Status Quo versus Outside Option in Parliamentary Politics

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Abstract

This paper is built on the work of Baron and Diermeier (2001, Quarterly Journal of Economics), which highlights the role of the status quo policy in parliamentary politics. Our main modification is to introduce “outside options” to make a distinction between voters and members of parliament whose parties are not in government. As the value of outside options increases, the status quo relative to outside options becomes less important in terms of parties’ bargaining strength. It is shown among others that if the value of outside options becomes sufficiently high, the status quo no longer has its bite on outcomes and the median-voter theorem holds in essence.

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1 Introduction

In an important paper, Baron and Diermeier (2001) (hereafter BD) consider a sequential model of election, government formation, and legislation in a parliamentary system with three parties, proportional representation, and a two-dimensional policy space. Parties are allowed to make transfers of office-holding benefits to implement efficient coalition bargaining. As noted by BD, this is a key assumption that distinguishes their model from the previous literature.

BD highlight the role of the status quo in proto-coalition bargaining, showing that the formateur (i.e., the party selected to try to form a government) prefers to form a government with the party that is more disadvantaged by the status quo policy. This is because the more disadvantaged party will be more willing to make office-holding concessions to the formateur in order to obtain policy changes from the status quo. Indeed, as BD demonstrate, if both the other parties are substantially disadvantaged by the status quo, it would be attractive for the formateur to form a consensus government.

BD assume that when the government fails to form, all the office-holding benefits vanish and all parties obtain nothing except for the utility derived from the status quo policy. This assumption implies that the reservation price of any party is completely determined by the status quo policy. In this paper we relax BD’s assumption and allow for a different possibility. More specifically, we assume that while a party can enjoy office-holding benefits if it is in government, the party can still receive some nonpolicy benefits if it is not in government. For convenience, we call these nonpolicy benefits “outside options.”

In the BD model, voters’ nonpolicy benefit is normalized to zero and, at the same time, a party’s nonpolicy benefit is also set to zero if it is not in government. This setting makes no distinction between voters and members of parliament whose parties are not in government. Our introduction of outside options allows for a distinction, in that members of parliament may obtain something beyond what voters can obtain even if their belonged parties are not in
government. We incorporate this distinction into the BD model and explore its implications for parliamentary politics.

As the value of outside options increases, the status quo relative to outside options becomes less important in terms of parties’ bargaining strength. We derive new results on the basis of this key insight. It is shown among others that if the value of outside options becomes sufficiently high, the status quo no longer has its bite on outcomes and the median-voter theorem holds in essence.

2 Model

The model is built on BD. As noted in the Introduction, our main modification is to allow for the incorporation of parties’ outside options. The value of outside options can be determined by common shocks (say, macro fluctuations across the whole economy so that three parties all face the same outside option), or idiosyncratic shocks (say, fluctuations only in some region of the economy so that three parties face different outside options). In this paper we confine our analysis to the common shocks. As will be seen, the analysis is somewhat involved even with this confinement. Perhaps more importantly, the confinement still enables us to derive some novel, interesting results regarding parliamentary politics. We let $\sigma \geq 0$ denote the common value of three parties’ outside options.

BD consider a two-dimensional policy space, assuming that the parties’ ideal policies are symmetrically located so that party ideology plays no role in their framework. We instead follow the common assumption with a one-dimensional policy space so that party ideology such as left-wing, right-wing, or centrist can play its role. As will be seen, this modification enriches the original BD model without much loss of generality.

There are finite number $N$ (even) voters and three parties in an economy. As noted by BD, increasing the number of parties increases the number of possible coalitions and hence
the complexity of the analysis. To compromise between the abstract model and the complex
real world in which there are often more than three parties in parliament with proportional
representation, the so-called “left-wing” party in our model may be interpreted to include
several parties whose ideologies are located on the left end of the unidimensional policy space.
Similar interpretation applies to the right-wing and the centrist party.

Voters are concerned only with policy and their preferences over policy $x \in \mathbb{R}$ are repre-
sented by a quadratic utility function

$$u(x; z) = -(x - z)^2,$$

where $z$ denotes a voter’s ideology or ideal policy, which is uniformly distributed on $[0, 1]$.

Parties are concerned with both policy and nonpolicy benefits. The preference of party $j$
is represented by:

$$U_j(x, \theta) = u_j(x) + \theta_j = -(x - z_j)^2 + \theta_j, \quad j = 1, 2, 3,$$

where $z_j$ is party $j$’s ideology or ideal policy, and $\theta_j$ is party $j$’s received nonpolicy benefit.
$\theta_j = y_j$, the party $j$’s office-holding benefits with $\sum_{j=1}^{3} y_j = Y$ if party $j$ is in government.
$\theta_j = \sigma$, the common outside option if party $j$ is not in government. BD assume that $y_j = 0$
and $\sigma = 0$ if party $j$ is not in government. This assumption makes no distinction between
voters and members of parliament whose parties are not in government. We allow for the
possibility that $y_j = 0$ but $\sigma \geq 0$ if party $j$ is not in government. Similar to BD, parties
are allowed to exchange office-holding benefits and hence $y_j$ can be either positive (a benefit
received by party $j$) or negative (a payment paid by party $j$).

For simplicity and to make our problem interesting, we assume that a party never prefers
seizing its outside option to joining in government.

Let $z_1 = \frac{1}{4}$, $z_2 = \frac{1}{2}$, and $z_3 = \frac{3}{4}$. Thus, party 1, 2, 3 represent a left-wing, centrist,
right-wing party, respectively. We assume as in BD that no voters are exactly indifferent
between any two parties’ ideology and that no party has any inherent advantage in winning the election when voters vote sincerely.

The timing and structure of the game follows BD.

The first stage: There is an election that determines the seat shares of the three parties in parliament by proportional representation.

The second stage: One party is selected as a formateur. If there is a party that wins a majority of seats in parliament, it automatically becomes the formateur. If no party wins a majority of seats, the formateur is selected with a probability equal to each party’s seat share in parliament. As noted by BD, this selection procedure is empirically well supported (Diermeier and Merlo, 1999). The formateur selects a “proto-coalition” in an attempt to form the government.

The last stage: The formateur makes a take-it-or-leave-it proposal to the coalition partners. The proposal includes the policy to be implemented and the distribution of the office-holding benefits among the coalition members. If all the parties in the coalition agree on the formateur’s proposal, the proposed policy is implemented and transfers are realized; if the proposal is rejected, then the status quo policy, \( q \in [0, 1] \), remains in effect.

3 Government formation and legislation

In this section, we consider government formation and policymaking in parliament. There are two separate cases to address. In the case of a minority parliament, none of the three parties wins a majority of seats in parliament and so a formateur is selected randomly with its probability equal to each party’s share of seats in parliament. In this case, the selected formateur can either choose one of the other two parties to form a minimum winning coalition (MWC) government, or include all of the other two parties to form a consensus government. In the case of a majority parliament, one single party wins a majority of seats in parliament.
The majority party automatically becomes the formateur and can either choose one of the other two parties to form a surplus government, or include all of the other two parties to form a consensus government, or propose its ideal policy to form a government by itself.

3.1 Minority parliaments

3.1.1 Leftist or rightist as formateur

We first consider the situation where one of the two extreme parties is selected as the formateur in a minority parliament.

Let \( y = (y_1, y_2, y_3) \). When party 1 (the left-wing party) is the formateur and makes a proposal \( (x, y) \), party \( j \neq 1 \) will accept it if

\[
 u_j(x) + y_j \geq u_j(q) + \sigma.
\]

Thus, if party 1 forms a coalition only with party \( j \neq 1 \) (the so-called "MWC government," it will choose \( y_j \) such that

\[
 y_j = u_j(q) - u_j(x) + \sigma.
\]

and hence party 1's utility will be equal to

\[
 u_1(x) + Y - y_j = u_1(x) + u_j(x) - u_j(q) + Y - \sigma. \tag{1}
\]

On the other hand, if party 1 forms a coalition with both party 2 and party 3 (the so-called "consensus government," it will choose \( y_j = u_j(q) - u_j(x) + \sigma \) for \( j = 2, 3 \) and hence party 1's utility will be equal to

\[
 u_1(x) + Y - y_j = \sum_{j=1}^{3} u_j(x) - u_j(q) - u_3(q) + Y - 2\sigma. \tag{2}
\]

Let \( x_{ij} \) denote the optimal policy when formateur \( i \) forms a coalition only with party \( j \neq i \). Based on (1), if party 1 forms a coalition only with party 2, the optimal policy will be
\[ x_{12} = \frac{z_1 + z_2}{2} = \frac{3}{8} \text{.} \] The resulting utility for party 1 in this case is
\[ -\frac{1}{32} + (q - \frac{1}{2})^2 + Y - \sigma. \] (3)

If party 1 forms a coalition only with party 3, the optimal policy will be \[ x_{13} = \frac{z_1 + z_3}{2} = \frac{1}{2} \text{.} \] The resulting utility for party 1 in this case is
\[ -\frac{1}{8} + (q - 3) - \frac{1}{8} + (q - 3)^2 + Y - 2\sigma. \] (4)

Based on (2), if party 1 forms a consensus government, the optimal policy will be \[ \bar{z} = \frac{z_1 + z_2 + z_3}{3} = \frac{1}{2} \text{.} \] The resulting utility for party 1 in this case is
\[ -\frac{1}{8} + (q - \frac{1}{2})^2 + (q - 3)^2 + Y - 2\sigma. \] (5)

Comparing (3), (4) and (5), government formation and legislation with the left-wing party as formateur in minority parliaments are summarized as follows.

**Lemma 1.** Suppose that party 1 is the formateur in a minority parliament.¹

a. \( \sigma \leq \frac{1}{256} : \)

(1) If \( q \in \left[0, \frac{3}{4} - \frac{1}{8}\sqrt{6 + 64\sigma}\right] \), party 1 forms a consensus government and proposes policy \( \bar{z} = \frac{1}{2} \).

(2) If \( q \in \left[\frac{3}{4} - \frac{1}{8}\sqrt{6 + 64\sigma}, 1\right] \), party 1 forms a MWC government with party 2 and proposes policy \( x_{12} = \frac{3}{8} \).

b. \( \sigma > \frac{1}{256} : \)

¹A coalition with party 2 is better than with party 3 if and only if \( q > \frac{7}{16} \). A grand coalition is better than a coalition with party 2 if and only if \( q < \frac{3}{4} - \frac{1}{8}\sqrt{6 + 64\sigma} \), and is better than a coalition with party 3 if and only if \( q < \frac{1}{2} - \sqrt{\sigma} \). We also note that \( \frac{7}{16} \leq \frac{3}{4} - \frac{1}{8}\sqrt{6 + 64\sigma} \leq \frac{1}{2} - \sqrt{\sigma} \) when \( \sigma \leq \frac{1}{256} \) (in which the equality holds if and only if \( \sigma = \frac{1}{256} \)), and \( \frac{1}{2} - \sqrt{\sigma} < \frac{3}{4} - \frac{1}{8}\sqrt{6 + 64\sigma} < \frac{7}{16} \) when \( \sigma > \frac{1}{256} \).
(1) If $q \in \left[0, \frac{1}{2} - \sqrt{\sigma}\right]$, party 1 forms a consensus government and proposes policy $\bar{z} = \frac{1}{2}$.

(2) If $q \in \left[\frac{1}{2} - \sqrt{\sigma}, \frac{7}{16}\right]$, party 1 forms a MWC government with party 3 and proposes policy $x_{13} = \bar{z} = \frac{1}{2}$.

(3) If $q \in \left[\frac{7}{16}, 1\right]$, party 1 forms a MWC government with party 2 and proposes policy $x_{12} = \frac{3}{8}$.

Note that $q \in \left[0, \frac{3}{4} - \frac{1}{8} \sqrt{6 + 64\sigma}\right] = \left[0, \frac{7}{16}\right]$ if $\sigma = \frac{1}{256}$.

When party 3 (the right-wing party) is the formateur, we obtain results symmetric to Lemma 1.

**Lemma 2.** Suppose that party 3 is the formateur in a minority parliament.

a. $\sigma \leq \frac{1}{256}$:

(1) If $q \in \left[\frac{1}{4} + \frac{1}{8} \sqrt{6 + 64\sigma}, 1\right]$, party 3 forms a consensus government and proposes policy $\bar{z} = \frac{1}{2}$.

(2) If $q \in \left[0, \frac{1}{4} + \frac{1}{8} \sqrt{6 + 64\sigma}\right]$, party 3 forms a MWC government with party 2 and proposes policy $x_{32} = \frac{5}{8}$.

b. $\sigma > \frac{1}{256}$:

(1) If $q \in \left[\frac{1}{2} + \sqrt{\sigma}, 1\right]$, party 3 forms a consensus government and proposes policy $\bar{z} = \frac{1}{2}$.

(2) If $q \in \left[\frac{9}{16}, \frac{1}{2} + \sqrt{\sigma}\right]$, party 3 forms a MWC government with party 1 and proposes policy $x_{31} = \bar{z} = \frac{1}{2}$.

(3) If $q \in \left[0, \frac{9}{16}\right]$, party 3 forms a MWC government with party 2 and proposes policy $x_{32} = \frac{5}{8}$. 

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Note that \( q \in \left[ \frac{1}{4} + \frac{1}{8}\sqrt{6 + 64\sigma}, 1 \right] = \left[ \frac{\sigma}{16}, 1 \right] \) if \( \sigma = \frac{1}{256} \).

We explain the results according to Lemma 1. The reasoning behind Lemma 2 is similar.

Lemma 1a-(1) corresponds to BD’s Proposition 2. When the value of outside options is relatively small (\( \sigma \leq \frac{1}{256} \)), party 1’s best choice is to form a consensus government if both the other parties are substantially disadvantaged by the status quo. As explained in BD, the formateur seeks the best bargain and so prefers to form a consensus government if both the other parties’ bargaining positions are substantially weaken by the status quo. Otherwise, party 1 forms a MWC government and seeks a coalition partner which has a weaker bargaining position, i.e., the party that is more disadvantaged by the status quo. This is the result of Lemma 1a-(2), which basically corresponds to BD’s Proposition 1.

The situation is different when the value of outside options becomes relatively large (\( \sigma > \frac{1}{256} \)). Since parties can obtain some significant compensation from outside options if no agreement is reached, it would be expensive to buy yea votes from both parties. As a result, the would-be consensus government when the value of outside options were low may no longer be attractive as the value of outside options becomes large. This explains why the case in Lemma 1b-(2) arises.

One may wonder why two extreme parties form a MWC government in this case. The logic underlying the result is exactly the BD argument. When the given \( q \) is neither large nor small as stated in Lemma 1b-(2), it would be most favorable to party 2 if the status quo persisted. This means that party 2 has a stronger bargaining position than party 3 and, therefore, party 1 as formateur prefers to form a government with party 3 than party 2. This result is absent in BD, due to the absence of outside options as well as extreme parties in the BD model.
3.1.2 Centrist as formateur

We next consider the situation where the centrist party is selected as the formateur in a minority parliament.

When party 2 (the centrist party) is the formateur, it can form a coalition with either party 1, or party 3, or both. Based on (1), if party 2 forms a coalition only with party 1, the optimal policy will be \( x_{21} = \frac{3}{8} \) and party 2 obtains a utility of \( -\frac{1}{32} + (q - \frac{1}{4})^2 + Y - \sigma \). If party 2 forms a coalition only with party 3, the optimal policy will be \( x_{23} = \frac{5}{8} \) and party 2 obtains a utility of \( -\frac{1}{32} + (q - \frac{3}{4})^2 + Y - \sigma \). Based on (2), if party 2 forms a consensus government with both parties 1 and 3, the optimal policy will be \( \bar{z} = \frac{1}{2} \) and party 2 obtains a utility of \(-\frac{1}{8} + (q - \frac{1}{4})^2 + (q - \frac{3}{4})^2 + Y - 2\sigma \).

Comparing these three cases, government formation and legislation with the centrist party as formateur in minority parliaments are summarized as follows.

**Lemma 3.** Suppose that party 2 is the formateur in a minority parliament. Then:

1. If \( q \in \left[0, \frac{1}{2}\right] \), party 2 forms a MWC government with party 3 and proposes policy \( x_{23} = \frac{5}{8} \).
2. If \( q \in \left[\frac{1}{2}, 1\right] \), party 2 forms a MWC government with party 1 and proposes policy \( x_{12} = \frac{3}{8} \).

Due to the common-value assumption, outside options exert no effect on the selection of a coalition partner for the centrist party. When party 2 is the formateur, it will form a MWC coalition either with party 3 or party 1, depending on whether the status quo is located closer to \( z_1 \) or \( z_3 \). A consensus government is never optimal for party 2.

3.2 Majority parliaments

As far as the formateur’s seeking one partner or two partners to form a government is concerned, a majority parliament does not differ from a minority one. Given the status quo and
the common outside option, both parliaments face the same incentives in this regard. However, the formateur in a majority parliament has an additional option to form a single-party government and choose its ideal policy without seeking other coalition partners. This is the main difference between majority and minority parliaments.

It is rare for one party to win a majority of seats in parliamentary systems with proportional representation. In their study on 255 governments over the period 1947-1999 in 9 West European countries (all of them elect their parliament according to proportional representation), Diermeier et al. (2003) observe 22 cases where a party controls an absolute majority of the parliamentary seats. However, if we interpret the three parties in our model loosely such that, say, the so-called “left-wing” party actually includes several parties whose ideologies are on the left end of the unidimensional policy space, then the observed frequency of majority parliaments may no longer be rare.

3.2.1 Leftist or rightist as formateur

When party 1 (the left-wing party) wins a majority in parliament, it can propose \( z_1 = \frac{1}{4} \) and consume all the benefits from holding office, in which case its utility will be equal to \( Y \). This is a choice better than forming a consensus government, a coalition only with party 2, and a coalition only with party 3, respectively, if

\[
Y \geq -\frac{1}{8} + (q - \frac{1}{2})^2 + (q - \frac{3}{4})^2 + Y - 2\sigma;
\]

\[
Y \geq -\frac{1}{32} + (q - \frac{1}{2})^2 + Y - \sigma;
\]

\[
Y \geq -\frac{1}{8} + (q - \frac{3}{4})^2 + Y - \sigma.
\]

Thus, we have the following results.\(^2\)

\(^2\)The condition for all the above inequalities to hold is \( q \in \left[\frac{5}{8} - \frac{1}{8} \sqrt{3 + 64\sigma}, \frac{5}{8} + \frac{1}{8} \sqrt{3 + 64\sigma}\right] \cap \left[\frac{1}{2} - \frac{1}{8} \sqrt{2 + 64\sigma}, \frac{1}{2} + \frac{1}{8} \sqrt{2 + 64\sigma}\right] \cap \left[\frac{3}{4} - \frac{1}{4} \sqrt{2 + 64\sigma}, \frac{3}{4} + \frac{1}{4} \sqrt{2 + 64\sigma}\right].\)
Lemma 4. Suppose that party 1 is the formateur in a majority parliament. Then $z_1$ is the best policy if

$$q \in \left[\frac{5}{8} - \frac{1}{8}\sqrt{3 + 64\sigma}, \frac{1}{2} + \frac{1}{8}\sqrt{2 + 64\sigma}\right] \text{ when } \sigma \leq \frac{1}{64};$$

$$q \in \left[\frac{3}{4} - \frac{1}{4}\sqrt{2 + 16\sigma}, \frac{1}{2} + \frac{1}{4}\sqrt{2 + 64\sigma}\right] \text{ when } \frac{1}{64} < \sigma \leq \frac{7}{16};$$

$$q \in [0, 1] \text{ when } \sigma > \frac{7}{16}.$$

Otherwise, party 1 behaves the same as those in Lemma 1.

When party 3 (the right-wing party) wins a majority in parliament, we have results symmetric to the above lemma.

Lemma 5. Suppose that party 3 is the formateur in a majority parliament. Then $z_3$ is the best policy if

$$q \in \left[\frac{1}{2} - \frac{1}{8}\sqrt{2 + 64\sigma}, \frac{3}{8} + \frac{1}{8}\sqrt{3 + 64\sigma}\right] \text{ when } \sigma \leq \frac{1}{64};$$

$$q \in \left[\frac{1}{3} - \frac{1}{8}\sqrt{2 + 64\sigma}, \frac{1}{4} + \frac{1}{4}\sqrt{2 + 16\sigma}\right] \text{ when } \frac{1}{64} < \sigma \leq \frac{7}{16};$$

$$q \in [0, 1] \text{ when } \sigma > \frac{7}{16}.$$

Otherwise, party 3 behaves the same as those in Lemma 2.

The Lemmas 4-5 correspond to Proposition 3 in BD. The main twist in the presence of outside options is that the range of the status quo supporting a single-party government will expand as the value of outside options becomes higher. This result is intuitive: as the value of outside options increases, the status quo relative to outside options becomes less important in terms of parties’ bargaining strength. Once the value of outside options is high enough with $\sigma > \frac{7}{16}$, the status quo no longer has its bite and any $q \in [0, 1]$ can support a single-party government in majority parliaments.
3.2.2 Centrist as formateur

When party 2 (the centrist party) wins a majority in parliament, it can propose its ideal policy \( z_2 = \frac{1}{2} \) and consume all the benefits from holding office, in which case its utility will be equal to \( Y \). This is a choice better than forming a coalition only with party 1, and a coalition only with party 3, respectively, if

\[
Y \geq -\frac{1}{32} + (q - \frac{1}{4})^2 + Y - \sigma; \\
Y \geq -\frac{1}{32} + (q - \frac{3}{4})^2 + Y - \sigma.
\]

Thus, we have the following result.\(^3\),

**Lemma 6.** Suppose that party 2 is the formateur in a majority parliament. Then \( z_2 \) is the best policy if

\[
q \in \emptyset \quad \text{when } \sigma \leq \frac{1}{32}; \\
q \in \left[\frac{3}{4} - \frac{1}{8}\sqrt{2 + 64\sigma}, \frac{1}{4} + \frac{1}{8}\sqrt{2 + 64\sigma}\right] \quad \text{when } \frac{1}{32} < \sigma \leq \frac{17}{32}; \\
q \in [0, 1] \quad \text{when } \sigma > \frac{17}{32}.
\]

(6)

Otherwise, party 2 behaves the same as those in Lemma 3.

\( q \in \emptyset \) means that there does not exist a \( q \) such that \( z_2 \) is the best policy. The results in Lemma 6 are similar to those in Lemmas 4-5. Note in particular that once \( \sigma > \frac{17}{32} \), any \( q \in [0, 1] \) can support a single-party government in majority parliaments.

3.3 Summary

As far as the formateur’s forming a MWC or surplus government and seeking which party as the coalition partner is concerned, outside options play no role. This is because both the

\[^3\text{The condition for both inequalities to hold is } q \in \left[\frac{1}{4} - \frac{1}{8}\sqrt{2 + 64\sigma}, \frac{1}{4} + \frac{1}{8}\sqrt{2 + 64\sigma}\right] \cap \left[\frac{3}{4} - \frac{1}{8}\sqrt{2 + 64\sigma}, \frac{3}{4} + \frac{1}{8}\sqrt{2 + 64\sigma}\right].\]
other parties have the same common value of outside options by assumption. Outside options exert their impact only if the coalition-formation decision has to do with (i) whether to form a consensus government in both minority and majority parliaments, and (ii) whether to form a single-party government than otherwise in majority parliaments.

When the value of outside options is high, buying two yea votes from both the other parties is expensive and so it can be optimal for a formateur to form a MWC/surplus rather than consensus government. This leads to the results of the extreme-party-coalition government in Lemmas 1b-(2) and 2b-(2) in the case of minority parliaments and the corresponding results of Lemmas 4-6 in the case of majority parliaments. This kind of results is absent in BD, since there is no role for outside options in their model.

When the value of outside options is low ($\sigma \leq \frac{1}{32}$), party 2 as formateur in majority parliament has no incentives to form a single-party government at any $q$ ($q \in \emptyset$). However, once the value of outside options becomes higher, seeking coalition partners gets more expensive and, consequently, it can be optimal for party 2 to form a single-party government rather than otherwise. This leads to the results of a single-party government in Lemma 6. Outside options exert their similar impact for parties 1 and 3 in Lemma 4-5. This kind of results is absent in BD, since again there is no role for outside options in their model.

4 Electoral equilibrium outcomes

When voters cast their votes in the stage of election, the seat share of each party in the parliament not only can determine the probability of each party being selected as formateur, but it can also affect the coalition formed in the legislative stage. As stated in Baron and Diermeier (2001), there are usually many Nash equilibria in the electoral stage. We follow their treatment to focus on the strong Nash equilibrium, in which the equilibrium outcome is robust to deviations by group of voters.
We first state a useful result.

**Lemma 7.** Majority parliaments will not form a single-party government in equilibrium if \( \sigma \leq \frac{1}{32} \).

This is clearly true for party 2 according to Lemma 6. We show in the Appendix that this is also true for parties 1 and 3.

With Lemmas 1-7 at hand, we turn to characterizing electoral outcomes. Depending on the value of outside options, we separate several cases to discuss. We provide a detailed proof for Proposition 1 in the Appendix. Other propositions can be proved using the same logic.

### 4.1 Case I: \( \sigma \leq \frac{1}{256} \)

Since the value of outside options is small, including \( \sigma = 0 \), we can view Case I as the unidimensional-policy-space version of the BD model.

**Proposition 1.** Let \( \sigma \leq \frac{1}{256} \). Then in a strong Nash equilibrium:

1. When \( q \in \left[ 0, \frac{5}{8} - \frac{1}{8} \sqrt{3 + 64\sigma} \right] \left( q \in \left[ \frac{3}{8} + \frac{1}{8} \sqrt{3 + 64\sigma}, 1 \right] \right) \), a majority parliament results, in which party 1 (party 3) wins a majority of seats in parliament. Party 1 (party 3) automatically becomes the formateur and forms a consensus government with policy \( \bar{z} = \frac{1}{2} \).

2. When \( q \in \left[ \frac{5}{8} - \frac{1}{8} \sqrt{3 + 64\sigma}, \frac{3}{4} - \frac{1}{8} \sqrt{6 + 64\sigma} \right] \left( q \in \left[ \frac{1}{4} + \frac{1}{8} \sqrt{6 + 64\sigma}, \frac{3}{8} + \frac{1}{8} \sqrt{3 + 64\sigma} \right] \right) \), a minority parliament results, in which party 1 (party 3) wins the half of seats in parliament, while the other two parties together win the other half of seats. Party 1 (party 3) is selected as a formateur with probability \( \frac{1}{2} \) and forms a consensus government with policy \( \bar{z} = \frac{1}{2} \); party 2 or 3 (party 1 or 2) are selected as a formateur with probability \( \frac{1}{2} \) and they seek each other as coalition partners and form a MWC government with policy \( x_{23} = \frac{5}{8} \) (\( x_{12} = \frac{3}{8} \)).
(3) When \( q \in \left[ \frac{3}{4} - \frac{1}{8} \sqrt{6 + 64\sigma}, \frac{1}{2} \right] \left( q \in \left[ \frac{1}{2}, \frac{1}{8} \sqrt{6 + 64\sigma} \right] \right) \), a minority parliament results, in which party 1 (party 3) wins the half of seats in parliament, while the other two parties together win the other half of seats. Party 1 (party 3) is selected as a formateur with probability \( \frac{1}{2} \) and it seeks party 2 as a coalition partner and forms a MWC government with policy \( x_{12} = \frac{3}{8} \); party 2 or 3 (party 1 or 2) are selected as a formateur with probability \( \frac{1}{2} \) and they seek each other as coalition partners and form a MWC government with policy \( x_{23} = \frac{5}{8} \) \( (x_{12} = \frac{3}{8}) \).

When the value of outsides options is small, our results are basically similar to those in the BD model. More precisely, our Proposition 1-(1) is similar to BD’s Proposition 4-(A) (our given \( q \) resembles BD’s condition that exactly one party as formateur would form a consensus government); our Proposition 1-(3) is similar to their Proposition 4-(B) (our given \( q \) resembles BD’s condition that no party as formateur would form a consensus government). BD’s Proposition 4-(C) depicts the resulting equilibrium outcome when all parties would like to form a consensus government, given \( q \). This case is absent in our model due to the restriction \( q \in [0, 1] \). BD’s Proposition 4-(D) depicts the equilibrium outcome in which some party as formateur randomizes between policies. This result arises due to the BD setting that the parties’ ideal policies are symmetrically located so as to eliminate the role of ideology. We allow the role of ideology and so there is no corresponding result in our model.

However, our Proposition 1-(2) is absent in BD. A consensus government is formed in this case even though a minority rather than majority parliament results. The reason underlying this result is that, in the regime stated in Proposition 1-(2), party 1 (party 3) will form a consensus government in a minority parliament, but it will instead propose its ideal policy and form a single-party government in a majority parliament. Since a majority of voters prefer policy \( \frac{1}{2} \) to \( \frac{1}{4} \) \( (\frac{3}{4}) \), voters will coordinate to maintain a minority rather than majority parliament. Due to the symmetry setup in ideologies, the formateur’s decision to form a consensus government is independent of whether the parliament is minority or majority in
BD.\textsuperscript{4} As a result, the case in Proposition 1-(2) would not arise in their model.

4.2 Case II: $\frac{1}{256} < \sigma \leq \frac{1}{32}$

There are two subcases to consider: $\frac{1}{256} < \sigma \leq \frac{1}{64}$ and $\frac{1}{64} < \sigma \leq \frac{1}{32}$.

**Proposition 2.** Let $\frac{1}{256} < \sigma \leq \frac{1}{64}$. Then in a strong Nash equilibrium:

1. When $q \in \left[0, \frac{5}{8} - \frac{1}{8}\sqrt{3 + 64\sigma} \right] \left( q \in \left[\frac{3}{8} + \frac{1}{8}\sqrt{3 + 64\sigma}, 1 \right] \right)$, the equilibrium outcome is the same as that in Proposition 1-(1).

2. When $q \in \left[\frac{5}{8} - \frac{1}{8}\sqrt{3 + 64\sigma}, \frac{1}{2} - \sqrt{\sigma} \right] \left( q \in \left[\frac{1}{2} + \sqrt{\sigma}, \frac{3}{8} + \frac{1}{8}\sqrt{3 + 64\sigma} \right] \right)$, the equilibrium outcome is the same as that in Proposition 1-(2).

3. When $q \in \left[\frac{1}{2} - \sqrt{\sigma}, \frac{7}{10} \right] \left( q \in \left[\frac{9}{16}, \frac{1}{2} + \sqrt{\sigma} \right] \right)$, a minority parliament results, in which party 1 (party 3) wins the half of seats in parliament, while the other two parties together win the other half of seats. Party 1 (party 3) is selected as a formateur with probability $\frac{1}{2}$ and it seeks party 3 (party 1) as a coalition partner and forms a MWC government with policy $x_{13} = \frac{1}{2}$ ($x_{31} = \frac{1}{2}$); party 2 or 3 (party 1 or 2) are selected as a formateur with probability $\frac{1}{2}$ and they seek each other as coalition partners and form a MWC government with policy $x_{23} = \frac{5}{8}$ ($x_{12} = \frac{3}{8}$).

4. When $q \in \left[\frac{7}{16}, \frac{1}{2} \right] \left( q \in \left[\frac{1}{2}, \frac{9}{16} \right] \right)$, the equilibrium outcome is the same as that in Proposition 1-(3).

The occurrence of Proposition 2-(3) is attributed to Lemmas 1b-(2) and 2b-(2).

**Proposition 3.** Let $\frac{1}{64} < \sigma \leq \frac{1}{32}$. Then in a strong Nash equilibrium:

\textsuperscript{4}To be precise, in BD, $\tilde{z}$ is better than $x_{12}$ for party 1 if and only if $-\frac{1}{2} - u_2(q) \geq 0$, and $\tilde{z}$ is better than $x_{13}$ if and only if $-\frac{1}{2} - u_3(q) \geq 0$. Moreover, $\tilde{z}$ is better than $z_1$ if and only if $-1 - u_2(q) - u_3(q) \geq 0$. Apparently, if $\tilde{z}$ is better than both $x_{12}$ and $x_{13}$, it is also better than $z_1$.\textsuperscript{17}
(1) When \( q \in \left[ 0, \frac{1}{2} - \sqrt{\sigma} \right] \left( q \in \left[ \frac{1}{2} + \sqrt{\sigma}, 1 \right] \right) \), the equilibrium outcome is the same as that in Proposition 1-(1).

(2) When \( q \in \left[ \frac{3}{4} - \frac{1}{4}\sqrt{2 + 16\sigma}, \frac{7}{16} \right] \left( q \in \left[ \frac{9}{16}, \frac{1}{4} + \frac{1}{4}\sqrt{2 + 16\sigma} \right] \right) \), a majority parliament results, in which party 1 (party 3) wins a majority of seats in parliament. Party 1 (party 3) automatically becomes the formateur and it seeks party 3 (party 1) as a coalition partner and forms a surplus government with policy \( x_{13} = \frac{1}{2} \) (\( x_{31} = \frac{1}{2} \)).

(3) When \( q \in \left[ \frac{3}{4} - \frac{1}{4}\sqrt{2 + 16\sigma}, \frac{7}{16} \right] \left( q \in \left[ \frac{9}{16}, \frac{1}{4} + \frac{1}{4}\sqrt{2 + 16\sigma} \right] \right) \), the equilibrium outcome is the same as that in Proposition 2-(3).

(4) When \( q \in \left[ \frac{7}{16}, \frac{1}{2} \right] \left( q \in \left[ \frac{9}{16}, \frac{9}{16} \right] \right) \), the equilibrium outcome is the same as that in Proposition 1-(3).

The occurrence of Proposition 3-(2) is attributed to the corresponding results of Lemmas 1b-(2) and 2b-(2) in Lemmas 4–5.

Compared to Case I, the new outcomes in Case II are those where two extreme parties form a government. Axelrod (1970) suggests that a formateur would form a government containing ideologically adjacent parties. The new outcomes in Case II provide an opposite alternative. As explained after Lemmas 1-2, this new type of outcomes arises due to the presence of both the status quo and the outside option.

### 4.3 Case III: \( \sigma > \frac{1}{32} \)

There are two subcases to consider: \( \frac{1}{32} < \sigma < \frac{17}{256} \) and \( \sigma \geq \frac{17}{256} \).

**Proposition 4.** Let \( \frac{1}{32} < \sigma < \frac{17}{256} \). Then in a strong Nash equilibrium:

(1) When \( q \in \left[ 0, \frac{1}{2} - \sqrt{\sigma} \right] \left( q \in \left[ \frac{1}{2} + \sqrt{\sigma}, 1 \right] \right) \), the equilibrium outcome is the same as that in Proposition 1-(1).
When \( q \in \left[ \frac{1}{2} - \sqrt{\sigma}, \frac{3}{4} - \frac{1}{4} \sqrt{2 + 16\sigma} \right] \left( q \in \left[ \frac{1}{4} + \frac{1}{4} \sqrt{2 + 16\sigma}, \frac{1}{2} + \sqrt{\sigma} \right] \right) \), the equilibrium outcome is the same as that in Proposition 3-(2).

When \( q \in \left[ \frac{3}{4} - \frac{1}{4} \sqrt{2 + 16\sigma}, \frac{7}{16} \right] \left( q \in \left[ \frac{9}{16}, \frac{1}{4} + \frac{1}{4} \sqrt{2 + 16\sigma} \right] \right) \), the equilibrium outcome is the same as that in Proposition 2-(3).

When \( q \in \left[ \frac{7}{16}, \frac{3}{4} - \frac{1}{8} \sqrt{2 + 64\sigma} \right] \left( q \in \left[ \frac{1}{4} + \frac{1}{8} \sqrt{2 + 64\sigma}, \frac{9}{16} \right] \right) \), the equilibrium outcome is the same as that in Proposition 1-(3).

When \( q \in \left[ \frac{3}{4} - \frac{1}{8} \sqrt{2 + 64\sigma}, \frac{1}{4} + \frac{1}{8} \sqrt{2 + 64\sigma} \right] \), a majority parliament results, in which party 2 wins a majority of seats in parliament. Party 2 automatically becomes the formateur and forms a single-party government with policy \( z_2 = \frac{1}{2} \).

Compared with previous Propositions, the new outcome is the case stated in Proposition 4-(5). It arises obviously due to Lemma 6. In the case of majority parliaments, BD’s Proposition 4-(A) resembles our Proposition 4-(1), predicting that the formateur will propose to form a consensus government. This is the only possible equilibrium outcome for majority parliaments in BD. Our Proposition 4-(5) includes an additional possibility: the formateur may would like to form a government by itself.

When the value of outside options gets large, the bargaining positions of both the other parties become strong, given any \( q \). It may then be optimal for a formateur in majority parliaments to form a government by itself. Although all of the three parties would like to form a single-party government in such a situation according to Lemmas 4-6, only the centrist party with its ideal policy can win a majority support of voters. This is because the ideal policy of the centrist party happens to be the median voter’s ideal policy, which is the Condorcet winner in our unidimensional policy space. The ideal policy of the left- or

\(^5\text{One can check that these regimes exist when } \sigma < \frac{17}{256}. \text{ That is, } \frac{7}{16} < \frac{3}{4} - \frac{1}{8} \sqrt{2 + 64\sigma} \text{ and } \frac{1}{4} + \frac{1}{8} \sqrt{2 + 64\sigma} < \frac{9}{16} \text{ if and only if } \sigma < \frac{17}{256}.\)
right-wing party is not the Condorcet winner and hence forming a single-party government could not be sustained in a strong Nash equilibrium.\(^6\)

The Condorcet winner is built on pairwise voting over policy alternatives. This pairwise feature holds in our model, since voters in their voting always make comparison between the equilibrium policy and the resulting alternative policy when some of them deviate from the equilibrium. The existence of the Condorcet winner is clearly crucial for our Proposition 4-(5). The same arguments cannot apply to the BD model because there does not exist a Condorcet winner in their two-dimentional policy space.

The status quo to support an equilibrium outcome varies with the value of outside options according to Propositions 1-4. When \(\sigma = \frac{3}{16}, q \in \left[\frac{7}{16}, \frac{3}{8} - \frac{1}{3}\sqrt{2} + 64\sigma \right] = \left[\frac{7}{16}, \frac{7}{16}\right]\) and hence the case in Proposition 4-(4) ceases to exist. When \(\sigma = \frac{1}{4}, q \in \left[0, \frac{1}{2} - \sqrt{\sigma} \right] = [0, 0]\) and hence the case in Proposition 4-(1) ceases to exist. When \(\sigma = \frac{7}{16}, q \in \left[\frac{1}{2} - \sqrt{\sigma}, \frac{3}{4} - \frac{1}{4}\sqrt{2} + 16\sigma \right] = 0\) and hence the case in Proposition 4-(2) ceases to exist. Finally, when \(\sigma = \frac{17}{32}, q \in \left[\frac{3}{4} - \frac{1}{3}\sqrt{2} + 64\sigma, \frac{1}{4} + \frac{1}{3}\sqrt{2} + 64\sigma \right] = [0, 1]\) and the case in Proposition 4-(5) is dominating and hence the case in Proposition 4-(3) ceases to exist.

**Proposition 5.** Let \(\sigma \geq \frac{17}{256}\). Then in a strong Nash equilibrium:

1. The case in Proposition 4-(4) ceases to exist if \(\frac{17}{256} \leq \sigma < \frac{1}{4}\).
2. The case in Proposition 4-(1) ceases to exist if \(\frac{1}{4} \leq \sigma < \frac{7}{16}\).
3. The case in Proposition 4-(2) ceases to exist if \(\frac{7}{16} \leq \sigma < \frac{17}{32}\).
4. The case in Proposition 4-(3) ceases to exist, and the unique equilibrium outcome is the case in Proposition 4-(5) if \(\sigma \geq \frac{17}{32}\).

As the value of outside options increases, the status quo relative to outside options becomes less important in terms of parties’ bargaining strength and so in terms of the formateur’s

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\(^6\)See Persson and Tabellini (2000, chapter 2) on the Condorcet winner for the detail.
coalition-formation decision. Note in particular that the range of $q$ stated in Proposition 4-(5) will expand as $\sigma$ becomes larger. Indeed, when the value of outside options is sufficiently high with $\sigma \geq \frac{17}{32}$, the status quo has no bite at all on equilibrium outcomes since $q \in [0, 1]$. Instead, the only strong Nash equilibrium outcome is the case in Proposition 4-(5) and the median-voter theorem emerges from parliamentary politics. This results because the median voter’s ideal policy is the Condorcet winner in our unidimensional policy space and so it beats all other possible policies in voters’ pairwise comparison.

Proposition 5 is an interesting result, suggesting a way to restore the median-voter theorem in parliamentary systems with proportional representation. However, it may not be easy to fully restore the theorem. Given $q \in [0, 1]$, the condition $\sigma \geq \frac{17}{32}$ stated in Proposition 5-(4) could be too stringent to meet in the real world.

5 Concluding remarks

Propositions 1-5 show that equilibrium outcomes vary substantially across $q$ and $\sigma$. Thus, a major message coming out of our paper is that outcomes in parliamentary democracies are sensitive to outside options as well as to the status quo. In explaining the formation of oversized coalition in parliamentary democracies, Volden and Carrubba (2004) fail to find evidence in support of the role of the status quo as suggested by BD. This finding may not be surprising in light of our model since, without controlling for outside options, it will be difficult to identify the impact of the status quo.

What are the nonpolicy benefits for members of parliament if their belonged parties are not in government? BD assume none, which makes no distinction between voters and members of parliament whose parties are not in government. Our introduction of outside options allows for a distinction. We incorporate this distinction into the BD model and explore its implications for parliamentary politics. As a first step, our analysis is admittedly highly preliminary and, in
particular, it is confined to common outside options, leaving out idiosyncratic outside options. In view of this, our derived results should be viewed with caution. Still, it is hoped that our accounting for both the status quo and the outside option may well serve as a stepping-stone for further study on parliamentary politics.

References


Appendix

Proof of Lemma 7.

Suppose that party 1 wins a majority of seats in parliament and forms a single-party government. Based on Lemma 4, it proposes $z_1$ if $q \in \left[ \frac{5}{8} - \frac{1}{8}\sqrt{3 + 64\sigma}, \frac{1}{2} + \frac{1}{8}\sqrt{2 + 64\sigma} \right]$ when $\sigma \leq \frac{1}{64}$, and if $q \in \left[ \frac{3}{4} - \frac{1}{4}\sqrt{2 + 16\sigma}, \frac{1}{2} + \frac{1}{8}\sqrt{2 + 64\sigma} \right]$ when $\sigma > \frac{1}{64}$. 
Without loss, consider a situation where there are exactly \( N = 2 + 1 \) voters cast their votes for party 1. Let voter \( h \) with ideal point \( z(h) \) be the pivotal voter, i.e., his vote can determine whether or not party 1 wins a majority of votes. If he votes for party 1, his utility will be \(-\left(\frac{1}{4} - z(h)\right)^2\). If he instead votes for party 2 or party 3, such a deviation would lead to a minority parliament. Depending on the location of \( q \) and the value of \( \sigma \), the resulting outcomes vary.

For example, consider that \( \sigma \leq \frac{1}{64} \) and \( q \in \left[\frac{5}{8} - \frac{1}{8}\sqrt{3 + 64\sigma}, \min\left\{\frac{3}{4} - \frac{1}{8}\sqrt{6 + 64\sigma}, \frac{1}{2} - \sqrt{\sigma}\right\}\right] \). Then in a minority parliament, there are three possibilities: (i) party 1 forms a consensus government with policy \( \bar{z} \) according to Lemmas 1a-(1) and 1b-(1), (ii) party 2 seeks party 3 as a coalition partner and forms a MWC government with policy \( x_{23} = \frac{5}{8} \) according to Lemma 3-(1), and (iii) party 3 seeks party 2 as a coalition partner and forms a MWC government with policy \( x_{32} = \frac{5}{8} \) according to Lemmas 2a-(2) and 2b-(3). Once a deviation occurs, party 1 obtains exactly half of the votes, so that from (i)-(iii) the outcome would be an even lottery over \( \frac{5}{8} \) and \( \frac{1}{2} \), in which case voter \( h \) obtains a utility \(-\frac{1}{2}(\frac{1}{2} - z(h))^2 - \frac{1}{2}(\frac{5}{8} - z(h))^2\).

Thus, voter \( h \) votes for party 1 if and only if \(-\left(\frac{1}{4} - z(h)\right)^2 \geq -\frac{1}{2}(\frac{1}{2} - z(h))^2 - \frac{1}{2}(\frac{5}{8} - z(h))^2\), or equivalently
\[
z(h) \leq \frac{33}{80}. \tag{A.1}
\]
Since voters distribute uniformly on \([0, 1]\) by assumption, (A.1) implies that there are more than half of the voters who can be better off if they deviate. As a result, a parliament where party 1 wins a majority of seats and proposes policy \( z_1 \) is not robust to a deviations by a group of voters in the regime considered. In other words, it cannot be sustained as a strong Nash equilibrium. The cases in the other regimes can be checked in a similar way.

The same logic applies to party 3.

**Proof of Proposition 1.**

(1) \( q \in \left[0, \frac{5}{8} - \frac{1}{8}\sqrt{3 + 64\sigma}\right] \)

**Step 1:** Government formation
(i) Since $\frac{5}{8} - \frac{1}{8}\sqrt{3 + 64\sigma} < \frac{3}{4} - \frac{1}{8}\sqrt{6 + 64\sigma}$ for any $\sigma \geq 0$, according to Lemmas 1a-(1) and 4, party 1 as formateur will form a consensus government with policy $\bar{z}$ both in a minority and a majority parliament.

(ii) According to Lemmas 3-(1) and 6, party 2 as formateur will seek party 3 as a coalition partner and form a MWC government with policy $x_{23}$ both in a minority and majority parliament.

(iii) According to Lemma 2a-(2), since $\frac{5}{8} - \frac{1}{8}\sqrt{3 + 64\sigma} < \frac{1}{2} < \frac{1}{4} + \frac{1}{8}\sqrt{6 + 64\sigma}$ for any $\sigma \geq 0$, party 3 as formateur will seek party 2 as a coalition partner and form a MWC government with policy $x_{32}$ in a minority parliament; according to Lemma 5, since $\frac{1}{2} - \frac{1}{8}\sqrt{2 + 64\sigma} < \frac{5}{8} - \frac{1}{8}\sqrt{3 + 64\sigma}$ for any $\sigma \geq 0$, in a majority parliament, party 3 as formateur will seek party 2 as a coalition partner and form a MWC government with policy $x_{32}$ when $q \in [0, \frac{1}{2} - \frac{1}{8}\sqrt{2 + 64\sigma}]$, and proposes its ideal policy when $q \in [\frac{1}{2} - \frac{1}{8}\sqrt{2 + 64\sigma}, \frac{5}{8} - \frac{1}{8}\sqrt{3 + 64\sigma}]$.

Step 2: Existence of a strong Nash equilibrium

We can construct a Nash equilibrium in which party 1 obtains a majority of votes. It is sufficient to consider the following situation: there are exactly $\frac{N}{2} + 1$ people vote for party 1, and the rest vote for party 2 or party 3. Let voter $h$ with ideal point $z(h)$ be the pivotal voter. Then his utility is $-(\frac{1}{2} - z(h))^2$ if he votes for party 1. If he instead votes for party 2 or party 3, a minority parliament would result. Based on the results in Step 1, as a formateur, party 1 proposes $\frac{1}{2}$, and both parties 2 and 3 propose $\frac{5}{8}$. Since party 1 obtains exactly half of the seats after the deviation, the resulting policy outcome would be an even lottery over $\frac{5}{8}$ and $\frac{1}{2}$. Voter $h$ has no incentives to deviate and votes for party 1 if and only if $-(\frac{1}{2} - z(h))^2 \geq -(\frac{5}{8} - z(h))^2$, or equivalently

$$z(h) \leq \frac{9}{16}. \quad (A.2)$$

Since we assume that voters distribute uniformly on $[0, 1]$, (A.2) implies that there are a majority of voters who are willing to vote for party 1. This is also a strong Nash equilibrium.
because the size of those who want to deviate are not large enough to change the outcome and so it is robust to deviations by groups of voters.

**Step 3: Uniqueness**

We show that the policy $\bar{z} = \frac{1}{2}$ is the unique strong Nash equilibrium outcome. Within this regime, the other possible outcomes are: (i) a lottery over $x_{23}$ and $\bar{z}$ with probability $\rho$ and $1-\rho$ ($\rho \geq \frac{1}{2}$), in which case either a minority parliament results and the total seats of parties 2 and 3 in the parliament is equal to $\rho$, or party 2 wins a majority of seats ($\rho = 1$); and (ii) $z_3$, in which case party 3 wins a majority of votes. Voter $h$ obtains utility $-\rho(\frac{1}{2} - z(h))^2 - (1-\rho)(\frac{5}{8} - z(h))^2$, and $-(\frac{3}{4} - z(h))^2$, respectively. One can check that as long as

$$\frac{5}{16} \leq z(h) \leq \frac{9}{16}, \quad (A.3)$$

voter $h$ has no incentives to deviate from $\frac{1}{2}$. (A.3) implies that there are a majority of voters who prefer $\frac{1}{2}$ to the other possible outcomes. Thus, $\frac{1}{2}$ is the unique strong Nash equilibrium outcome because the size of those who want to deviate is not large enough to change the outcome and so it is robust to deviations by groups of voters.

(2) $q \in \left[\frac{5}{8} - \frac{1}{8}\sqrt{3 + 64\sigma}, \frac{3}{4} - \frac{1}{8}\sqrt{6 + 64\sigma}\right]$:

**Step 1: Government formation**

In this regime, party 1 proposes its ideal policy if it obtains a majority of seats according to Lemma 4, and forms a consensus government in a minority parliament according to Lemma 1a-(1). Party 2 seeks party 3 as a coalition partner and forms a MWC government both in a minority and a majority parliament according to Lemmas 3-(1) and 6. Party 3 seeks party 2 as a coalition partner and forms a MWC government in a minority parliament according to Lemma 2a-(2), and proposes its ideal policy in a majority parliament according to Lemma 5.

**Step 2: Existence of a strong Nash equilibrium**
If party 1 or party 3 wins a majority of seats in parliament, it proposes its ideal policy. However, according to Lemma 7, this case cannot be sustained as an equilibrium. A minority government must form in equilibrium.\footnote{If party 2 wins a majority of seats, it forms a MWC government with either party 1 or 3. This case cannot be sustained as a strong Nash equilibrium. See the following argument.}

We can construct an equilibrium where there are exactly $N/2$ voters vote for party 1. That is, the equilibrium policy is an even lottery over $\bar{z}$ and $x_{23}$. Consider a pivotal voter $h$. Suppose $h$ votes for party 1 in equilibrium, from which he obtains $-\frac{1}{2}(\frac{1}{2} - z(h))^2 - \frac{1}{2}(\frac{5}{8} - z(h))^2$. If he instead votes for party 2 or 3, then party 2 and 3 together wins a majority of votes, and so the policy outcome will be $x_{23}$ and his utility becomes $-(\frac{5}{8} - z(h))^2$. However, voter $h$ has no incentives to deviate if (A.2) holds.

On the other hand, consider a voter who votes for party 2 or 3. Again, he obtains $-\frac{1}{2}(\frac{1}{2} - z(h))^2 - \frac{1}{2}(\frac{5}{8} - z(h))^2$. If he instead votes for party 1, party 1 will win a majority of votes and thus will implement its ideal policy $\frac{1}{4}$. However, this deviation is not beneficial if $-\frac{1}{2}(\frac{1}{2} - z(h))^2 - \frac{1}{2}(\frac{5}{8} - z(h))^2 \geq -(\frac{1}{4} - z(h))^2$, or equivalently

$$z(h) \geq \frac{33}{80}. \quad (A.4)$$

That is, there are more than half of the voters who have no incentives to deviate. Clearly, $z(h) = \frac{1}{2}$ satisfies both (A.2) and (A.4). Thus, every voter to the left of $\frac{1}{2}$ votes for party 1 and every voter to the right $\frac{1}{2}$ votes for either party 2 or party 3, and each group comprises a half of the entire population.

**Step 3: Uniqueness**

Within this regime, the possible outcomes include: a lottery over $x_{23}$ and $\bar{z}$ with probability $\rho$ and $1 - \rho$, $\rho > \frac{1}{2}$, in which case either a minority parliament results and the total seats of parties 2 and 3 in the parliament is equal to $\rho$, or party 2 wins a majority of seats; $z_1$, in which case party 1 wins a majority of votes; and $z_3$, in which case where party 3 wins a
majority of votes. Voter \( h \) obtains utility 
\[-\rho\left(\frac{5}{8} - z(h)\right)^2 - (1 - \rho)\left(\frac{1}{2} - z(h)\right)^2, -\left(\frac{1}{4} - z(h)\right)^2,\]
and 
\[-\left(\frac{3}{4} - z(h)\right)^2,\]
respectively. One can check an even lottery over \( x_{23} \) and \( \bar{z} \) is still better than the above ones as long as
\[
\frac{33}{80} \leq z(h) \leq \frac{9}{16}. \tag{A.5}
\]
That is, a majority of voters have no incentives to deviate from this equilibrium, which means that it is the unique strong Nash equilibrium outcome.

\[
(3) \quad q \in \left[\frac{3}{4} - \frac{1}{5}\sqrt{6 + 64\sigma}, \frac{1}{2}\right]:
\]

**Step 1:** Government formation

In a minority parliament, party 1 seeks party 2 as a coalition partner and forms MWC government with policy \( x_{12} \) according to Lemma 1a-(2), and party 2 (3) seeks 3 (2) as a coalition partner according to Lemma 3-(1) (Lemma 2a-(2)). In a majority parliament, party 2 behaves the same, while party 1 (3) proposes its ideal policy.

**Step 2:** Existence of a strong Nash equilibrium

As noted in Step 1, if party 1 or party 3 wins a majority of seats in parliament, it proposes its ideal policy. However, according to Lemma 7, this case cannot be sustained as an equilibrium. Again, a minority parliament must form in this regime.

We can construct an equilibrium where there are exactly \( N/2 \) voters vote for party 1, and the other \( N/2 \) voters vote or either party 2 or 3. This leads to an even lottery over \( \frac{3}{8} \) and \( \frac{5}{8} \). Consider a voter \( h \) who votes for party 1 in the equilibrium. Then he obtains
\[-\frac{1}{2}\left(\frac{3}{8} - z(h)\right)^2 - \frac{1}{2}\left(\frac{5}{8} - z(h)\right)^2.\]
If he instead votes for party 2 or 3, then party 2 and 3 together win a majority of seats. The resulting policy outcome will be another lottery over \( \frac{3}{8} \) and \( \frac{5}{8} \), from which he obtains a utility
\[-\rho\left(\frac{5}{8} - z(h)\right)^2 - (1 - \rho)\left(\frac{3}{8} - z(h)\right)^2,\]
where \( \rho > \frac{1}{2} \). However, one can check that voter \( h \) has no incentives to deviate if \( z(h) \leq \frac{1}{2} \).

On the other hand, consider a voter who votes for party 2 or 3 in the equilibrium. Again, he obtains
\[-\frac{1}{2}\left(\frac{3}{8} - z(h)\right)^2 - \frac{1}{2}\left(\frac{5}{8} - z(h)\right)^2\]
in the equilibrium. If he instead deviates to party 1,
party 1 will win a majority of seats and propose \( z_1 \). However, such a deviation is not beneficial if \( z(h) \geq \frac{5}{16} \). Therefore, for this to be sustained as a Nash equilibrium, the following must hold:

\[
\frac{5}{16} \leq z(h) \leq \frac{1}{2}.
\]

(A.6)

Clearly, \( z(h) = \frac{1}{2} \) satisfies (A.6). Thus, there is a voter such that everyone to the left of \( \frac{1}{2} \) votes for party 1 and everyone to the right of \( \frac{1}{2} \) votes for either party 2 or party 3, and each group comprises a half of the entire population.

**Step 3: Uniqueness**

The other possible outcomes in this regime are: a lottery over \( x_{23} \) and \( x_{12} \) with probability \( \rho \) and \( 1 - \rho \), \( \rho > \frac{1}{2} \), in which case either a minority parliament results and the total seats of parties 2 and 3 in the parliament is equal to \( \rho \), or party 2 wins a majority of seats; \( z_1 \), in which case party 1 wins a majority of votes; and \( z_3 \), in which case party 3 wins a majority of votes. One can check an even lottery over \( x_{23} \) and \( x_{12} \) is better than the above ones as long as

\[
\frac{13}{32} \leq z(h) \leq \frac{1}{2}.
\]

(A.7)

That is, a majority of voter have no incentives to deviate from this equilibrium, which means that it is the unique strong Nash equilibrium outcome.

The results for the other half of status quo \( q \in [\frac{1}{2}, 1] \) can be derived in a similar way.