will help the PC win in the downstream competition, and could, in turn, be beneficial for the SC.

If the upstream and downstream competitions are linked through the SC’s response to the downstream competition, the next question is that whether the existing auction theories under the independent private value (IPV) assumption can apply to the downstream auction. If the number of bidders or the reservation price in the downstream auction affects the SCs’ behavior, the downstream auction will be affected since the bidders’ costs include the winning subcontract bids. Prime contractors may have stronger bargaining power as competition increases in the downstream auction. Thus, it is worthwhile to verify whether the existing auction theorem (such as revenue equivalence or optimal reservation price) holds in the downstream auction.

The third question is regard to the PC’s mechanism choice in upstream auctions. The existing literature gives unclear guidance concerning the optimal mechanism for a PC to select SCs, especially regarding the selection between the first- and second price rules. Taking into account the widespread use of first-price auctions in real world subcontract competitions, it is upon by the prime contractor in computing her overall bid (the Drennan Rule). However, the prime contractor is free to lower the subcontract price by disclosing the current lowest subcontract bids on some subcontracts to obtain an even lower price (bid shopping). See Grosskopf and Medina (2007) for more details.

\textsuperscript{2}For instance, the state of Oregon requires bidders in public projects to submit a list of first-tier subcontractors and their subcontract bids if the amount of the bid is greater than five percent of the total project bid or $15,000 (ORS 279C.370).
is interesting to examine whether the first-price auction dominates the second-price rule from
the viewpoint of the PC’s profit maximization.

The final and most basic question lies in whether there exists a symmetric increasing
equilibrium which supports the above arguments. Since the bidder’s objective function differs
from the standard auction setting, especially in upstream auctions, a new model must be
established in order to analyze the procurement auction including pre-award subcontract
competitions.

To answer these questions, a two-stage game is established. In the first stage, each PC
solicits irrevocable price offers from her own SCs and selects the lowest one, assuming that
PCs know the distribution, but not the values of the SCs’ marginal costs. With the selected
SC, each PC makes a pre-award subcontract agreement which specifies the scope of the job
performed by the SC and the payment made by the PC to the SC. In the second stage, the
PC computes the total project cost by adding up her own marginal cost and the subcontract
payment. Then, each PC bids in the procurement auction assuming that each bidder does
not know the other bidders’ project costs. The winning PC undertakes the contract with the
selected SC, incurring her own marginal cost and making the subcontract payment.

After the existence of a symmetric increasing equilibrium in both the upstream and down-
stream auctions is verified, PCs have greater bargaining power than SCs in response to in-
creasing competition in the downstream auction; SCs strategically provide larger discounts on
their price offers as the number of PCs rises or the reservation price declines in the downstream
competition. Such behavior results in an endogenous downward shift in the distribution of
the bidders’ private information as the number of bidders increases or the reservation price
drops in the downstream auction. This fact contradicts the assumption employed in standard
auction models that the distribution of the players is exogenously given.

Furthermore, PCs’ expected payoffs may be greater when the first-price auction rather
than the second-price auction is used in upstream competitions. The use of the first-price
rule in upstream auctions benefits not only the procurement buyer but also society, yielding a
more \textit{ex post} efficient allocation. These results are fairly in line with the conventional wisdom
on the widespread use of the first-price auction in real-world procurement auctions.

Given bidders with endogenously distributed private information, what is the optimal
mechanism for the procurement buyer? The Revenue Equivalence Theorem still holds in the
downstream auction due to the fact that the IPV property is well maintained. However,
because of the endogeneity of the PC’s private information distribution, the optimal reserva-
tion price becomes a function of the number of bidders, unlike that under the standard IPV
environment, which is thoroughly examined in Riley and Samuelson (1981).

Our model is closest to that of Hansen (1988), who argues that bidders bid more aggressively if there is a downstream market in which the quantity demanded is determined by the winning bid price. A non-trivial extension made here to Hansen lies in modeling the downstream competition using an auction game, which enables us to provide a qualitative examination of the downstream market, including optimal design and efficiency analysis.

On the other hand, there is a fairly large volume of existing literature which deals with procurement auctions involving subcontracting. For instance, Kamien et al. (1989) and Gale et al. (2000) analyze post-award subcontracting in which the winning firm may split the award and subcontract parts of it to the best partners. Because the SCs’ marginal costs are essentially unobservable for PCs, there exists an adverse selection problem between SCs and PCs. A great deal of literature, therefore, investigates the principal-agency relationship between SCs and PCs (e.g. Kawasaki and McMillan (1987)). Furthermore, the variations in subcontracting regulations across nations is discussed in detail in Marechal and Morand (2003). However, the existing literature does not model both upstream and downstream competitions, nor does it analyze both at the same time.

The remaining part of this paper is organized as follows. Section 2 describes the model of procurement auctions with pre-award subcontracting. Section 3 examines the equilibrium bidding behavior in upstream auctions. Section 4 analyzes the equilibrium in the downstream auction. Section 6 provides further discussion, and the final section is the conclusion. Proofs are given in the Appendix.

2 The model

Consider that a procurement buyer solicits bids from $N$ prime contractors (PC) denoted by $i = \{1, \ldots, N\}$ to purchase a project. The value for the procurement buyer is assumed to be equal to $V$.

Suppose the $i$th PC’s cost of producing the project is characterized by

$$c_i = \bar{t} + \theta_i,$$

where $\bar{t} \in T$ is a positive constant value whereas $\theta_i \in \Theta$ is a random variable drawn from the publicly known distribution $F_\theta$. In other words, the work to complete the project consists of

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3 A variety of issues on procurement contracts are discussed in Bajari and Tadelis (2001), such as contract forms (fixed-price or cost-plus), tendering or negotiating, change orders, default or non-performance.
two components, \( T \) and \( \Theta \), each of which involves the project costs \( \bar{t} \) and \( \theta_i \). Suppose the \( i \)th PC knows \( n \) subcontractors (SCs), denoted by \( SC_{i,j} \) with \( j \in \{1, \ldots, n\} \), who can complete the work \( T \) with the production cost \( t_{i,j} \in [t, \bar{t}] \). Prior to the procurement auction held by the procurement buyer, the \( i \)th PC solicits subcontract bids through a first-price sealed bid auction from the \( n \) subcontractors. Upon solicitation by the \( i \)th PC, each SC draws its production cost \( t_{i,j} \) from the publicly known distribution \( F_t \) with \( F_t(\bar{t}) = 0 \) and \( F_t(t_{\bar{t}}) = 1 \) and submits a subcontract bid \( s_{i,j} \) to the \( i \)th PC.

Let \( s_i \) be the lowest subcontract bid received by the \( i \)th PC. Then, after calculating the marginal costs \( c_i = s_i + \theta_i \), the \( i \)th PC submits a bid \( b_i \) to the procurement buyer.

Throughout the paper, private information is assumed. The SC’s marginal cost \( t_{i,j} \), which is drawn from the publicly known distribution \( F_t \), is privately known to \( SC_{i,j} \). The PC’s cost of completing \( \Theta \) work, denoted by \( \theta_i \), is also privately known only to the \( i \)th PC.

To simplify the analysis for the existence of an increasing symmetric equilibrium, the cumulative distribution functions of \( \theta \) and \( t \) are assumed to satisfy the increasing hazard rate (IHR), i.e., \( \frac{d}{dt} \frac{f_\theta(t)}{F_\theta(t)} \leq 0 \) and \( \frac{d}{dt} \frac{f_t(t)}{F_t(t)} \leq 0 \). Many distribution functions, including uniform, normal and chi distributions, meet the criteria.

### 3 Equilibrium in upstream auctions

From here on, we analyze a symmetric equilibrium in upstream auctions based on two cases. The first subsection is a discussion of a simple case in which a PC’s costs depend only on the selected subcontract bids and the number of subcontract bids solicited is equal to one. The second subsection is devoted to the analysis without these restrictions. Both analyses
will proceed backward, assuming symmetric equilibrium in the downstream (second-stage) auction.

### 3.1 A simple case

A simple model assumes that \( n = 1 \) and that the \( i \)th PC’s marginal cost \( \theta_i \) is constant and is normalized to zero.

To simplify a notation, let \( t_i \) denote the cost signal of the SC who submits a subcontract bid to the \( i \)th PC. Because the first-price rule is used in a subcontract auction, the subcontract bid is equal to the amount of payment for the SC. Hence, assuming that all the SCs follow an increasing strategy \( \sigma \) in upstream auctions, the PC’s marginal cost is equal to

\[
c = \sigma(t_i).
\]

Based on the characterization of private information, each PC strategically determines the optimal bid for the downstream auction. Clearly, the \( i \)th PC’s cost is independently and identically distributed for all \( i \) if the \( i \)’s subcontractor follows the symmetric increasing strategy \( \sigma \). Hence, standard arguments can be applied to claim that there exists a symmetric increasing equilibrium \( \beta \) in the PC’s strategy if a standard auction mechanism is used in the downstream auction.\(^4\)

Now, let us go back to the analysis of upstream auctions. Suppose all SCs follow the symmetric increasing strategy \( \sigma \) and that all the rival PCs follow the symmetric increasing bidding strategy \( \beta \). Then, the probability that \( i \)th PC will beat the remaining \( N - 1 \) rival PCs is given by

\[
Pr(\text{Prime wins}|P(s), s) = [1 - F_t(\sigma^{-1}(s))]^{N-1}.
\]

Since the \( i \)th SC receives the payoff \( s_i - t_i \) conditional on the \( i \)th PC’s successful bid, the SC’s objective function is given as

\[
\pi(t_i|N, b_r) = \max_s (s - t_i) \left[ 1 - F_t(\sigma^{-1}(s)) \right]^{N-1}.
\]

To solve the SC’s symmetric equilibrium strategy in the upstream auction, take the derivative

\(^4\)Standard auctions include first- and second-price auctions in which the PC with the lowest marginal cost is awarded.
of (1) with respect to \(s\). Solving the differential equation and suppressing some notations, the candidate of the equilibrium strategy \(\sigma\) is obtained as

\[
\sigma(t_i|N, b_r) = t_i + \frac{\int_{t_i}^{b_r} [1 - F_t(\hat{t})]^{N-1} d\hat{t}}{[1 - F_t(t_i)]^{N-1}}.
\]

(2)

Notice that this strategy is identical to the equilibrium bidding strategy in a symmetric independent private value procurement auction with \(N\) bidders. Recall that an upstream auction has only one bidder, whose optimal strategy should be to bid as high as possible in the case of a standard procurement auction. However, the obtained SCs’ bidding function shows that SCs bid strictly lower than the highest price. In particular, each SC bids as if all other SCs, including the ones bidding for other PCs, were also his rivals.

It is easy to see that SCs bid more aggressively, i.e., bid lower as competition increases among PCs. Consider first that there is one more PC in the downstream procurement auction. Then, for any \(t\), the second term on the left-hand side of equation (2) declines to

\[
\frac{\int_{t_i}^{b_r} [1 - F_t(\hat{t})]^{N-1} d\hat{t}}{[1 - F_t(t_i)]^{N-1}}.
\]

Since the term is also decreasing in \(b_r\), the informational rent of SCs drops as the reservation price in the downstream auction declines.

Obviously, these results are observed only when a PC uses the first-price rule in the upstream auction. Truth-telling is the SC’s dominant strategy if the second-price rule is used in the upstream competition. Since the subcontract payment is always the same as \(\bar{t}\) if \(n = 1\), the PC who has the lowest subcontractor among all, including the ones who bid to the other PCs, may not get a job. It follows that the second-price rule in the upstream competition can create \textit{ex post} inefficiency, while the first-price rule cannot in this simple case.

When this simple case is generalized, the SCs’ bidding strategy and, hence, the PCs’ costs are endogenously determined by the number of rivals and the reservation price in the downstream auction. The qualitative features of the simple example are therefore robust.

### 3.2 Generalized cases

Now, let us investigate the generalized case where \(n \geq 1\) and \(\theta\) is a random variable.\(^6\) As in the simple case, the PC’s marginal cost is independently and identically distributed if there exists a symmetric increasing equilibrium strategy \(\sigma\) in upstream auctions. Therefore, the

\(^5\)For any \(\hat{t} > t_i\),

\[
\left(\frac{1 - F_t(\hat{t})}{1 - F_t(t_i)}\right)^N - \left(\frac{1 - F_t(t_i)}{1 - F_t(t_i)}\right)^{N-1} = \left(\frac{1 - F_t(\hat{t})}{1 - F_t(t_i)}\right)^{N-1} \left[\frac{1 - F_t(t_i)}{1 - F_t(t_i)} - 1\right] < 0.
\]

Thus, for any \(t_i < b_r\), the bidder’s informational rent decreases as the number of PCs increases from \(N\) to \(N + 1\).

\(^6\)The randomized \(\theta\) can not only capture the private information of the PCs but also take into consideration the case in which PCs may subcontract other parts of the work and these subcontracts are competitively distributed to other sets of SCs.
PC with the lowest cost wins in the downstream auction if a standard auction is used.

Given the outcome in the downstream auction, we examine upstream auctions and investigate the SCs’ symmetric equilibrium strategy $\sigma$. If other SCs who submit a subcontract bid to the $i^{th}$ PC follow $\sigma$, then $P(s|n) = \left[1 - F_i(\sigma^{-1}(s))\right]^{n-1}$ is the probability that the SC who submits a subcontract bid $s$ will be the lowest bidder in an upstream auction. Let $Q(s|\cdot)$ denote the probability that the PC who receives $s$, the lowest subcontract bid, wins in the downstream auction given that all the SCs submitting a bid to the $i^{th}$ PC with $\hat{i} \neq i$ follow $\sigma$. Then, the objective function of the $j^{th}$ SC in the $i^{th}$ upstream auction is given as

$$\pi(t_{i,j}|N, b_r, \sigma) = \max_s (s - t_{i,j}) \left[1 - F_i(\sigma^{-1}(s|\cdot))\right]^{n-1} Q(s|N, b_r, \sigma).$$ (3)

To obtain $Q(\cdot)$, we first examine the probability that the $i^{th}$ PC’s marginal cost $c_i$ is lower than another PC’s cost, provided that the $i^{th}$ PC’s cost equals $\theta_i$ and the lowest subcontract bid submitted to the $i^{th}$ PC equals $s_i$. Denoting by $F_c(\cdot)$ the cumulative distribution function of $c_i$, the Convolution Theorem gives the probability by $^7$

$$1 - F_c(s_i + \theta_i|\sigma) = \int_t^N n f_i(t) \left[1 - F_i(t)\right]^{n-1} \left[1 - F_\theta(s_i + \theta_i - \sigma(t|\cdot))\right] dt.$$ (4)

Since the number of PCs equals $N$, the probability that the $i^{th}$ PC who receives $s_i$ as the lowest subcontract bid will win in the downstream auction is equal to

$$Q(s_i|N, b_r, \sigma) = \int_{\theta}^{b_r-s_i} \left[1 - F_c(s_i + \theta_i|\sigma)\right]^{N-1} f_\theta(\theta_i) d\theta_i,$$ (5)

and the derivative is given by

$$Q'(s_i|N, b_r, \sigma) = -\int_{\theta}^{b_r-s_i} \left[1 - F_c(s_i + \theta_i|\sigma)\right]^{N-1} f_\theta'(\theta_i) d\theta_i,$$ (6)

implying that $Q$ is decreasing in $s_i$.

The decreasing function $Q(s|\cdot)$ is a reminder of the decreasing function $q(\cdot)$, included in the bidder’s objective function in Hansen (1988). He examines an upstream procurement auction in which the winner obtains the right to sell to a downstream market characterized by a downward sloping demand schedule. If $Q(s|\cdot)$ is interpreted as a demand schedule, i.e., the probability that the item is sold in a downstream market, these two models coincide.

$^7$Another way to obtain $1 - F_c(\cdot)$ is given in the Appendix.
A non-trivial extension made here is that the downstream demand schedule, which is exogenously given in Hansen, is endogenously derived from the downstream auction. The advantages of modeling an auction in the downstream competition are three-fold. First, modeling allows greater applicability of the model to real-world business practice. Second, modeling also allows an analysis of the extent to which the environmental variables in the downstream auction, such as the number of bidders and the reservation price, affect the SC’s equilibrium strategy in upstream auctions. Finally, modeling allows an analysis of the optimal mechanism in the downstream auction.

Next, that the SC’s strategy is indeed increasing is verified. Although the verification is a bit more complicated than in the case discussed by Hansen because the downstream competition here is modeled as an auction, the following proposition claims that there exists a symmetric increasing equilibrium in the SCs’ strategy.

**Proposition 1.** The SCs’ symmetric equilibrium strategy $\sigma$ is strictly increasing in $t$. Hence, there exists a symmetric increasing equilibrium in the upstream auctions.

**Proof.** Taking the derivative of (3) with respect to $s$, we obtain

$$\sigma'(\sigma^{-1}(s|\cdot)) = \left[ s - t_{i,j} \right] (n-1) \times \frac{f_t(\sigma^{-1}(s|\cdot))}{1 - F_t(\sigma^{-1}(s|\cdot))} \frac{Q(s|\cdot)}{Q(s|\cdot)} + \left[ s - t_{i,j} \right] Q'(s|\cdot).$$

(7)

For any $t \in [\breve{t}, \bar{t}]$, $\sigma(t|\cdot) \geq t$ holds with the boundary condition $\sigma(\breve{t}|\cdot) = \breve{t}$. Suppose there is an interval $(t^-, t^+) \subset [\breve{t}, \bar{t}]$ such that $\sigma'(t^+) = 0$ and for all $t \in [t^-, t^+)$, $\sigma'(t^-) < 0$. Then, from (7), $\sigma(t^+|\cdot) = t^+$. First, we show that $t^- \leq t$. Suppose $t^-$ is strictly greater than $\breve{t}$. Then, we have $\sigma'(t^-) = 0$, implying that $\sigma(t^-) = t^-$. Since $\sigma$ is decreasing at some $t \in (t^-, t^+)$, $\sigma(t^-) < t$ for all $t \in (t^-, t^+)$. Thus, we have a contradiction. Second, we show that $t^+ \geq \bar{t}$. If this is not true, because of the differentiability of $\sigma$, there should exist $t \in (t^+, \bar{t})$ such that $\sigma(t) < t$, which also contradicts the condition that $\sigma(t|\cdot) \geq t$. Finally, we show that it is impossible for $\sigma'$ to be negative for all $t \in [\breve{t}, \bar{t})$. Suppose it were the case. Since $Q'$ is negative and $Q$ is nonnegative for any $t$, we obtain $Q(\sigma(\breve{t}|\cdot)) > Q(\sigma(t^+|\cdot)) > 0$ for all $t \in (\breve{t}, \bar{t})$. Since $\sigma$ and $Q$ are continuous and differentiable, there exists $\hat{t}$ which is close to $\bar{t}$ such that $Q(\hat{t}|\cdot) + [\hat{t} - \bar{t}] Q'(\hat{t}|\cdot) > 0$, where $\hat{t}$ is a maximizer of (3) if $t = \hat{t}$. On the other hand, by (7), there exists $\bar{t} < \hat{t}$ such that $\sigma'(\bar{t}|\cdot) < 0$, and $Q(\bar{t}|\cdot) + [\bar{t} - \hat{t}] Q'(\bar{t}|\cdot) < 0$, where $\bar{t}$ is a maximizer of (3) if $t = \bar{t}$. Since $\sigma$ is continuous and differentiable, there exists $t^0 \in (\bar{t}, \hat{t})$ such that $Q(s^0|\cdot) + (s^0 - t^0)Q'(s^0|\cdot) = 0$, where $s^0$ is a maximizer of (3) if $t = t^0$, implying that $\sigma'(t^0) \to +\infty$. A contradiction results. \qed
After confirming the strictly increasing $\sigma$, we investigate the SC’s bidding function. Suppressing subscripts and replacing $\sigma^{-1}(s|\cdot) = t$ in (7), we obtain

$$
\frac{1}{\sigma(t|N,b_r) - t} - (n - 1) \frac{f_t(t)}{1 - F_t(t)} \frac{1}{\sigma'(t|N,b_r)} = - \frac{Q'(\sigma(t|N,b_r)|N,b_r,\sigma)}{Q(\sigma(t|N,b_r)|N,b_r,\sigma)}.
$$

(8)

This equation holds for any $\hat{t} \in [t,\hat{t}]$ in equilibrium. Thus, taking the integral from $t$ through $\hat{t}$ and using the integral by parts on the right-hand side, we obtain

$$
\sigma(t|N,b_r) = t + \frac{\int_{t}^{\hat{t}} \left[1 - F_t(i)\right]^{n-1} Q(\sigma(i|\cdot)|N,b_r,\sigma) \, di}{\left[1 - F_t(t)\right]^{n-1} Q(\sigma(t|\cdot)|N,b_r,\sigma)}
$$

(9)

Solving for $\sigma(t|\cdot)$ gives the SC’s bidding function. Although not a closed form, it provides many insights. First, the optimal bidding strategy is to bid the next lowest cost conditional both on the SC’s winning in the upstream auction bid and on the PC’s winning in the downstream auction. This is more easily seen if (9) is rearranged as

$$
\sigma(t|N,b_r) = \frac{\int_{t}^{\hat{t}} \left[1 - F_t(i)\right]^{n-2} Q(\sigma(i|\cdot)|N,b_r,\sigma) \, di}{\left[1 - F_t(t)\right]^{n-1} Q(\sigma(t|\cdot)|N,b_r,\sigma)}
$$

(10)

Notice that the first term on the right-hand side is the expected cost of the lowest rival SC in the $i$th upstream auction conditional on the SC’s winning in the upstream auction and $i$th PC’s winning in the downstream auction. The second term is the conditional expected cost of the $i$th lowest rival PCs.

Second, the $\sigma$ is a function of the number of PCs and the reservation price in the downstream auction. The fact that these environmental variables affect the SC’s strategy $\sigma$ implies that both up- and downstream auctions are linked through the SCs’ strategic response. To see this link more formally, it is convenient to use the following three lemmas. The first one is to show that $Q(\cdot)$ has an increasing hazard rate (IHR).

**Lemma 1.** $Q(s)$ has IHR, i.e., $-\frac{Q'(s|\cdot)}{Q(\sigma)}$ is increasing in $s$.

**Proof.** Let $F_\sigma(\cdot)$ be the cumulative distribution of $\sigma(\cdot)$. Then, for any $s \equiv \sigma(t|\cdot)$, we obtain $F_\sigma(s) = F_t(\sigma^{-1}(s|\cdot))$ and

$$
\frac{f_\sigma(s|\cdot)}{1 - F_\sigma(s|\cdot)} = \frac{f_t(\sigma^{-1}(s|\cdot))}{1 - F_t(\sigma^{-1}(s|\cdot))} \frac{1}{\sigma'(\sigma^{-1}(s|\cdot))}.
$$

(10)
On the other hand, (7) can be rearranged as
\[
\frac{1}{s - \sigma^{-1}(s|\cdot)} = -\frac{Q'(s|\cdot)}{Q(s|\cdot)} + (n - 1) \frac{f_\theta(\sigma^{-1}(s|\cdot))}{1 - F_\theta(\sigma^{-1}(s|\cdot))} \frac{1}{\sigma'(\sigma^{-1}(s|\cdot))}.
\]
Substitute (10) into it, and we obtain
\[
\frac{1}{s - \sigma^{-1}(s|\cdot)} = -\frac{Q'(s|\cdot)}{Q(s|\cdot)} + (n - 1) \frac{f_\sigma(s|\cdot)}{1 - F_\sigma(s|\cdot)}.
\]
Suppose by contradiction that \(-\frac{Q'(s|\cdot)}{Q(s|\cdot)}\) is decreasing in \(s\). Since \(s - \sigma^{-1}(s|\cdot)\) is decreasing in \(s\), \(\frac{f_\sigma(s|\cdot)}{1 - F_\sigma(s|\cdot)}\) is increasing in \(s\) by (11). On the other hand, the log-concavity of \(f_\theta\) implies the IHR of \(F_\theta\). Since the convolution of two random variables with IHR is also IHR (Barlow and Proschan (1975)[1, Sect 4.4]), the distribution of the random variable \(c \equiv \theta + \sigma(t|\cdot)\) satisfies IHR. Now, let \(c_{1:N-1}\) denote the lowest order statistics among \(N - 1\) iid valuation samples of \(c\). Then, the hazard rate of the random variable \(c_{1:N-1}\) is given by
\[
\frac{(N - 1)f_c(c_{1:N-1})[1 - F_c(c_{1:N-1})]^{N-2}}{[1 - F_c(c_{1:N-1})]^{N-1}} = \frac{(N - 1)f_c(c_{1:N-1})}{[1 - F_c(c_{1:N-1})]},
\]
which is increasing in \(c_{1:N-1}\) if and only if \(F_c\) is IHR. Thus, the cumulative distribution of \(c_{1:N-1}\), i.e. \(F_{c_{1:N-1}}\), also satisfies IHR. Finally, define \(\delta \equiv c_{1:N-1} - \theta_i\). Since \(F_{c_{1:N-1}}\) and \(F_\theta\) are IHR, the cumulative distribution of \(\delta\), the convolution of these two random variables, must also satisfy IHR. Note that \(Q(s|\cdot)\) is the probability that the random variable \(\delta\) is greater than \(s\). Hence, denoting by \(F_\delta\) the cumulative distribution function of \(\delta\), we can rewrite \(Q(s|\cdot)\) as \(1 - F_\delta(s|\cdot)\) and \(Q'(s|\cdot)\) as \(-f_\delta(s|\cdot)\). Since \(F_\delta\) has IHR,
\[
\frac{-f_\delta(s|\cdot)}{1 - F_\delta(s|\cdot)} = \frac{Q'(s|\cdot)}{Q(s|\cdot)}
\]
is decreasing. We have reached a contradiction. Hence, \(Q(s)\) has IHR.

The next lemma ensures that the hazard function of \(Q\) is increasing in \(N\) and decreasing in the reservation price \(b_r\).

**Lemma 2.** Suppose that \(\sigma\) is independent of \(N\). Then, \(-\frac{Q'(s|N,b_r,\sigma^{-1})}{Q(s|N,b_r,\sigma^{-1})}\) is strictly increasing in \(N\) and weakly decreasing in \(b_r\).

**Proof.** See the Appendix for proof.

The next two lemmas ensure that the SC’s two bidding functions with different numbers of bidders or different reservation prices never cross each other for any \(t < \ell\).
Lemma 3. There exists no \( t < \tilde{t} \) such that \( \sigma(t|\tilde{m},\cdot) = \sigma(t|m,\cdot) \) for any \( \tilde{m} > m \geq 1 \).

Proof. Suppose there exists \( t \) such that \( \sigma(t|\tilde{m},\cdot) = \sigma(t|m,\cdot) = \xi(t) \). Then, by Equation (9),

\[
\int_t^{\tilde{t}} [1 - F_t(\tilde{t})]^{n-1} \frac{Q(\xi(\tilde{t})|\tilde{m}, b_r, \sigma)}{Q(\xi(\tilde{t})|m, b_r, \sigma)} \, d\tilde{t} = \int_t^{\tilde{t}} [1 - F_t(\tilde{t})]^{n-1} \frac{Q(\xi(\tilde{t})|m, b_r, \sigma)}{Q(\xi(\tilde{t})|m, b_r, \sigma)} \, d\tilde{t}.
\]

By Lemma 2, \(- \frac{Q'(s|m,\cdot)}{Q(s|m,\cdot)}\) is increasing in \( m \). Thus, we obtain

\[
0 > \frac{Q(s|\tilde{m},\cdot)}{Q(s|m,\cdot)} \left\{ \frac{Q'(s|\tilde{m},\cdot)}{Q(s|\tilde{m},\cdot)} - \frac{Q'(s|m,\cdot)}{Q(s|m,\cdot)} \right\} = \frac{Q'(s|\tilde{m},\cdot)Q(s|m,\cdot) - Q'(s|m,\cdot)Q(s|\tilde{m},\cdot)}{|Q(s|m,\cdot)|^2} = \frac{d}{ds} \left( \frac{Q(s|\tilde{m},\cdot)}{Q(s|m,\cdot)} \right)
\]

for any \( \tilde{m} > m \geq 1 \). This implies that for any \( \tilde{s} > s \),

\[
\frac{Q(\tilde{s}|\tilde{m},\cdot)}{Q(\tilde{s}|m,\cdot)} < \frac{Q(s|\tilde{m},\cdot)}{Q(s|m,\cdot)} \iff \frac{Q(\tilde{s}|\tilde{m},\cdot)}{Q(s|\tilde{m},\cdot)} < \frac{Q(\tilde{s}|m,\cdot)}{Q(s|m,\cdot)}.
\]

Replacing \( \tilde{s} = \xi(\tilde{t}) \) and \( \xi = \sigma(t) \) and multiplying by \([1 - F_t(\tilde{t})]^{n-1}\) on both sides will yield

\[
[1 - F_t(\tilde{t})]^{n-1} \frac{Q(\xi(\tilde{t})|\tilde{m},\cdot)}{Q(\xi(\tilde{t})|m,\cdot)} < [1 - F_t(\tilde{t})]^{n-1} \frac{Q(\xi(\tilde{t})|m,\cdot)}{Q(\xi(\tilde{t})|m,\cdot)}
\]

\[
\iff \int_t^{\tilde{t}} [1 - F_t(\tilde{t})]^{n-1} \frac{Q(\xi(\tilde{t})|\tilde{m},\cdot)}{Q(\xi(\tilde{t})|m,\cdot)} \, d\tilde{t} < \int_t^{\tilde{t}} [1 - F_t(\tilde{t})]^{n-1} \frac{Q(\xi(\tilde{t})|m,\cdot)}{Q(\xi(\tilde{t})|m,\cdot)} \, d\tilde{t}.
\]

Thus, we obtain a contradiction. \( \square \)

Lemma 4. There exists no \( t < \tilde{t} \) such that \( \sigma(t|\tilde{b}_r,\cdot) = \sigma(t|b_r,\cdot) \) for any \( b_r < \tilde{b}_r \).

Proof. Suppose there exists \( t \) such that \( \sigma(t|m,\tilde{b}_r,\cdot) = \sigma(t|m,b_r,\cdot) = \xi(t) \). Then, by Equation (9),

\[
\int_t^{\tilde{t}} [1 - F_t(\tilde{t})]^{n-1} \frac{Q(\xi(\tilde{t})|m, \tilde{b}_r, \sigma)}{Q(\xi(\tilde{t})|m, b_r, \sigma)} \, d\tilde{t} = \int_t^{\tilde{t}} [1 - F_t(\tilde{t})]^{n-1} \frac{Q(\xi(\tilde{t})|m, b_r, \sigma)}{Q(\xi(\tilde{t})|m, b_r, \sigma)} \, d\tilde{t}.
\]

Lemma 2 indicates that \(- \frac{Q'(s|b_r,\cdot)}{Q(s|b_r,\cdot)}\) is decreasing in \( b_r \). Thus, we obtain

\[
0 < \frac{Q(s|\tilde{b}_r,\cdot)}{Q(s|b_r,\cdot)} \left\{ \frac{Q'(s|\tilde{b}_r,\cdot)}{Q(s|\tilde{b}_r,\cdot)} - \frac{Q'(s|b_r,\cdot)}{Q(s|b_r,\cdot)} \right\} = \frac{d}{ds} \left( \frac{Q(s|\tilde{b}_r,\cdot)}{Q(s|b_r,\cdot)} \right).
\]
for any $b_r < \hat{b}_r$, which implies that for any $\hat{s} > s$,

$$\frac{Q(s|\hat{b}_r, \cdot)}{Q(s|b_r, \cdot)} < \frac{Q(\hat{s}|\hat{b}_r, \cdot)}{Q(\hat{s}|b_r, \cdot)} \iff \frac{Q(s|\hat{b}_r, \cdot)}{Q(s|b_r, \cdot)} < \frac{Q(\hat{s}|\hat{b}_r, \cdot)}{Q(\hat{s}|b_r, \cdot)}.$$ 

Replacing $\hat{s} = \xi(\hat{t})$ and $\xi = \sigma(t)$ and multiplying by $[1 - F_t(\hat{t})]^{n-1}$ on both sides will yield

$$[1 - F_t(\hat{t})]^{n-1} \frac{Q(\xi(\hat{t})|b_r, \cdot)}{Q(\xi(\hat{t})|b_r, \cdot)} < [1 - F_t(\hat{t})]^{n-1} \frac{Q(\xi(\hat{t})|\hat{b}_r, \cdot)}{Q(\xi(\hat{t})|b_r, \cdot)} \iff \int_t^\hat{t} [1 - F_t(\hat{t})]^{n-1} \frac{Q(\xi(\hat{t})|b_r, \cdot)}{Q(\xi(\hat{t})|b_r, \cdot)} d\hat{t} < \int_t^\hat{t} [1 - F_t(\hat{t})]^{n-1} \frac{Q(\xi(\hat{t})|\hat{b}_r, \cdot)}{Q(\xi(\hat{t})|b_r, \cdot)} d\hat{t}.$$ 

Thus, we have reached a contradiction. \hfill \Box

Now, whether the SC’s strategy is affected by the downstream competition is examined. The next proposition characterizes the SC’s equilibrium strategy $\sigma$.

**Proposition 2.** Subcontractors bid more aggressively in upstream auctions as the number of PCs rises or the reservation price drops in the downstream auction.

**Proof.** Since $\sigma(t|m)$ and $\sigma(t|m)$ never cross each other at $t < \hat{t}$ for any $\hat{m} > m \geq 1$, $\sigma(t|m)$ must be monotone in $m$, the number of bidders, even if $m$ is a real number instead of an integer. Suppose by contradiction that $\sigma(t|m)$ is increasing in $m \geq 1$. Then, replacing $N$ with a real number $m$ in (8) yields

$$\frac{1}{\sigma(t|m, b_r) - t} - (n - 1) = \frac{f_t(t)}{1 - F_t(t)} \frac{1}{\sigma'(t|m, b_r)} = \frac{Q'(\sigma(t|m, b_r)|m, b_r, \sigma)}{Q(\sigma(t|m, b_r)|m, b_r, \sigma)}.$$  

The right-hand side is positive if $m > 1$ and vanishes if $m = 1$. Therefore, if the number of bidders increases to $\hat{m} > 1$, then $\sigma'(\hat{t}|\hat{m}, \cdot) > \sigma'(t|m, \cdot)$ must hold for all $t \in [t, \hat{t}]$. Thus, $\int_t^\hat{t} \sigma'(\hat{t}|\hat{m}, \cdot) d\hat{t} = \hat{t} - \sigma(t|\hat{m}, \cdot) > \hat{t} - \sigma(t|m, \cdot) = \int_t^\hat{t} \sigma'(\hat{t}|\hat{m}, \cdot) d\hat{t}$, implying $\sigma(t|\hat{m}, \cdot) < \sigma(t|m, \cdot)$. A contradiction is reached.

Similarly, $\sigma(t|N, b_r)$ is strictly increasing in $b_r$. Let $\hat{b}_r \leq t + \theta < \hat{t} + \theta \leq b_r$. Suppose there exists $t < \hat{t}$ such that $\sigma(t|N, b_r) < \sigma(t|\hat{N}, b_r)$. Since an SC has no chance to get a job in the case of $\hat{b}_r$, his strategy is $\sigma(t|\hat{b}_r, \cdot) = t$ for all $t$, whereas in the case of $b_r$, an SC seeks a positive bid margin when the cost is strictly smaller than $\hat{t}$, i.e., $\sigma(t|\cdot,\hat{b}_r, \cdot) > t$ for any $t < \hat{t}$. A contradiction is reached. Thus, $\sigma(t|N, b_r)$ is strictly increasing in $b_r$. \hfill \Box

This result also implies that PCs have stronger bargaining power against SCs as the number of PCs increases or the reservation price declines in the downstream auction. Furthermore, SCs are squeezed only if the first-price mechanism is used in upstream auctions;
truth-telling is a dominant strategy for SCs in the upstream competitions if the second-price mechanism is used. In the next section we investigate which mechanism is beneficial for PCs and society.

4 The downstream auction

Finally we examine the downstream market given the symmetric equilibrium in the upstream market. The SC’s aggressive bids in upstream auctions affect the PC’s acquiring information and strategy in the downstream auction. In short, we obtain the following proposition.

Proposition 3. If first-price auctions are used in upstream competitions, then the distribution of the bidders’ private information in the downstream competition is provided endogenously by the number of bidders and reservation price, namely \( F_c(c_i|N, b_r) \). Furthermore, the distribution shifts in the sense of first-order stochastic dominance i.e., for all \( c_i \), \( F_c(c_i|N+1, b_r) < F_c(c_i|N, b_r) \) with any positive integer \( N \), and \( F_c(c_i|N, b_r) < F_c(c_i|N, \tilde{b}_r) \) for any \( b_r < \tilde{b}_r \).

Proof. Let \( F_\sigma \) be the cumulative distribution of \( \sigma(t|\cdot) \). Then, we obtain

\[
[1 - F_c(c|N, b_r)]^N \equiv [1 - F_\sigma(c|N, b_r)]^{nN} = [1 - F_t(\sigma^{-1}(c|N, b_r))]^{nN}.
\]

For any \( t \), \( \sigma(t|\cdot) \) rises as \( N \) decreases or \( b_r \) increases. Equivalently, for any \( s \), \( \sigma^{-1}(s|N, b_r) \) declines as \( N \) decreases or \( b_r \) increases. Hence, \( F_t(\sigma^{-1}(c|N+1, b_r)) \) is first-order stochastically dominated by \( F_t(\sigma^{-1}(c|N, b_r)) \); similarly, \( F_t(\sigma^{-1}(c|N, \tilde{b}_r)) \) is first-order stochastically dominated by \( F_t(\sigma^{-1}(c|N, b_r)) \) for any \( b_r < \tilde{b}_r \).

It is clear that the expected lowest subcontract price for each upstream auction is lower if the first-price auction is used. In this sense, the Revenue Equivalence Theorem never holds in upstream auctions.

Proposition 4. The expected lowest subcontract bid accepted by each PC is greater if the second-price auction is used in upstream competitions.

Proof. First, the expected subcontract price \( E(c_i) \) is equal to the expected second lowest cost \( E(t_i^{(2)}) \) if a second-price mechanism is used in the upstream auction. On the other hand, if a first-price mechanism is used in the upstream auction, the expected subcontract price equals the expected lowest bid \( \sigma(t^{(1)}) \), where \( t^{(m)} \) is the \( m \)th lowest order statistics among
n samples of $t$. Let $N = 1$. By equation (9), $E[\sigma(t|1, b_r)] = E(t^{(2)}|b_r)$. If $N \geq 2$, then $E[\sigma(t|1, b_r)] < E(t^{(2)}|b_r)$ by Proposition 2.

Hansen also suggests that the endogeneity of quantity reduces bids under the first-price rule, whereas the optimal strategy is unchanged in the second-price rule. Similar arguments hold in upstream subcontract auctions.

Would a PC prefer to use the second-price rule in the upstream? Obviously, the first-price auction induces the lower subcontract price on average if $n = 1$ since SCs bid aggressively even if they have no competitor in upstream auctions whereas the contract price equals the reservation price when the second-price auction is used.

The analysis becomes much more complicated if there are two or more SCs in an upstream auction. To illustrate, we focus on the case where $\theta$ is constant and no reservation price is used, i.e., $b_r = \theta + \bar{t}$. Then, the following proposition is obtained.

**Proposition 5.** Suppose the SCs’ bidding function $\sigma$ is non-concave. Then, PCs are better off using the first-price mechanism in upstream auctions.

**Proof.** See the Appendix for proof.

The non-concave function includes the linear bidding strategy observed if the random variable $t$ follows the uniform distribution. Thus, the sufficient condition is applicable to many situations. We propose the following corollary to the efficiency argument.

**Corollary 1.** The second-price mechanism in upstream auctions likely leads to an inefficient allocation, i.e., the SC who is not lowest is picked more frequently.

If the first-price auction is used, the subcontract payment described in the pre-award subcontract agreement is likely to be too high for the PC to be cost-disadvantaged in the downstream auction. This is true for the case where the SC is the most efficient supplier among all the SCs, including those of other PCs. That is, the second-price auction is likely to induce ex post inefficient allocation in the subcontract.

The endogenously determined distribution of the bidders’ private information affects the procurement buyer’s mechanism design problem. To conclude this section, we turn to a detailed analysis of the optimal reservation price, i.e., the cost-minimizing reservation price. First, we examine the following lemma regarding order statistics.

**Lemma 5.** Suppose $x$ is an i.i.d. random variable following the density function $f(x|\cdot)$ and the cumulative distribution function $F(x|\cdot)$. Let $x^{(2)}$ be the second-lowest order statistics
among the $N$ samples of $x$ with a cumulative distribution function $F^{(2)}(x^{(2)})$. Suppose that $F(x|\bar{\mu})$ first-order stochastically dominates $F(x|\mu)$, namely $F(x|\bar{\mu}) < F(x|\mu)$ for any $\mu < \bar{\mu}$. Then, $F^{(2)}(\cdot|\bar{\mu}) < F^{(2)}(\cdot|\mu)$ holds, or equivalently, $E(x^{(2)}|\mu) < E(x^{(2)}|\bar{\mu})$ holds for any $x^{(2)}$.

**Proof.** The cumulative distribution function of $x^{(2)}$ is given by $F^{(2)}(x|\mu) = (N - 1)[1 - F(x|\mu)]^N - N[1 - F(x|\mu)]^{N-1}$, and its density is $f^{(2)}(x|\mu) = N(N - 1)f(x|\mu)[1 - F(c|\mu)]^{N-2}F(x|\mu)dx$. Taking derivative with respect to $\mu$, we obtain

$$\frac{\partial F^{(2)}(x|\mu)}{\partial \mu} = \left(\frac{\partial F(x|\mu)}{\partial \mu}\right) N(N - 1)F(x|\mu)[1 - F(x|\mu)]^{N-2} < 0$$

Since this holds for any $x$, $F^{(2)}(x|\bar{\mu})$ first-order stochastically dominates $F(x|\mu)$ for any $\mu < \bar{\mu}$.

**Proposition 6.** The optimal reservation price in the downstream auction is decreasing in the number of bidders.

**Proof.** The expected return to the procurement buyer is given by

$$V \left[1 - (1 - F_c(b_r|N,b_r))^N\right] - N \int_{c}^{b_r} \left[\dot{c}f_c(\dot{c}|N,b_r) + F_c(\dot{c}|N,b_r)\right][1 - F_c(\dot{c}|N,b_r)]^{N-1}d\dot{c}$$

Taking the derivative with respect to $b_r$ gives the optimal reservation price $b^*_r$:

$$N \left[V f_c(b^*_r|N,b^*_r) - b^*_r f_c(b^*_r|N,b^*_r) - F_c(b^*_r|N,b^*_r)[1 - F_c(b^*_r|N,b^*_r)]^{N-1}\right]$$

$$+ \frac{\partial}{\partial b_r} VN [1 - F_c(\cdot|N,b_r)]^{N-1}$$

$$- \frac{\partial}{\partial \mu} N \int_{c}^{b_r} [\dot{c}f_c(\dot{c}|N,\mu) + F_c(\dot{c}|N,\mu)][1 - F_c(\dot{c}|N,\mu)]^{N-1}d\dot{c}\bigg|_{\mu=b_r} = 0 \quad (13)$$

The last term is equal to $\frac{\partial}{\partial b_r} E(c^{(2)}|N,b_r)$, which is positive from Lemma 5. Let $-\Phi = \frac{\partial}{\partial b_r} VN [1 - F_c(\cdot|N,b_r)]^{N-1} - \frac{\partial}{\partial b_r} E(c^{(2)}|N,b_r) \geq 0$. Then, (13) becomes

$$b^*_r = V - \frac{F_c(b^*_r|N,b^*_r)}{f_c(b^*_r|N,b^*_r)} - \frac{\Phi}{N [1 - F_c(b^*_r|N,b^*_r)]^{N-1}}.$$ 

The optimal reservation price is in general a function of the number of bidders.
5 Empirics

Now we examine whether the theoretical prediction is in line with the empirical data. The data contain the bid results of the procurement auctions for civil engineering projects from April 2005 through March 2008 held by the Ministry of Land, Infrastructure and Transportation (MLIT) in Japan. The number of contracts awarded was 11,114 during this period and 87,160 bids are observed.

To control the auction-specific heterogeneity such as the project size, a regression is undertaken in which the observed bid is on the left-hand side and the engineer’s estimated costs as well as other auction-specific variables such as auction date and the auction format are on the right hand-side. The equation (14) shows the regression.

\[
\text{Percentage } \text{Bid}_t = \alpha + \beta x_{t},
\]

where \( x_{t} \) includes the engineer’s estimated cost, the auction date, the auction format dummy and the scoring auction dummy.\(^8\)

<table>
<thead>
<tr>
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<tr>
<td>Engineer’s estimated cost</td>
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</tr>
<tr>
<td>Observations</td>
</tr>
<tr>
<td>R-squared</td>
</tr>
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</table>

Table 1: Regression result for estimated costs on the number of bidders

The residuals created on this regression can be considered normalized bids assumed to be independently distributed. Then, the nonparametric estimation methodology proposed by Guerre et al. (2000) yields bidders’ costs from the normalized bid.

\(^8\)In the scoring auction, bidders submit not only the price bid but also some other factors such as the completion time or the level of noise arising from construction work. The winner is determined based on the scoring bid which is typically computed as the weighted average of the price bid and the factor bids.
Figure 1 shows the scattered plot for the normalized bids and the estimated costs versus the number of observed bids in each auction.

Since the government procurement auctions in the data disclose the number of bidders after the auction is over, the number of competitors is unknown for each bidder. Such a case would be the most plausible reason that the negative relationship never holds if the number of bidders is much smaller than the average: 11.6. If bidders consider that the number of rivals is smaller than the actual number of bidders, then the estimated latent variables are biased positively, whereas the costs are likely to be overestimated if bidders believe that the number of participants is greater than the actual number of bidders. Therefore, if it is true that the former situation occurs more frequently when the number of actual bidders is more than 11.6 and the latter occurs more frequently if the number is less than 11.6, then the true value of the PC’s costs should be lower than the estimates if the number of bidders is relatively low in the sample, but the true value should be higher if the number is relatively high.

Taking this information into account, it is still statistically significant that the number
of participants in the auction negatively affects the bidder’s cost on average. Table 2 shows the result of regression analysis of estimated costs based on the number of bidders. In the samples with relatively large numbers of bidders, namely, more than 11, the PCs’ costs on average are statistically likely to be low as the number of bidders decreases.

<table>
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<td>(0.76)</td>
<td>(1.17)</td>
<td>(3.22)**</td>
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<td>(24.00)**</td>
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<td>0.76</td>
<td>0.62</td>
<td>1.37</td>
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<td>(2.06)**</td>
<td></td>
<td>(0.96)</td>
<td>(0.98)</td>
<td>(3.57)**</td>
<td>(2.69)**</td>
<td>(5.36)**</td>
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<td>≥ 9</td>
<td>≥ 11</td>
<td>≥ 13</td>
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<td>≥ 17</td>
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</table>

Table 2: Regression result for estimated costs on the number of bidders

6 Discussion

The model discussed in this paper focuses on the symmetric case where the number of SCs solicited in the \( i \)th upstream auction is identical for all \( i = 1, \ldots, N \). It is easily shown that the obtained results are unchanged even in cases in which some PCs solicit more subcontract bids than the others. Since it is natural to think that each PC does not know the number of subcontract bids solicited by other PCs, the procurement auction still satisfies the symmetric assumption such that each bidder is \textit{ex ante} the same.

Subcontractors typically bid to a specific PC. The major reason for the practice comes from past difficulties in working with other PCs, which also makes PCs reject low bids from some SCs. Working with SCs who are considered to be insufficiently qualified results in higher risk for PCs. Therefore, SCs bid their favorite PCs first and, if time permits, get through to the rest (Dyer and Kagel (1996)).
What if SCs bid to multiple PCs? Our results are robust in such a case as long as there is at least one SC who submits a bid only to a particular PC. Consider first that there exists a set of SCs who bid only once to particular PCs. Since these SCs bid aggressively to help the PC accept their bids, the remaining SCs who bid to all PCs still have to bid aggressively.

Only in the case where all SCs bid for all PCs will SCs not bid aggressively in accordance with the increase in the number of PCs in the downstream auctions. Recall, however, that \( \frac{\partial \sigma}{\partial b} < 0 \) still holds, regardless of \( N \). It follows that SCs still bid more aggressively if the downstream auction has a lower reservation price.

Finally, the theory applies to the empirical analysis of procurement auctions. For instance, Guerre et al. (2000), the seminal paper of structural analysis of first-price auctions, identifies the bidders’ distribution of private information, assuming that it is independent of the number of bidders. However, if it shifts downward as the number of bidders increases, the cost estimates obtained from the non-parametric estimation must be pooled separately according to the number of bidders to estimate the distribution.

7 Conclusion

Most of the auction literature implicitly assumes that the players are agents who send a message directly to the principal (auctioneer). In reality, and especially in procurement auctions, goods and services are produced by a team of firms (main and subfirms). The lower tiered subfirms and suppliers can be non-negligible players who also possess private information in the Bayesian game.

Taking into account the lower tiered producers and suppliers, it may be obvious that the intensity of competition in the downstream auction affects not only the primary contractor’s profit but also the subcontractors’ profits. The aggressive bidding of the subcontractors in the upstream auction helps their PC win in the downstream auctions.

The main contribution of this paper lies in formulating an auction model including the vertically related production system that can be seen in most of industries. Our theory suggests that an additional entrant to the downstream auction results in the primary contractor’s stronger bargaining power against the subcontractors. In other words, prime contractors have the cost distribution endogenously determined by the number of bidders and the reservation price in the downstream auction.

The application of the theory is widespread from joint bidding or bid consortium. Even if member firms have a close and trusting relationship with each other, each member owns
private information, creating the possibility for each member firm to obtain rent against the representative firm of the consortium.

Throughout, we rule out the ex post negotiation between a prime and subcontractors. Even if a pre-bid price quote is received, the prime contractors is still able to negotiate a lower price from the subcontractors. The prime contractor may solicit companies to perform irrelevant categories of work or in an irrelevant geographic area. However, as noted in the Availability and Disparity Study by Caltrans in 2007, good-faith efforts are critical to achieve successful subcontracting, which obviously leads to more efficient production and greater profits for the winning prime contractors. Hence, our model does not consider the ex post negotiation between sub- and prime contractors.

The incentive for subcontracting is not only to reduce costs. For instance, Marechal and Morand (2003) point out that subcontracting can reduce the risk of potential change orders. Given the sheer volume of procurement, it is clear that more serious research and evaluation are needed to investigate the effect of subcontracting.

Appendix

An alternative way to obtain $1 - F_c(\cdot)$

Since the range of $\sigma$ is $[\sigma(t), \sigma(\hat{t})]$, there exists $\sigma^{-1}(s + \theta_i - \theta)$ if and only if $\sigma(t) \leq s + \theta_i - \theta \leq \sigma(\hat{t})$ for some $s$. This implies that his PC beats her rival in the procurement auction with the probability $[1 - F_{\hat{t}}(\sigma^{-1}(s + \theta_i - \theta))]^n$ if $\theta \in [s + \theta_i - \sigma(\hat{t}), s + \theta_i - \sigma(t)]$ for some $s$. On the other hand, if $s + \theta_i - \sigma(c) \leq \theta$ for some $s$ and $t_i$, his PC wins, which happens with a probability equal to $\int_{s + \theta_i - \sigma(\hat{t})}^{\theta} f_\theta(\theta)d\theta = 1 - F_\theta[s + \theta_i - \sigma(t)].$ Finally, if $s + \theta_i - \sigma(\hat{t}) \geq \theta$, his PC will lose.

Let us define $\sigma(t_{1:n}) = s + \theta_i - \theta$. Then we have $\int_{s + \theta_i - \sigma(\hat{t})}^{\sigma(\hat{t})} f_\theta(\theta)d\theta = \int_{\sigma(\hat{t})}^{\sigma(t_{1:n})} f_\theta(s + \theta_i - \sigma(t_{1:n}))d\sigma(t_{1:n}) = \int_{\sigma(t_{1:n})}^{\sigma(\hat{t})} f_\theta(s + \theta_i - \sigma(t_{1:n}))d\sigma(t_{1:n})$. Therefore, we obtains

$$1 - F_c(s + \theta_i) = 1 - F_\theta[s + \theta_i - \sigma(t)] + \int_{\sigma(t_{1:n})}^{\sigma(\hat{t})} [1 - F_{t(t_{1:n})}]^n f_\theta(s + \theta_i - \sigma(t_{1:n}))d\sigma(t_{1:n}).$$

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9The effect of such ex post changes on procurement contracts is thoroughly analyzed in Bajari and Tadelis (2001).
From the integral by parts,

\[ 1 - F_c(s + \theta_i) = 1 - \mathcal{F}_\theta[s + \theta_i - \sigma(t)] - \left[(1 - F_\theta(t_{1:n}))^{n} F_\theta(s + \theta_i - \sigma(t_{1:n}))\right]_t^t 
- \int_t^t n f_t [1 - F_\theta(t_{1:n})]^{n-1} F_\theta(s + \theta_i - \sigma(t_{1:n})) dt_{1:n} \\
= \int_t^t n f_t [1 - F_\theta(t_{1:n})]^{n-1} \left[1 - F_\theta(s + \theta_i - \sigma(t_{1:n}))\right] dt_{1:n}, \]

where the last equality holds from the fact that \( \int_t^t n f_t [1 - F_\theta(t_{1:n})]^{n-1} dt_{1:n} = 1 \).

**Proof that** \(-\frac{Q'(s|N,b_r,\sigma)}{Q(s|N,b_r,\sigma)}\) **is strictly increasing in** \(N\)

*Proof.* Define \( s \equiv \sigma(t|b_r) \). The derivative of \( Q(s|N,b_r,\sigma) \) with respect to \( s \) is given by

\[ Q'(s|N,b_r,\sigma) = - \int_\theta^{b_r-s} [1 - F_c(s + \theta_i)]^{N-1} f_\theta(\theta_i) d\theta_i. \]

Together with (5), we obtain

\[ -\frac{Q'(s|N,b_r,\sigma)}{Q(s|N,b_r,\sigma)} = \frac{\int_\theta^{b_r-s} [1 - F_c(s + \theta_i)]^{N-1} f_\theta(\theta_i) d\theta_i}{\int_\theta^{b_r-s} [1 - F_c(s + \theta_i)]^{N-1} f_\theta(\theta_i) d\theta_i} \tag{A-1} \]

Let \( k_0 \) be a positive real number such that \( \frac{f'_\theta(\theta)}{f_\theta(\theta)} = k_0 \). Since \( \frac{f'_\theta(\theta)}{f_\theta(\theta)} \) is non-increasing in \( \theta \), there exists a non-negative and non-increasing function \( k(\theta) \) such that for all \( \theta \in [\theta, b_r - s] \)

\[ \frac{f'_\theta(\theta)}{f_\theta(\theta)} - k_0 = k(\theta), \]

with \( k(b_r - s) = 0 \). Thus, we have \( f'_\theta(\theta) = [k_0 + k(\theta)] f_\theta(\theta) \). Substituting it into (A-1) gives

\[ -\frac{Q'(s|N,b_r,\sigma)}{Q(s|N,b_r,\sigma)} = k_0 + \left[ \frac{\int_\theta^{b_r-s} [1 - F_c(s + \theta_i)]^{N-1} k(\theta_i) f_\theta(\theta_i) d\theta_i}{\int_\theta^{b_r-s} [1 - F_c(s + \theta_i)]^{N-1} f_\theta(\theta_i) d\theta_i} \right] \\
= k_0 + \int_\theta^{b_r-s} g(\theta_i|N) k(\theta_i) d\theta_i, \tag{A-2} \]
where \( g(\theta|N, b_r) \equiv [1 - F_c(s + \theta)]^{N-1}f_\theta(\theta)/\int_{\theta}^{b_r-s}[1 - F_c(s + \theta)]^{N-1}f_\theta(\theta)d\theta \). Define \( G(\theta|N, b_r) = \int_{\theta}^{\theta^*} g(\theta|\cdot)d\theta \). Then, for any \( N > 1 \) we obtain

\[
G(\theta|N + 1) - G(\theta|N) = \frac{1}{\Delta} \left\{ \int_{\theta}^{\theta^*} [1 - F_c(s + \hat{\theta})]^N f_\hat{\theta}(\hat{\theta})d\hat{\theta} \int_{\theta}^{b_r-s}[1 - F_c(s + \hat{\theta})]^{N-1}f_\theta(\hat{\theta})d\hat{\theta} 
- \int_{\theta}^{\theta^*} [1 - F_c(s + \hat{\theta})]^{N-1}f_\theta(\hat{\theta})d\hat{\theta} \int_{\theta}^{b_r-s}[1 - F_c(s + \hat{\theta})]^N f_\theta(\hat{\theta})d\hat{\theta} \right\}
\]

where \( \Delta \) is a positive number. By the Mean Value Theorem, we obtain

\[
= \frac{1}{\Delta} \left\{ \left[ (1 - F_c(\theta^-)) - (1 - F_c(\theta^+)) \right] \right.
\times \left. \int_{\theta}^{\theta^*} [1 - F_c(s + \hat{\theta})]^{N-1}f_\theta(\hat{\theta})d\hat{\theta} \int_{\theta}^{b_r-s}[1 - F_c(s + \hat{\theta})]^{N-1}f_\theta(\hat{\theta})d\hat{\theta} \right\},
\]

where \( \theta^- \in [\theta, \bar{\theta}] \) and \( \theta^+ \in [\theta, b_r - s] \). Since \( F_c \) is strictly increasing, the whole terms are strictly positive. Hence, given an increasing function \( \sigma(t|b_r) \) independent of \( N \), \( G(\theta|N + 1) \) is first-order stochastically dominated by \( G(\theta|N) \). Since \( k(\theta_i) \) is non-increasing, \( -Q'(s|N, b_r, \sigma)/Q(s|N, b_r, \sigma) \) is strictly increasing in \( N \). Hence, for any \( t \) and \( b_r, -Q'(\sigma(t|b_r)|N, b_r, \sigma)/Q(\sigma(t|b_r)|N, b_r, \sigma) \) is strictly increasing in \( N \).

Proof that \( -Q'(s|N, b_r, \sigma)/Q(s|N, b_r, \sigma) \) is weakly decreasing in \( b_r \)

Define \( s \equiv \sigma(t|b_r) \). Let \( b_r < \bar{b}_r \). From Equation (A-2), we obtain

\[
G(\theta|N, \bar{b}_r) - G(\theta|N, b_r)
= \frac{1}{\Delta} \left\{ \left[ (1 - F_c(\theta^-)) - (1 - F_c(\theta^+)) \right] \right.
\times \left. \int_{\theta}^{\theta^*} [1 - F_c(s + \hat{\theta})]^{N-1}f_\theta(\hat{\theta})d\hat{\theta} \int_{\theta}^{b_r-s}[1 - F_c(s + \hat{\theta})]^{N-1}f_\theta(\hat{\theta})d\hat{\theta} \right\}
\]

\[
\leq 0.
\]

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This result implies that $G(\theta | N, \tilde{b}_r)$ first-order stochastically dominates $G(\theta | N, b_r)$ and that equality holds if and only if $F_i(b_r) = 1$. Hence, from Equation (A-2), we obtain

$$\frac{Q'(s | N, \tilde{b}_r)}{Q(s | N, \tilde{b}_r)} - \left( - \frac{Q'(s | N, b_r)}{Q(s | N, b_r)} \right)$$

$$= \int_{\theta}^{\hat{b}_r - s} g(\theta_i | N, \tilde{b}_r) k(\theta_i) d\theta_i - \int_{\theta}^{\hat{b}_r - s} g(\theta_i | N, b_r) k(\theta_i) d\theta_i$$

$$= \int_{\theta}^{\hat{b}_r - s} [g(\theta_i | N, \tilde{b}_r) - g(\theta_i | N, b_r)] k(\theta_i) d\theta_i + \int_{\hat{b}_r - s}^{\hat{b}_r - s} g(\theta_i | N, \tilde{b}_r) k(\theta_i) d\theta_i$$

$$\leq 0,$$

where $\theta^- < \theta^+$. The third equality is obtained by the Mean Value Theorem. Furthermore, $\int_{\theta}^{\hat{b}_r - s} g(\cdot | N, \tilde{b}_r) = \int_{\theta}^{\hat{b}_r - s} g(\cdot | N, b_r) = 1$, or equivalently

$$\int_{\theta}^{\hat{b}_r - s} [g(\theta_i | N, \tilde{b}_r) - g(\theta_i | N, b_r)] d\theta_i + \int_{\theta}^{\hat{b}_r - s} g(\theta_i | N, \tilde{b}_r) d\theta_i = 0.$$

Together with the fact that $k(\cdot)$ is non-increasing, we obtain the last inequality.

**Proof for Proposition 5**

*Proof.* Without the loss of generality, one can normalize $\theta = 0$. Then, the objective function of the PC who selects a subcontract with the first-price auction is given by

$$u(c(t_i) | N, b_r) = \max_{b} (b - c_i) \left[1 - F_{\sigma}(\beta^{-1}(b) | N)\right]^{n(N-1)},$$

for any $c_i \leq b_r$ with boundary condition $u(b_r) = 0$. From the envelope integral formula, we have

$$u(c(t_i) | N, b_r) = \int_{c(t_i)}^{b_r} [1 - F_{\sigma}(\hat{c} | N)]^{n(N-1)} d\hat{c}$$

$$= \int_{t_i}^{b_r} [1 - F_{\sigma}(\hat{c} | N)]^{N-1} d\hat{c} - \int_{t_i}^{c(t_i)} [1 - F_{\sigma}(\hat{c} | N)]^{n(N-1)} d\hat{c}.$$

On the other hand, the objective function of the PC who selects a subcontract with the second-price subcontract auction and whose opponents select their subcontract with the first-price
subcontract auctions is given by

\[
u^{SP}(t_i|N,b_r) = \int_{t_i}^{b_r} \max(b - t_i^{(2)}) \left[ 1 - F_\sigma(\beta^{-1}(b|N)) \right]^{n(N-1)} dt_i^{(2)} \]

\[	imes \frac{(n-1) f_i(t_i^{(2)}) \left[ 1 - F_i(t_i^{(2)}) \right]^{n-2}}{[1 - F_i(t_i)]^{n-1}} dt_i^{(2)}, \tag{A-3}
\]

where \(t_i^{(2)}\) denotes the signal of the second lowest SC in the upstream auction. For any \(t_i^{(2)}\), the PC maximizes its own expected payoff by choosing \(b\). Thus,

\[
\frac{dw(t_i^{(2)})}{dt_i^{(2)}} = - \left[ 1 - F_\sigma(t_i^{(2)}) \right]^{n(N-1)}
\]

must hold where \(w(\cdot) = (\beta(c(t_i^{(2)})) - t_i^{(2)}) \left[ 1 - F_\sigma(t_i^{(2)}) \right]^{n(N-1)}\). Since this holds for any \(\hat{t} \in [t_i^{(2)}, b_r]\), we obtain

\[(\beta(\sigma(t_i^{(2)})) - t_i^{(2)}) \left[ 1 - F_\sigma(t_i^{(2)}) \right]^{n(N-1)} = \int_{t_i^{(2)}}^{b_r} [1 - F_\sigma(\hat{t})]^{n(N-1)} d\hat{t}.
\]

Plugging the formula back into (A-3), we obtain

\[
u^{SP}(c_i|N,b_r) = \int_{t_i}^{b_r} \int_{t_i^{(2)}}^{b_r} \left[ 1 - F_\sigma(\hat{t}) \right]^{n(N-1)} d\hat{t} \frac{(n-1) f_i(t_i^{(2)}) \left[ 1 - F_i(t_i^{(2)}) \right]^{n-2}}{[1 - F_i(t_i)]^{n-1}} dt_i^{(2)}
\]

\[
= \int_{t_i}^{b_r} \int_{t_i^{(2)} = t_i}^{\hat{t}} \left[ 1 - F_\sigma(\hat{t}) \right]^{n(N-1)} \frac{(n-1) f_i(t_i^{(2)}) \left[ 1 - F_i(t_i^{(2)}) \right]^{n-2}}{[1 - F_i(t_i)]^{n-1}} dt_i^{(2)} d\hat{t}
\]

\[
= \int_{t_i}^{b_r} \left[ 1 - F_\sigma(\hat{t}) \right]^{n(N-1)} d\hat{t} - \int_{t_i}^{b_r} \left( \frac{1 - F_i(\hat{t})}{1 - F_i(t_i)} \right)^{n-1} \left[ 1 - F_\sigma(\hat{t}) \right]^{n(N-1)} d\hat{t}.
\]
Now, suppose that $\sigma$ is non-concave for any $t$. Then,

\[
\int_{t_{(1)}^i}^{\sigma(t_{(1)}^i)} [1 - F_{\sigma}(c)]^{n(N-1)} \, dc \\
\leq \left[ 1 - F_{\sigma}(t_{(1)}^i) \right]^{n(N-1)} \left( \sigma(t_{(1)}^i) - t_{(1)}^i \right) \\
= \left[ 1 - F_{\sigma}(t_{(1)}^i) \right]^{n(N-1)} \int_{i = t_{(1)}^i}^{i} \left( \frac{1 - F_i(\hat{t})}{1 - F_i(t_{(1)}^i)} \right)^{n-1} \left[ \frac{1 - F_{\sigma}(\hat{t})}{1 - F_{\sigma}(t_{(1)}^i)} \right]^{n(N-1)} \, d\hat{t} \\
\leq \left[ 1 - F_{\sigma}(t_{(1)}^i) \right]^{n(N-1)} \int_{i = t_{(1)}^i}^{i} \left( \frac{1 - F_i(\hat{t})}{1 - F_i(t_{(1)}^i)} \right)^{n-1} \left[ \frac{1 - F_{\sigma}(\hat{t})}{1 - F_{\sigma}(t_{(1)}^i)} \right]^{n(N-1)} \, d\hat{t} \\
= \int_{i = t_{(1)}^i}^{i} \left( \frac{1 - F_i(\hat{t})}{1 - F_i(t_{(1)}^i)} \right)^{n-1} \left[ 1 - F_{\sigma}(\hat{t}) \right]^{N-1} \, d\hat{t}.
\]

The second inequality holds due to the convexity of $\sigma$.$^{10}$

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$^{10}$The convexity of $\sigma$ implies $\sigma(t|\cdot) - t \leq \sigma'(t)[t - \sigma^{-1}(t)]$, which is equivalent to $\frac{1}{t - \sigma^{-1}(t)} \leq \frac{\sigma'(t)}{\sigma(t|\cdot) - t}$. Thus,

\[
\frac{1}{t - \sigma^{-1}(t)} \leq \frac{\sigma'(t)}{\sigma(t|\cdot) - t}.
\]

On the other hand, from the SC’s objective function

\[
(s - t)[1 - F_{\sigma}(s|\cdot)]^{Nn-1} \\
(s - t)[1 - F_{\sigma}(s)]^{Nn-1}.
\]

The first-order condition of (A-5) is

\[
\frac{\sigma(t|\cdot) - t}{\sigma'(t)} = \frac{1}{Nn - 1} \frac{1 - F_i(t)}{f_i(t)}.
\]

Similarly, the first-order condition of (A-6) is

\[
\sigma(t|\cdot) - t = \frac{1}{Nn - 1} \frac{1 - F_{\sigma}(\sigma(t|\cdot))}{f_{\sigma}(\sigma(t|\cdot))}.
\]

Therefore, (A-4) can be rewritten as

\[
\frac{f_{\sigma}(t)}{1 - F_{\sigma}(t)} \leq \frac{f_i(t)}{1 - F_i(t)} = \frac{f_{\sigma}(\sigma(t|\cdot))}{1 - F_{\sigma}(\sigma(t|\cdot))}
\]

\[
\Leftrightarrow -f_{\sigma}(\sigma(t|\cdot))\sigma'(t)[1 - F_{\sigma}(t)] + [1 - F_{\sigma}(\sigma(t|\cdot))]f_{\sigma}(t) \leq 0 \text{ for all } t \geq t_{(1)}^i
\]

\[
\Leftrightarrow \frac{\partial}{\partial t} \frac{1 - F_{\sigma}(\sigma(t))}{1 - F_{\sigma}(t)} \leq 0 \text{ for all } t \geq t_{(1)}^i
\]

\[
\Leftrightarrow \frac{1 - F_{\sigma}(\sigma(t))}{1 - F_{\sigma}(t)} \leq \frac{1 - F_{\sigma}(\sigma(t_{(1)}^i))}{1 - F_{\sigma}(t_{(1)}^i)}
\]

\[
\Leftrightarrow \frac{1 - F_{\sigma}(\sigma(t))}{1 - F_{\sigma}(\sigma(t_{(1)}^i))} \leq \frac{1 - F_{\sigma}(t)}{1 - F_{\sigma}(t_{(1)}^i)}
\]
Proof that $F_c$ has a monotone increasing hazard rate (IHR)

**Proof.** Define $\delta \equiv c_{1:N-1} - \theta_i$, where $c_{1:N-1}$ denotes the lowest order statistics among $N - 1$ iid valuation samples of $c$. Since $-\frac{Q'(s)}{Q(s)} = \frac{f_{\delta}(s)}{1-F_{\delta}(s)}$ is increasing in $s$ by Lemma 1, $F_3$ has IHR. Now we consider the random variable $c_{1:N-1} = \delta + \theta_i$. Since the IHR is closed in convolution, $F_{c_{1:N-1}}$ has IHR. Then, from (12), $F_c$ has IHR if and only if $F_{c_{1:N-1}}$ has IHR. Hence, $\frac{f_c}{1-F_c}$ is increasing.

**References**


