Trading of Bad Reputation and Endogenous Cost of Control*

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Abstract

For a firm to have incentives to produce high quality products, its profit must suffer following failure to maintain high quality. This punishment generates a negative externality because all shareholders, including those with no control rights and thus not responsible for the bad outcomes, are punished. In a dynamic model of an experience-goods firm whose control rights are tradeable, we identify equilibria in which buyers forgive the firm’s bad outcomes as soon as its control rights change hands. Through control-right turnover, the firm’s profit and the values of the noncontrolling shares can be preserved without undermining the incentives of the controlling shareholder. Buyers’ differential treatment of the existing and new owners following poor outcomes gives rise to an equilibrium trade of ownership of firms with damaged reputations. Our analysis identifies an endogenous cost of corporate control and provides a rationale for the separation of ownership and control. We also derive the founder’s optimal firm-ownership structure.

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Since it is costly to produce high-quality goods, sellers need incentives to maintain high-quality production. The high profits and price premiums associated with a good reputation have been considered by economists as effective incentives for sellers to engage in costly quality improvement. The basic idea behind this reputation mechanism, which dates back to Benjamin Klein and Keith Leffler (1981), is that if a firm continues to produce at a high quality level, customers will pay a premium for its products; otherwise, they will punish the firm by either asking for a large discount or not purchasing from the company at least for a period of time. A notable recent contribution based on this reputation mechanism is by Hörner (2002), who shows that competition helps incentivize high-quality production because in a competitive environment, consumers can readily walk away from a firm that has failed to produce high-quality goods.\footnote{See Bar-Isaac and Tadelis (2008) for a comprehensive review of the literature on seller reputation.}

A firm having to be punished following bad performance means that whenever a firm’s reputation is tarnished by a bad outcome, its profit and value must suffer. When a firm’s control for quality is imperfect, punishment is a necessary part of the equilibrium path because, otherwise, the firm’s owner has no incentive to produce high-quality goods in the first place.\footnote{The nature of the punishment is similar to that in Green and Porter (1984).} Such punishment imposes a \textit{negative externality} on diffused shareholders who have no control rights and are not responsible for the firm’s bad outcome. These unfortunate consequences on “innocent” shareholders seems both “unjust” and harmful to the firm’s value. A natural question is whether such externality can be mitigated. The first part of our paper shows that the negative externality of punishment can indeed be mitigated and the firm’s profit can be improved when the turnover of firm ownership is allowed.

The basic idea behind this finding is as follows. Consider a firm that is owned by one controlling shareholder and a continuum of noncontrolling shareholders. Suppose that following a bad outcome, the controlling shareholder sells her block of shares to a new entrepreneur. As long as the sale price is low enough, the incumbent controlling shareholder is sufficiently punished for the incentive to exert effort to be maintained. Since the new owner and the noncontrolling shareholders are not responsible for the bad outcome, once the ownership changes hands, consumers no longer have to punish the firm, allowing the company’s damaged reputation to be repaired through the turnover of ownership.\footnote{This may explain why only two months after Hebei Sanyuan paid USD90 million to purchase most of the production capacity of the Sanlu Group—China’s largest milk power producer, which went bankrupt after the melamine contamination scandal broke in September 2008—the company’s revenue quickly grew in May 2009 to USD10 million, which was equal to the total revenue over the first four months of the year, and the revenue for the year was expected to reach USD176 million. Sources: “Sanyuan to employ all staff of scandal-hit Sanlu by November,” http://en.hnedu.cn/read/7545.html; “Sanyuan reports sales surge after takeover of Sanlu,” http://www.chinadaily.com.cn/bizchina/2009-06/12/content_8279233.htm.}
This paper’s formal analysis considers a dynamic model of an experience-goods firm whose product quality is stochastically determined by both the monetary costs incurred by the firm and the effort exerted by the controlling shareholder.\(^4\) We distinguish between two types of owners, a controlling shareholder and a continuum of noncontrolling shareholders. The main objective of our analysis is to study how the turnover of controlling shareholders impacts the firm’s profit and shareholder values. When the turnover of ownership is not allowed, during the normal phase, small and anonymous consumers pay their reservation value for the monopolist’s experience good. But the firm has to offer a substantial discount following the customers’ bad experience. Punishment for a bad outcome on the equilibrium path prevents the firm from capturing the full surplus from production, which requires the firm to charge customers their reservation value every period.

Next, we allow the ownership of controlling shares to be traded. We identify equilibria in which, following a bad outcome, buyers forgive the firm and continue to pay their reservation value if and only if the firm’s controlling shares change hands. The customers' differential treatment of the existing and new controlling shareholders following bad outcomes gives rise to the equilibrium trading of controlling shares. The ownership turnover of controlling shares raises the firm’s profit and the values of the noncontrolling shares because the firm is spared from the otherwise necessary equilibrium punishment. Contrary to conventional wisdom, the turnaround of a firm with a damaged reputation does not necessarily require the new owner to be better at running the company.\(^5\)

Note that in order to ensure the incumbent controlling shareholder’s incentive to maintain high quality, the equilibrium transaction price following a bad outcome must be low enough for the incumbent owner to be sufficiently punished for the bad outcome. We assume that the incumbent controlling shareholder and the acquirer Nash bargain over the transaction price. If bargaining breaks down, the incumbent owner continues to run the company. To ensure that the Nash-bargained transaction price is sufficiently low, the off-the-equilibrium punishment path following a breakdown in the bargaining must be more severe than the punishment path in a game where turnover is not permitted. In our formal analysis, we show that if the incumbent owner’s bargaining power is too strong, then even with ownership turnover, the firm cannot capture the entire surplus from a trade. However, we also show that this problem is resolved when players' discount factor is sufficiently large.

In the above scenario, turnover raises the values of the noncontrolling shares but the controlling shareholder still has to be punished when the product quality fails. This implies that the total shareholder value can be raised by converting some of the controlling shares into noncontrolling shares. In other words, the founder of the company can benefit by issuing noncontrolling shares after setting up the company. This provides a rationale for the (partial) separation of ownership

\(^4\)The idea is that the firm has to pay its workers a higher salary and use more-expensive materials to improve product quality. The pursuit of high-quality production also relies on the controlling shareholder’s effortful monitoring.

\(^5\)But it will still requires real talent to bring a successful company to the next level.
from control. Note that despite this benefit of issuing noncontrolling shares, the total shareholder value does not monotonically increase in the fraction of noncontrolling shares. As more shares are converted into noncontrolling shares, the controlling shareholder’s incentive to exert effort weakens because he now receives a smaller share of the profit but is required to put forth the same amount of effort to maintain high-quality production. In our framework, the optimal share structure is the outcome of a tradeoff between reducing the externality of punishment and managing the controlling shareholder’s moral-hazard problem.

Our theory generates equilibrium predictions consistent with the empirical findings which show that poor company performance is associated with CEO and/or ownership turnover (see, e.g., Coughlan and Schmidt 1985, Warner, Watts, and Wruck 1988, and Weisbach 1988) and that most successful turnarounds involve the replacement of the CEO and/or a change in the ownership and the board directors (see, e.g., Clapham, Schwenk, and Caldwell 2005, Kanter 2003, and Goodstein and Boeker 1991). The management literature has emphasized the importance of strategic change in turnaround and views executive and ownership turnover as a catalyst for the strategic change. However, several empirical studies have found that strategic change is often not an integral part of turnaround (see, e.g., Hambrick and Schecter 1983 and Robbins and Pearce 1992), suggesting that the new CEO may not have to pursue a different strategy to turn around the company. Kanter (2003) points out that even when both the incumbent and new leaders understand the problem and have a solution, the new leader seems to have an edge over the incumbent leader in implementing the solution. As an example, Kanter (2003) quotes director from the European branch of Gillette group who stated, “I’m absolutely certain there’s not one person in the whole company who for one moment thought that we should do anything other than get out of the trade loading” (p.63) and explains that it still took new CEO Jim Kilts to turn around the company by getting out of trade loading. Our theory goes beyond existing management theories by showing that turnover alone can lead to improved performance, even when the new CEO and the incumbent CEO have identical turnaround strategies and the new CEO is no more capable than the incumbent.

If acquisition occurs because the new owner is able to create more value than the existing owner, one would expect that a higher acquisition premium would result in or would signal better performance following acquisition. However, our theory predicts a negative relationship between the acquisition premium paid by the acquirer and the post-acquisition performance of the company if the variation in acquisition premiums is due to different allocations of bargaining power between the incumbent and new controlling shareholders. Interestingly, the negative relationship predicted by our theory is empirically identified by Krishnan, Hitt, and Park (2007).

One key implication of our theory is that there exists an endogenous cost of corporate control; namely, shareholders with control rights have to be punished for bad outcomes on the equilibrium path but noncontrolling shareholders do not. We set up our model in a way that allows the
private benefit of control, netting the effort cost of monitoring, to be positive so that when the controlling shareholder and noncontrolling shareholders receive the same stream of income per share, the control premium is positive. In the equilibria with a turnover of the ownership of controlling shares, however, there is an endogenous cost of control. The reason is that following a bad outcome, the controlling shareholder has to sell her shares at a low price. Since she has been punished, the firm can continue to sell the product at the customers' reservation value. Therefore, the noncontrolling shareholders can earn the entire stream of high profits while the controlling shareholder cannot, thus creating a wedge between the income streams received by the controlling shareholder and the noncontrolling shareholders. When the net private benefit is insufficient to overcome the endogenous cost of control, the control premium may become negative.

Our model’s ability to account for a negative control premium is interesting because conventional wisdom suggests that shares with more control rights are valued (weakly) higher than shares with less or no control rights. Yet examples of negative control premiums have been documented empirically. Moreover, our theory’s specific prediction that the control premium is lower and more likely to be negative during downturns is also consistent with the empirical finding that negative control premiums are more frequently observed at financially distressed companies.

Our theory of turnover is related to the literature studying the trading of firms’ good reputations pioneered by Tadelis (1999, 2002, 2003) and Mailath and Samuelson (2001). According to their theories, when the ownership of a firm is unobservable and a firm’s reputation is determined by its past performance, new entrants can buy firms with good reputations that have to exit for exogenous reasons, giving value to company names. The benefit of selling the firm at a high price also motivates finitely lived firms to maintain their high performance. One limitation of these theories, as pointed out by Deb (2007), is that trademarks have value only when their transactions are unobservable by consumers. Our theory of firm-ownership turnover is significantly different from these theories in many ways. The above mentioned theories are predominantly ones of adverse selection, while ours is purely one of moral hazard; firm owners in our theory do not have types. The above theories predict that firm-ownership transactions take place following good outcomes, there is no improvement of performance following a transaction, and the transactions create value only if they are unobservable by customers. In sharp contrast to these theories, we predict that firm-ownership transactions take place following bad outcomes, performance is expected to improve following ownership change, and these transactions create value only if they are observable by customers. This suggests that our theory complements these theories in an important way. Höfler and Sliwka (2003) point out that replacing the existing manager with one who is less informed

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6See, e.g., Dyck and Zingales (2004), Lease, McConnell, and Mikkelsen (1983), Pinegar and Ravichandran (2003), Chen (2004), Kruse, Kyono, and Suzuki (2006), and Valero, Gomez, and Reyes (2008), which are further discussed in Section 3.3.


8Moral hazard can be introduced into these theories but adverse selection is necessary.
of the workers’ abilities has the short-run benefit (in second period of their model) of motivating the workers to work harder. In contrast, we study the long-run benefit of ownership turnover in mitigating the externality of on-the-equilibrium path punishment. Höffler and Sliwka (2003) do not analyze the long-run effect on effort, but the long-run effect would be unclear because workers anticipate such management turnover (in the first period).

In the existing literature on repeated games, the possibility that players can break away from a relationship and form a new one is often viewed as a threat to the sustainability of the relationship. Several studies have focused on how to mitigate the effect of this threat. See, for example, Kandori (1992), Ghosh and Ray (1996), and Kranton (1996). By contrast, we focus on how allowing the non-deviator (the customers and noncontrolling shareholders in our framework) to form a new relationship (with a new controlling shareholder) reduces the inefficiency of punishment.9

The paper that is closest to ours in spirit is by Fong and Li (2009). While Fong and Li (2009) also explore efficiency gain through player turnover, their theory focuses on identifying the optimal rules of turnover in different environments.

The rest of the paper is organized as follows. We set up the model in Section 2. We analyze the model in Section 3. Various generalizations are discussed in Section 4. Section 5 concludes.

2 Model

Players Time is discrete and infinite, \( t = 1, 2, \ldots \). There are three kinds of players in the game: customers, entrepreneurs, and investors. All players share the same discount factor, \( \delta \in (0, 1) \), across periods. There is a continuum of anonymous customers of measure one. The market is served by a monopoly firm possessing a technology of producing experience goods, i.e., goods of which the quality cannot be observed at the time of purchase. While there is only one firm, the firm is owned by one entrepreneur who has full control rights over the firm’s business decisions and a continuum of investors who own the company’s shares but have no control rights. We call the entrepreneur with control rights the \textit{controlling shareholder} and the other investors the \textit{noncontrolling shareholders}. Suppose the controlling shareholder owns a fraction, \( \theta \), of the firm’s shares and the remaining fraction, \( (1 - \theta) \), is owned by the noncontrolling shareholders. We assume that direct transfers between controlling and noncontrolling shareholders are not feasible.10 For

9 Also notice that key to the other studies’ results—which suggest that the outside opportunity to form a new relationship threatens the existing relationship—is the assumption that new relationships should be started the same way regardless of how the previous relationship ends. If we remove this restriction, then opportunities to form new relationships does not necessarily threaten the existing one. This is because there exists an equilibrium in which, apart from the first relationship, all relationships result in the worst possible equilibrium outcome. The opportunity to form a new relationship then becomes irrelevant.

10 We discuss how allowing such transfers to occur and allowing the controlling shareholder to publicly burn money impact the analysis in Section 4.
now, treat \( \theta \) as exogenous but in Section 4 we will endogenize \( \theta \) by considering it as optimally chosen by the founder of the company. The share structure \( \theta \) and the identities of the shareholders are perfectly observable to all players in the game.

Production Technology
In every period, \( t \), the production technology may yield two possible outcomes, \( y_t \in \{0,1\} \), with each outcome representing the utility received by customers upon consumption. The realization of the outcome is publicly observable and perfectly correlated among customers consuming the goods in period \( t \). The probability of each outcome depends on both the monetary production cost the firm incurs, \( c_t \in \{c^L,c^H\} \), and the controlling shareholder’s effort choice in monitoring and managing, \( e_t \in \{e^H,e^L\} \), and we assume

\[
1 > \Pr(y_t = 1|e_t = e^H \land c_t = c^H) \equiv p > q \equiv \Pr(y_t = 1|e_t \neq e^H \lor c_t \neq c^H) > 0.
\]

While \( c_t \) is born by all shareholders, both \( c_t \) and \( e_t \) are chosen by the controlling shareholder. Both \( c_t \) and \( e_t \) are unobservable by consumers. Since \( p < 1 \) and \( q > 0 \), this is a game of imperfect public monitoring. Effort and monetary costs are perfect complements in the sense that both have to be high to result in a high likelihood of a good outcome; neither \( e^H \) nor \( c^H \) alone will result in high likelihood of good outcome. When \( e_t = e^H \) and \( c_t = c^H \), we say the firm engage in high-quality production, even though doing so does not guarantee high quality; otherwise, we say it engages in low-quality production. We assume that quality improvement is socially efficient:

\[
e^H + c^H - (e^L + c^L) < p - q.
\]

The interpretation of the production technology is that quality improvement requires purchasing expensive production inputs and providing incentives for workers (who are not explicitly modelled here). To implement high-quality production, it is also necessary for the controlling shareholder to engage in effortful management and monitoring. The assumption of perfect complementarity is made for simplicity.

Payoffs
Denote the price the firm charges by \( P_t \). Denote the (normalized) values of each unit of controlling shares and noncontrolling shares in period \( t \) by \( V_t \) and \( U_t \), respectively. Each consumer receives an instantaneous payoff of

\[
Pr(y_t = 1) - P_t.
\]

The controlling shareholder receives fraction \( \theta \) of the firm’s profit and incurs effort cost \( e_t \). We assume she also receives an exogenous private benefit of control, \( B \). Our assumption that the private benefit is independent of \( \theta \) is in line with the model in Zingales (1995). This results in a total payoff of

\[
\theta V_t = B - e_t + \theta (P_t - c_t).
\]
Noncontrolling shareholders simply receive fraction \((1 - \theta)\) of the firm’s profit:

\[
(1 - \theta) U_t = (1 - \theta) (P_t - c_t).
\]

We assume that \(B - c^L > B - c^H > 0\) so that when the controlling shareholder and noncontrolling shareholders receive the same stream of income per share, the net benefit of controlling the company is positive regardless of the controlling shareholder’s effort.\(^{11}\) Finally, we assume that a noncontrolling shareholder always earns a positive profit, i.e., \(p - c^H > q - c^L \geq 0\), which implies that the controlling shareholder also always earns a positive profit:

\[
B - c^L + \theta (q - c^L) > 0.
\]

**Turnover of Controlling Shareholder** An important element of our analysis is that every period an entrepreneur arrives and may purchase the entire block of shares from the incumbent controlling shareholder. If acquisition does not take place in a period, the potential acquirer exits forever. When acquisition takes place, it is publicly observable. However, the actual transfer price can neither be publicly observed nor credibly disclosed by the transacting parties.\(^{12}\) We assume that the transaction price is determined by Nash bargaining and we denote the incumbent’s bargaining power by \(\beta \in (0, 1)\).\(^{13}\)

\(^{11}\)It will be clear that in the context of our model, these two types of shareholders will receive the same stream of income when ownership turnover is not allowed or when consumers treat the incumbent controlling shareholder and new controlling shareholder symmetrically. However, when ownership turnover is allowed, they may not receive the same stream of income.

\(^{12}\)If the transaction price of the controlling shares is observable, then a low equilibrium transaction price can be easily enforced by consumers’ belief that the new owner will engage in high quality production if and only if the transaction price is sufficiently low.

\(^{13}\)It is quite natural to assume that the incumbent’s bargaining power is less than 1. Zingales (1995) also makes a similar assumption.
Timeline The following figure illustrates the timeline within each period:

![Timeline Figure]

Finally, we assume that if there are transfers between the controlling shareholder and noncontrolling shareholders, such transfers can neither be publicly observed nor credibly disclosed by the transacting parties. If any transfer between the controlling shareholder and any noncontrolling shareholder is creating any value, we assume that the value will be fully captured by the controlling shareholder. In other words, the controlling shareholder has 100% bargaining power over the noncontrolling shareholders.

3 Analysis

The main objective of our analysis is to characterize the optimal relational contract of the game. We define the optimal relational contract as the perfect public equilibrium (PPE) that maximizes the total (normalized) shareholder value:

\[ S = \theta V + (1 - \theta) U. \]

It will become clear in Section 3.3 that \( S \) is also the value of the company to the founder if she can sell noncontrolling shares to perfectly competitive investors. Note that the total shareholder value is bounded from above by \( \overline{S} := B + p - (e^H + c^H) \), which is achieved when the firm engages in high-quality production every period and consumers pay \( p \) every period. The lower bound of the total shareholder value is \( \underline{S} := B + q - (e^L + c^L) \), which is achieved when the firm engages in low-quality production every period and consumers pay \( q \) every period. We are particularly interested in the condition under which the firm can achieve the highest possible total shareholder value.
Before proceeding with our characterization, we consider the benchmark in which the transfer of the controlling shares is not allowed.

3.1 Benchmark Case: No Transfer of Control Rights

In this subsection, we consider the case in which there is only one player who can be the controlling shareholder, i.e., the firm’s control rights cannot be transferred. The purpose of this section is to show that any equilibrium in which the firm engages in high-quality production necessarily entails the destruction of the firm’s profit. We show that when ownership turnover is not allowed, the optimal relational contract yields a total shareholder value strictly less than the theoretical upper bound, i.e. $S < \bar{S}$, the firm profit (which is also the value per noncontrolling share) is strictly less than $p - c^H$, and the value of each controlling share is strictly less than $p - c^H + (B - e^H) / \theta$.

Recall that $V$ and $U$ are the per-unit market values of the controlling and noncontrolling shares, respectively. When the controlling shares cannot be traded, both the controlling and noncontrolling shareholders receive the present discounted value of the firm’s profit stream and the values of the two classes of shares differ only due to the private benefits and effort costs:

$$V = U + \frac{B - e^H}{\theta} > U.$$  

In other words, there is a positive control premium of $(B - e^H) / \theta$.

Let $W$ be the (normalized) market value per controlling share following a bad outcome. If $W$ is strictly less than $V$, it is attained by the firm lowering the price until the outcome turns good. Notice that every time the controlling shareholder is punished, the noncontrolling shareholders are punished to the same extent on a per-share basis. The controlling shareholder has the option of perpetually engaging in low-quality production and selling the product at $P_t = q$. When the controlling shareholder exercises this option, each noncontrolling share receives a payoff of $q - c^L$ and each controlling share receives $(B - e^L) / \theta + (q - c^L) > 0$. Note that our assumption that each individual consumer has zero measure and is anonymous implies that the firm may not charge higher than the expected value of the product, i.e., $P_t \leq p$. We do not impose a lower bound on $P_t$ in the formal analysis.

The value of each controlling share, $V$, is given by

$$\theta V = (1 - \delta) \left[ B - e^H + \theta (P - c^H) \right] + \delta (p \theta V + (1 - p) \theta W).$$

Since $U$ and $V$ differ only by a constant, both $U$ and $V$ increase in $W$. To induce effort, the following incentive constraint is needed:

$$\theta V \geq (1 - \delta) \left( B - e^L + \theta (P - c^L) \right) + \delta (q \theta V + (1 - q) \theta W).$$ (1)
Combining the two to eliminate $V$, we obtain
\[
\frac{(1 - \delta) \left[ B - e^H + \theta \left( P - e^H \right) \right] + \delta (1 - p) \theta W}{\theta (1 - \delta p)} \geq \frac{(1 - \delta) \left( B - e^L + \theta \left( P - e^L \right) \right) + \delta (1 - q) \theta W}{\theta (1 - \delta q)}.
\]
(2)

One immediate observation is that since $1 / (1 - \delta p) > 1 / (1 - \delta q)$, for any given $W$, the incentive constraint is easier to satisfy with a higher $P$. Setting a higher $P$ also raises both $U$ and $V$. Therefore, in the optimal relational contract, $P = p$.

For the analysis to be nontrivial, it is necessary that the moral hazard problem is not too severe. Specifically, we need
\[
e^H - e^L + (e^H - e^L) \leq \frac{(p - q)^2}{(1 - q)}.
\]
(3)

We will adopt this assumption throughout the paper. Since $(p - q)^2 / (1 - q) < (p - q)$, (3) implies our earlier assumption that quality improvement is socially efficient. In fact, it means that the efficiency gain from quality improvement must be large enough for high-quality production to be sustainable.

The proposition below states the result of this section formally.

**Proposition 0** Let $\tilde{V}^0 (\theta)$ be the maximum equilibrium value per controlling share, $\tilde{U}^0 (\theta)$ be the maximum equilibrium value per noncontrolling share, and $\tilde{S}^0 (\theta)$ be the maximum equilibrium total shareholder value. Suppose (3) holds and
\[
\theta > \theta \equiv \frac{(1 - q) (e^H - e^L)}{(p - q)^2 - (1 - q) (e^H - e^L)}.
\]

Then, if
\[
\delta \geq \hat{\delta} (\theta) \equiv \frac{e^H - e^L + \theta (e^H - e^L)}{q (e^H - e^L + \theta (e^H - e^L)) + \theta (p - q)^2},
\]
then
\[
\tilde{V}^0 (\theta) = \frac{B - e^H}{\theta} + (p - e^H) - \frac{1 - p}{p - q} \left( \frac{e^H - e^L}{\theta} + e^H - e^L \right),
\]
\[
\tilde{U}^0 (\theta) = (p - c^H) - \frac{1 - p}{p - q} \left( \frac{e^H - e^L}{\theta} + e^H - e^L \right),
\]
\[
\tilde{S}^0 (\theta) = B + p - (e^H + c^H) - \frac{1 - p}{p - q} \left( \frac{e^H - e^L}{\theta} + e^H - e^L \right);
\]
and if $\delta < \hat{\delta}(\theta)$, then

\[
\begin{align*}
V^0(\theta) &= \frac{B - e^L}{\theta} + q - c^L, \\
U^0(\theta) &= q - c^L, \\
S^0(\theta) &= B + q - (e^L + c^L).
\end{align*}
\]

Clearly, when the discount factor is too low, i.e., when $\delta < \hat{\delta}(\theta)$, high-quality production will not be sustainable. Proposition 0 points out that even when the discount factor is high enough, i.e., when $\delta \geq \hat{\delta}(\theta)$, the monopolist is still unable to charge consumers the expected value of its product every period. This is due to the fact that the firm can only charge consumers their reservation value $p$ during the normal phase; whenever the firm has produced at the low quality, which happens with a positive probability, it has to offer consumers a discount even if they continue to produce at high quality.$^{14}$ This loss in profit is similar in nature to the loss in profits of collusive firms under imperfect monitoring identified by Green and Porter (1984). Focusing on the case of $\delta \geq \hat{\delta}(\theta)$, the first term $(p - c^H)$ in $U^0(\theta)$, is the expected accounting profit of the firm if the firm always operates in the absence of an agency problem. Similarly, the first term in $V^0(\theta)$ is the sum of the same expected accounting profit and the net private benefit per share. The second terms in $U^0(\theta)$ and $V^0(\theta)$ are the profits that must be destroyed to provide incentives for the controlling shareholder to improve output quality. Notice that the noncontrolling shareholders suffer the same loss in profits as does the controlling shareholder. Following a bad outcome, the controlling shareholder must be punished or she will have no incentive to exert high effort and incur high monetary costs to increase the chance of producing high-quality goods. However, the punishment imposes a negative externality on the noncontrolling shareholders, who are also punished despite the fact that they are not responsible for the bad outcome. Perhaps more importantly, noncontrolling shareholders do not suffer from a moral hazard so it is wasteful in terms of shareholder value to punish them for a bad outcome.

Another point worth noting is that the severity of the agency problem is related to the share structure of the firm. It can be verified that the cutoff discount factor $\hat{\delta}$ is decreasing in the fraction of controlling shares, $\theta$. Figure 2 depicts how $\hat{\delta}$ changes in $\theta$.

$^{14}$An alternative punishment is that with a certain probability consumers believe that the firm forever engage in low-quality produces in the future.
When the controlling shareholder owns too few shares, i.e., when $\theta \leq \theta_0$, there is no discount factor at which high-quality production is sustainable. Moreover, $\delta(\theta) = \theta V^0(\theta) + (1 - \theta) U^0(\theta)$ is the lowest for $\theta < \theta(\delta)$ and increases in $\theta$ for $\theta \geq \theta(\delta)$. The analysis so far implies that the optimal share structure is to set $\theta = 1$. In other words, all the shares should be owned by the controlling shareholder. Doing so maximizes $\bar{S}(\theta)$ and minimizes $\hat{\delta}$. Note that in the optimal relational contract, $U$ and $V$ are also maximized respectively.

### 3.2 The Effect of Ownership Turnover

We now return to the case in which each period an entrepreneur arrives and may acquire the block of controlling shares.

Consider the following equilibrium, which consists of four phases: a normal phase, an on-the-equilibrium-path punishment phase, and two off-the-equilibrium-path punishment phases.

- The game begins in the normal phase. In the normal phase, the controlling shareholder sets the price at $p$ and engages in high-quality production, i.e., exerts effort $e^H$ and incurs monetary cost $c^H$ on the firm’s behalf. If the outcome is good, there will be no turnover of ownership and the game stays in the normal phase.\[^{15}\] If the outcome is bad, the game switches to the on-the-equilibrium-path punishment phase.

- In the on-the-equilibrium-path punishment phase, the controlling shareholder sells the entire block of controlling shares to a new entrepreneur at the takeover price $T$ through Nash\[^{15}\]

\[^{15}\text{In Section 3.3, when we analyze the company’s control premium, we will discuss an payoff equivalent equilibrium in which turnover also takes place following a good outcome.}\]
bargaining. The firm under the new ownership may or may not have to offer the good at a discounted price. The new controlling shareholder engages in high-quality production in the on-the-equilibrium-path punishment phase. The game switches back to the normal phase if the outcome is good and stays in the on-the-equilibrium-path punishment phase if the outcome is bad.

- If the Nash bargaining breaks down, then the game switches to the first off-the-equilibrium-path punishment phase in which the incumbent controlling shareholder continues to engage in high-quality production and offers a one-period discount to customers for the experience good.

- Any other publicly observable deviations, including a deviation from the above-mentioned punishment phases, will trigger the second off-the-equilibrium-path punishment phase in which the controlling shareholder forever engages in low-quality production and sets price equal to $q$.

In the search of the optimal relational contract, it is without loss of generality to focus on the class of equilibria outlined above. The only two feasible ways to punish the incumbent controlling shareholder for bad outcomes: (i) a price cut to customers (or equivalently coordinating on a certain probability of forever reverting to the low-quality-low-price equilibrium), and (ii) an outright sale of controlling shares to the newly arrived entrepreneur. In what follows, we will show that using (ii) in the punishment phase on the equilibrium path, instead of relying on price cut alone as in the benchmark case, strictly increases the total shareholder value for some parameter values.

Denote by $W$ the value of a controlling share in the first off-the-punishment-path punishment phase, i.e., when ownership is retained by the incumbent controlling shareholder. Denote by $\hat{W}$ the corresponding value when ownership is transferred to the new owner. The transaction price per share, $T$, is given by

$$T = W + \beta(\hat{W} - W). \quad (4)$$

A shortcut to account for the value of a controlling share is to imagine hypothetically that every time a bad outcome arises, the controlling shareholder, instead of realizing the loss of $\theta (V - T)$ by selling her block of shares, realized the loss of $\theta (V - T)$ but then continued to hold on to the

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16 Here, the sale of controlling shares must be outright simply because $\theta$ is assumed to be fixed. If we do not assume $\theta$ is fixed, the optimal relational contract may require only a partial sale of controlling shares to the newly arrived entrepreneur while the remaining controlling shares are sold as non-controlling shares to outside investors. However, the optimal relational contract always requires an outright sale of controlling shares when $\theta$ is chosen optimally by the founder of the company, the case we analyze in Section 3.3.

17 We will discuss what happens if we modify the model to allow the controlling shareholder to either burn money or make transfers to the noncontrolling shareholders in section 4.
controlling shares. With this interpretation, the value per controlling share can be expressed as

$$V = \frac{B - e^H}{\theta} + (p - c^H) - \delta (1 - p) \frac{V - T}{1 - \delta}. \quad (5)$$

To account for the value of a noncontrolling share, notice that the company’s profit per share loses the amount $V - \hat{W}$ every time a bad outcome is realized and ownership subsequently changes hands. Both the new controlling shareholder and the noncontrolling shareholders suffer the same loss. Therefore,

$$U = (p - c^H) - \delta (1 - p) \frac{V - \hat{W}}{1 - \delta}. \quad (6)$$

We show in the following proposition that by allowing the turnover of the controlling shares, the value of the noncontrolling shares can be increased and the highest possible value of the noncontrolling shares, $p - c^H$, can be attained if the discount factor is large enough. Recall we focus on the optimal relational contract, i.e., the equilibrium that maximizes the total shareholder value, $\theta V + (1 - \theta) U$. Let $\bar{U}$ and $\bar{V}$ be the values of noncontrolling shares and the controlling shares in the optimal relational contract, respectively.

**Proposition 1** Suppose (3) holds, $\theta > \bar{\theta}$, and $\beta \in (0, 1)$. For each $\beta$, there exists $\hat{\delta} (\beta, \theta) \in (\hat{\delta} (\theta), 1)$ such that

(i) if $\delta \in [0, \hat{\delta} (\theta))$, then

$$\bar{U} = q - c^L \text{ and } \bar{V} = \bar{U} + (B - e^L) / \theta;$$

(ii) if $\delta = \hat{\delta} (\theta)$, then

$$\bar{U} = \bar{U}^0 (\theta) \text{ and } \bar{V} = \bar{V}^0 (\theta);$$

(iii) if $\delta \in (\hat{\delta} (\theta), \hat{\delta} (\beta, \theta))$, then

$$\bar{U} = (p - c^H) - \delta \frac{1 - p}{1 - \delta} \left\{ \frac{c^L + c^H - c^L}{p - q} \left[ \beta^{-1} (\delta - 1 - q) - (1 - q) \right] - (p - q) \left( \beta^{-1} - 1 \right) \right\} \in (\bar{U}^0 (\theta), p - c^H)$$

and

$$\bar{V} = \bar{V}^0 (\theta);$$

(iv) if $\delta \in [\hat{\delta} (\beta, \theta), 1)$, then

$$\bar{U} = p - c^H \text{ and } \bar{V} = \bar{V}^0 (\theta).$$

According to Part (i) of Proposition 1, if high-quality production is not sustainable in the absence of ownership turnover, then ownership turnover cannot increase firm profits. This is because
ownership turnover cannot change the fact that the worst possible punishment payoff to the controlling shareholder is \( B - e^L + \theta (q - c^L) \) and that such punishment is not enough to incentivize her. However, Parts (ii)-(iv) of the proposition suggest that as long as high-quality production is sustainable in the original game without turnover, then turnover can improve the noncontrolling shareholders’ value and such improvement is increasing in \( \delta \).\(^{18}\) When \( \delta \) is sufficiently high, or more specifically when \( \delta \geq \tilde{\delta} (\beta, \theta) \), noncontrolling shareholders can gain a full surplus. Figure 2 depicts what cutoff \( \tilde{\delta} (\beta, \theta) \) looks like.

![Figure 3](image_url)

Although the controlling shareholder’s equilibrium payoff remains unchanged, and in both cases she earns less than \( B - e^H + \theta (p - c^H) \), there is a notable difference in the way she earns that payoff. When the turnover of control rights is not allowed, the controlling shareholder earns the net private benefit and her share of the firm’s stream of profits, which is less than \( p - c^H \) per period, because the firm has to offer a price discount to customers in the period following a bad outcome. With a turnover of control rights, although the firm’s profit is \( p - c^H \) each period, the controlling shareholder does not capture the entire stream of profits because, once a bad outcome is observed, she is required to sell her controlling shares at the discounted price of \( \tilde{W} (\delta, \theta) \) per share.

As we pointed out in the introduction, the equilibrium predictions of our theory are consistent with the empirical findings that poor company performance is associated with CEO/ownership turnover and that most successful turnarounds involve the replacement of the CEO and/or a change

\(^{18}\)In Section 4, we discuss how introducing private monitoring into the model can allow turnover to both increase firm profits and improve production efficiency. Fong and Li (09) analyze how the efficiency is affected by the protocol of turnover in general games.
in the ownership and the board directors. Our model is also able to account for the empirical studies that have found that strategic change is often not an integral part of turnaround. Another noteworthy feature of the equilibrium, distinct from existing management theories, is that while the owners’ abilities of running the firm are identical, we see an improvement in the firm’s performance following an ownership turnover.\textsuperscript{19}

Next, we discuss the model’s implication on the relationship between the acquisition premium and the post-acquisition performance of the company. Acquisition premiums involved in mergers and acquisitions are often sizeable, so those involved in these activities are naturally interested in whether a higher acquisition premium is associated with a better post-acquisition performance. Traditional management theories suggest a positive relationship between the acquisition premium and the post-acquisition performance. This is because a manager who is more capable or has identified a higher valuable in the target is willing to pay a higher acquisition premium and the company is also expected to perform better. However, this view cannot account for the negative relationships between the acquisition premium and the post-acquisition performance identified in some empirical studies (see, e.g., Sirower, 1994 and Krishnan, Hitt, and Park, 2007).

In our analysis here, we focus on the parameter range in which case (iii) of Proposition 1 is satisfied. Holding fixed the other parameters, we study how changes in the bargaining power of the incumbent, $\beta$, affects the transfer price and the firm’s profit margin immediately after the acquisition.

In the equilibrium constructed in the previous section, following a bad outcome of production, the block of controlling shares is sold to a new owner at a (per-share) price of $T$ through Nash bargaining. If $\delta \in (\hat{\delta}(\theta), \tilde{\delta}(\beta, \theta))$, the firm has to offer a discount to customers after the new owner takes control. From Proposition 1, the discount is given by $\left(\bar{V}^0(\theta) - \bar{W}\right) / (1 - \delta)$. Thus, the value of a noncontrolling share following a bad outcome, denoted by $\bar{U}$, is below its value during the normal phase, $\bar{U}$:

\[
(p - c^H) - (1 - \delta p) \frac{\bar{V}^0(\theta) - \bar{W}}{1 - \delta} = \bar{U} < \bar{U} = (p - c^H) - \delta (1 - p) \frac{\bar{V}^0(\theta) - \bar{W}}{1 - \delta}.
\]  

(7)

Define the acquisition premium as the (per-share) transaction price of the block of controlling shares minus the value of the noncontrolling shares during a downturn, i.e., $T - \bar{U}$.

\begin{corollary}
An increase in the bargaining power of the incumbent controlling shareholder $\beta$ has the following effect on the firm’s optimal relational contract:
\end{corollary}

\textsuperscript{19}In our model, because a takeover always occurs after a bad outcome, on the equilibrium path, the incumbent owner never runs the company after a bad outcome. This is an improvement over the off-the-equilibrium path on which takeover does not occur. If we introduce some friction during takeover so that ownership does not change hands immediately following a bad outcome, we will see low on-the-equilibrium-path performance following a bad outcome and improvement following the takeover.
(i) The acquisition premium, $T - U$, weakly increases.

(ii) The firm’s accounting profit in the period after the turnover of the controlling shares, given by $p - e^{H - \frac{V^0(\theta) - W}{1 - \delta}}$, weakly decreases.

Moreover, if $\delta > \hat{\delta}(\theta)$, the above relations are strict when $\beta$ is sufficiently large.

Therefore, if the source of variable is different allocations of the bargaining power between the incumbent and new controlling shareholders, then our model predicts a (weakly) negative relationship between the acquisition premium paid by the acquirer and the firm’s post-acquisition performance. One descriptive argument used by the authors who empirically identified this negative relationship is that a high acquisition premium is an indication of bad managerial decision or managerial hubris so the manager also tends to make bad decisions when running the company. Our explanation is different; in our model all managers have the same managerial ability. The negative relationship is a necessary part of the equilibrium to ensure the incumbent controlling shareholder with a higher bargaining power will not be overpaid so that she has the proper incentive to maintain the company’s reputation.

### 3.3 Endogenous Cost of Control and (Partial) Separation of Ownership and Control

In this subsection, we explore the *endogenous cost of control* in the model and analyze how the cost of control and the controlling shareholder’s moral hazard pin down the optimal ownership structure. This cost of control arises because the controlling shareholder must be punished following a bad outcome, while the noncontrolling shareholders either do not have to be punished (for $\delta \geq \tilde{\delta}(\beta, \theta)$), or they are punished less severely when they have to be punished (for $\delta \in (\tilde{\delta}(\theta), \tilde{\delta}(\beta, \theta))$). We will show that because of this cost of control, although the net private benefit per share, $(B - e^{H})/\theta$, is positive, the control premium, defined as the difference between the market value of a controlling share and the market value of a noncontrolling share, may be negative. Moreover, because the punishments targeted at the controlling shareholder take place during difficult times, the control premium is lower and more likely to be negative.

These implications are interesting because negative control premiums have been identified empirically. Dyck and Zingales (2004) found that some companies’ privately negotiated controlling blocks were traded at a price below the prevailing price on the market and Lease, McConnell, and Mikkelson (1983), Pinegar and Ravichandran (2003), Chen (2004), Kruse, Kyono, and Suzuki (2006), and Valero, Gomez, and Reyes (2008) found that some companies’ shares with superior voting rights were traded at a discount compared to the shares with inferior voting rights. Some informal arguments for the observed negative control premiums are that shares with inferior control rights are more liquid and that the controlling shareholder may have to bear legal liabilities. Nevertheless, the empirical observation of negative control premiums is considered by some to be puzzling because there is no formal theory that rationalizes it. Lease, McConnell, and Mikkelson

Moreover, our theory’s specific prediction that the control premium is lower and more likely to be negative during downturns is also consistent with Barclay and Holderness’s (1989) empirical finding that the average premium is lower following poor performance and with Kruse, Kyono, and Suzuki’s (2006) finding that the estimated private benefits of control in their data are the most negative when the target firm is financially distressed.

Let \(K\) be the control premium following a good outcome and \(O\) be the control premium following a bad outcome. For ease of exposition, in the previous section, we focused on equilibria in which controlling shares are traded only following a bad outcome and noncontrolling shares are never traded. One can easily construct payoff-equivalent equilibria in which controlling shares are traded following a good outcome and noncontrolling shares are traded as well. If controlling shares were traded following a good outcome, the market price would be \(\bar{V}\). If noncontrolling shares were traded, the market price would be \(\bar{U}\) following a good outcome and \(\underline{U}\) following a bad outcome. Therefore, we define \(\Delta^H \equiv \bar{V} - \bar{U}\) and \(\Delta^L \equiv \bar{T} - \underline{U}\).

**Claim 1** For \(\delta \geq \tilde{\delta} (\beta, \theta)\),

\[
\Delta^H = \frac{B - e^H}{\theta} - (1-p) \frac{e^H - e^L}{\theta} + \frac{c^H - c^L}{p-q} \\
\Delta^L = \frac{B - e^H}{\theta} - (\delta^{-1} - p) \frac{e^H - e^L}{\theta} + \frac{c^H - c^L}{p-q}.
\]

For \(\delta \in (\hat{\delta} (\theta), \tilde{\delta} (\beta, \theta))\),

\[
\Delta^H = \frac{B - e^H}{\theta} + (1-p) \frac{(\beta^{-1} - 1)}{1-\delta} \left[ \left( \frac{e^H - e^L}{\theta} + c^H - c^L \right) \frac{1-\delta q}{p-q} - \delta (p-q) \right] \\
\Delta^L = \frac{B - e^H}{\theta} + (1-\delta p) \frac{(\beta^{-1} - 1)}{\delta (1-\delta)} \left[ \left( \frac{e^H - e^L}{\theta} + c^H - c^L \right) \frac{1-\delta q}{p-q} - \delta (p-q) \right]
\]

In both cases, \(\Delta^H > \Delta^L\), and for some parameter values, \(\Delta^H < 0\).

Using the term \(\left( \frac{e^H - e^L}{\theta} + c^H - c^L \right) / (p-q)\) to measure the severity of the moral hazard problem, we find that the moral hazard problem has a negative impact on the control premium if \(\delta \geq \tilde{\delta} (\beta, \theta)\). However, when \(\delta \in (\hat{\delta}(\theta), \tilde{\delta} (\beta, \theta))\), the impact of moral hazard on control premium is positive. The reason is that when \(\delta \geq \tilde{\delta} (\beta, \theta)\), the value of non-controlling shares is constant at \(p - e^H\) and hence independent of the severity of moral hazard problem. On the other hand, when the moral hazard problem is severe, the punishment following bad outcome must be larger, and
thus a smaller value of $\hat{W}(\delta, \theta)$ and $\hat{V}$. However, when $\delta \in (\hat{\delta}(\theta), \tilde{\delta}(\beta, \theta))$, a more severe moral hazard problem hurts the value of non-controlling shares more than that of the controlling shares. The intuition is that because the new controlling shareholder has a positive bargaining power, when he must be punished after the acquisition, i.e. $\delta \in (\hat{\delta}(\theta), \tilde{\delta}(\beta, \theta))$, his continuation value $\hat{W}$ must decrease by more than that of $\hat{W}(\delta, \theta)$ when the moral hazard problem is more severe. A reduction in $\hat{W}$ translates into a larger price discount after acquisition.

Next, we consider from the company founder’s perspective the optimal value of $\theta$. The total payoff of the founder, denoted by $S$, consists of two components, the value of the shares that he retains and the proceeds from the sales of the noncontrolling shares. We assume there is perfect competition for noncontrolling shares among investors which allows the company founder to fully capture the value of the value of the noncontrolling shares. Therefore, the total payoff of the founder is

$$S = \theta V + (1 - \theta) U.$$ 

Note that this is the total shareholder value of the company at the time when the ownership structure is determined. However, it is less than the sum of the net private benefit and company profit. This is because part of the value of the company is captured through Nash bargaining by future controlling shareholders who take over the company’s control.

The basic trade-off here is the moral hazard problem versus surplus extraction: a smaller $\theta$ makes the moral hazard problem more severe, which lowers the value of both kinds of shares; on the other hand, issuing more noncontrolling shares enables the founder to capture a larger fraction of the benefits resulting from firm-ownership turnover.

Define $\underline{\theta}(\delta)$ as the solution to $\delta = \underline{\hat{\delta}}(\theta)$ and $\overline{\theta}(\delta, \beta)$ as the solution to $\delta = \overline{\hat{\delta}}(\beta, \theta)$.\footnote{\underline{\theta}(\delta) and \overline{\theta}(\delta, \beta) are both well-defined because both \underline{\hat{\delta}}(\theta) and \overline{\hat{\delta}}(\beta, \theta) are strictly decreasing as a function of \theta.} It is easy to verify that for $\delta \in (\underline{\hat{\delta}}(1), 1)$, $\underline{\theta}(\delta) < \theta(\delta) < \overline{\theta}(\delta, \beta) < 1$ (see figure 3).
Proposition 2 Let $\theta^*$ be the optimal fraction of the controlling shares.

(i) If $\delta \in [\hat{\delta}(1), 1)$, then $\theta^*$ is unique and is in the interval $(\underline{\theta}(\delta), \min \{\bar{\theta}(\delta, \beta), 1\}]$. Moreover, $\theta^* \to \underline{\theta}$ as $\delta \to 1$.

(ii) If $c^H - c^L = 0$ and $\delta \in [\hat{\delta}(1), 1)$, then $\theta^* = \min \{\bar{\theta}(\delta, \beta), 1\}$. Moreover, there exists a non-empty convex set $M(\delta, \beta) \subset \mathbb{R}_+^2$ such that $\theta^*(\delta)$ is unique and in the interval $(\underline{\theta}(\delta), \min \{\bar{\theta}(\delta, \beta), 1\})$ if and only if $(c^H - c^L, c^H - c^L) \in M(\delta, \beta)$.

(iii) If $\delta \in (\hat{\delta}(1), 1)$, then $\theta^*$ is (locally) weakly increasing in $c^H - c^L$ and $c^H - c^L$.

The main message of part (i) of Proposition 2 is that the optimal share structure is to convert some controlling shares, but not as many of them as possible, into noncontrolling shares.

Part (ii) of the above proposition shows that the assumption that high-quality production requires monetary cost $c^H$ affects the results of the model in a non-trivial way. If $c^H - c^L = 0$, then the optimal ownership structure is always to convert just as much shares into non-controlling shares without having to invoke a price cut in the punishment phase. Consequently, the model predicts that if the ownership structure is optimally chosen by the founder of the company, a price cut or a drop in post-acquisition profit never occurs. On the other hand, if $c^H - c^L > 0$, a price cut or a drop in post-acquisition profit may occur for certain parameter values, i.e., when $(c^H - c^L, c^H - c^L) \in M(\delta, \beta)$.

Part (iii) confirms the intuition that the optimal ownership structure is chosen to trade off mitigating moral hazard problem and extracting surplus generated from ownership turnover. When moral hazard is severe, i.e., $c^H - c^L$ or $c^H - c^L$ is high, the owner should retain a larger fraction of shares.
The analysis in this section is related to Zingales (1995), who derives the practice of selling cash-flow rights to disperse shareholders and selling control rights through direct bargaining as the outcome of maximization of total proceeds from the sale of a company. Our model has the similar feature that disperse shareholders are perfectly competitive and the acquirer of control rights has substantial bargaining power. Other than that, our analysis is different in several important ways. In our model, the firm’s decision to sell the shares is endogenized while the firm’s decision to sell shares in Zingales’s model is exogenously given. The optimal ownership structure in our model is the outcome of the tradeoff between managing the controlling shareholder’s moral hazard and reducing the externality of punishment and our analysis allows us to derive an endogenous cost of control, while these elements are all absent in Zingales’s analysis.

4 Discussion

For tractability, we have abstracted away from many issues in our analysis. Below, we discuss some of them.

**Competition Among Potential Owners** We have ignored the issue of competition among potential acquirers in the analysis by assuming that every period only one potential acquirer enters the game. Notice that if the transaction price of firm ownership is publicly observable, then competition among buyers has little impact on our equilibrium construction. Any equilibrium transaction price of the controlling shares can be supported by the belief that if any potential owner pays an amount other than the equilibrium price, then the new owner will receive a continuation payoff equal to the lowest possible equilibrium payoff, which is further supported by the consumers’ self-fulfilling belief that the firm will only engage in low-quality production. This is sufficient to deter any deviation. If the transaction price is unobservable, then more-intense competition can be modelled as a higher bargaining power for the incumbent owner. In the extreme case where competition is so fierce that the incumbent has all the bargaining power, i.e., $\beta = 1$, firm-ownership turnover does not at all increase the firm’s value. We take the view that it is unlikely that the incumbent has 100% of the bargaining power. Even when there are simultaneously multiple buyers seeking control of the firm, as long as the incumbent owner cannot commit to a grand mechanism (say by holding an auction), but instead has to sequentially bargain with one buyer at a time, there is a bargaining protocol that ensures the seller only receives a fraction of the total surplus. Moreover, oftentimes the incumbent owner has to face competition from owners of other companies trying to sell control rights in the M&A market. We find it comforting that for any interior split of bargaining power, i.e., for $\beta \in (0,1)$, allowing turnover expands the payoff set for the shareholders for a range of sufficiently high discount factors.

**Alternative Turnaround Mechanisms** We have focused on how the turnover of ownership can enhance the company’s profitability and shareholder value. We have done so not because this
is the only way for the firm to mitigate externality of equilibrium punishment. Rather, our focus was motivated by the fact that firm-ownership turnover is common and empirical findings that ownership and management turnover is an integral part of a successful turnaround. In fact, if we allow the controlling shareholder to burn money or make transfers to noncontrolling shareholders, then there exist equilibria in which consumers forgive the firm’s bad outcome if and only if the controlling shareholder burns a large enough amount of money or transfers a large enough amount of money to the noncontrolling shareholders. We do not find such equilibria interesting because they are not empirically relevant. Also, we will argue below that under plausible conditions, these equilibria are dominated by those supported by ownership turnover.

Money burning has to be credible in order to be a useful punishment device. In the model, the only channel to “burn money” is to cut the price charged to customers. It is relatively easy to convince the customers that the controlling shareholder suffered a loss from a price cut. However, transfers to a third party (say, a charity) may not be an effective way to punish the controlling shareholder if secret side-payments can be made between the controlling shareholder and the third party. This is because every time the controlling shareholder is required to make a side payment to the third party, she can ask the third party to secretly give her a kickback. Furthermore, if the controlling shareholder derives pleasure from the transfer (say, when the third party is a charity), then it is even harder to use such transfer to punish the controlling shareholder. Finally, if consumers’ demand for the firm’s product is downward sloping, then punishment via turnover is more socially efficient than turnover via money burning or transfer to the third party.

Using transfers to noncontrolling shareholders as punishment for the controlling shareholder’s poor performance also has its problems. First, noncontrolling shareholders may have a perverse incentive to sabotage the firm’s operation as they stand to gain in case of product failure. Second, and relatedly, this punishment device is susceptible to collusion between shareholders. For instance, if noncontrolling shareholders can monitor the controlling shareholder’s action, some noncontrolling shareholders may form a relational side contract with the controlling shareholder in which these noncontrolling shareholders secretly pay the controlling shareholder to shirk. Our turnover mechanism is less susceptible to these problems because noncontrolling shareholders do not gain from failure of quality and turnover of the controlling shareholder makes relational side contract between the controlling shareholder and noncontrolling shareholders difficult to implement.

Finally, requiring the controlling shareholder to make a transfer to either a third party or the noncontrolling shareholder following every bad outcome may cause her to run into her liquidity constraint because bad outcomes are associated with low (possibly negative) profits. Even if a few bad outcomes may not cause any trouble, a sufficiently long streak of bad outcomes, which always happens with a positive probability, will cause the controlling shareholder’s liquidity constraint to fail and the equilibrium to unravel. By contrast, under the ownership turnover mechanism, the
controlling shareholder will be receiving a payment for selling the controlling shares so she will not run into her liquidity constraint.

Efficiency-Enhancing Role of Turnover In our analysis, while ownership turnover eliminates the negative externality imposed on the noncontrolling shareholders, improves firm profit, and creates shareholder value, it does not impact production efficiency because our model is simplified in a way that whenever high-quality production is sustainable, the equilibrium outcome is socially efficient. However, it is not difficult to construct alternative models in which ownership turnover also plays a efficiency-enhancing role. One possibility is to introduce private monitoring. To be more concrete, suppose each unit of the product has the same probability of being high-quality but the quality of each unit is an independent random draw. Also suppose each customer privately observes the realization of the unit he consumes and then simultaneously makes an announcement about the quality realization. The firm’s reputation, which is assumed to be either good or bad for simplicity, is stochastically determined by these reports and the probability of a good reputation increases in the number of positive reports. If every time the firm’s reputation is bad it must offer a discount to its customers while continuing to exert high effort and incur high monetary cost in production, then customers will be tempted to misreport high quality as low quality. This implies there is a limitation on using price cuts as a disciplinary device for the firm. We can show that in this modified model, full production efficiency is unattainable without ownership turnover, and that allowing for ownership turnover can improve efficiency.\footnote{Alternatively, one can consider a model in which it is very costly to maintain product safety, yet product failure leads to a negative utility for customers. In such an environment, some moderate price cuts would not constitute severe enough punishment of the firm for the purpose of incentivizing it to engage in high-quality production. If there is no effective way for the firm to rebate its profits to consumers, then the only way for the consumers to punish the firm may be to coordinate on an inefficient equilibrium path on which consumers stop purchasing from the firm, believing it does not exert socially efficient effort to avoid further product failure.}

Turnover during Good vs. Difficult Times In our formal analysis, we focus on equilibria in which turnover takes place only during difficult times where both the incumbent owner and new owner benefit from the transaction. However, payoff-equivalent equilibria can be constructed such that turnover takes place during good times as well, except that during good times there are no strict gains from firm-ownership turnover. To generate strict gain from transaction of ownership during good times, one can introduce exogenous arrivals of entrepreneurs who are better at running the company than the existing owner. During good times, firm ownership will be changed only if a more capable entrepreneur shows up. On the other hand, during difficult times, firm ownership may be changed and improvement can be brought about even if the acquirer does not possess superior management ability. This suggests that taking a successful business to the next level is more difficult and deserves more praise than turning around a failing business.

Probabilistic Availability of New Owners and Costly Turnover In our formal analysis, ownership turnover is frictionless in the sense that there is a potential owner available to take over
the firm every period. In a more realistic setting, following a bad outcome, there may not be any potential owners immediately available. In this case, on-the-equilibrium-path punishment of the firm will continue to take place until ownership changes hands. This will give rise to a more natural empirical implication that when a firm’s reputation is tarnished, the firm’s profitability will decrease and stay low until a new owner takes over. Another natural extension of our model is to introduce turnover cost. When turnover is costly, the firm will necessarily fail to capture the full surplus as part of the surplus will be dissipated through the transaction cost of ownership turnover. The heterogeneity of transaction costs with different potential owners can also lead to delay in turnover as the incumbent owner may wait until she meets a potential owner with a low-enough transaction cost.

5 Conclusion

This paper considers an experience-good producer who is subject to a moral hazard problem: the producer’s choice of effort and other costly inputs stochastically determines the output quality. We introduce player turnover into this otherwise-standard relational contract setting with imperfect public monitoring. Punishment must be imposed by consumers on the firm following poor-quality output, but this hurts every shareholder of the firm, including the noncontrolling shareholders, who are not responsible for the decision making. We have shown that this negative externality of punishment on innocent parties can be mitigated if we allow the ownership of controlling shares to be traded. Trading of firm ownership occurs because consumers treat the new and incumbent owners differently even though these owners have identical incentives and abilities. When the discount factor is large enough and/or the bargaining power of the incumbent controlling shareholder is weak enough, it is possible to reduce or eliminate the otherwise necessary punishment imposed on the firm and the noncontrolling shareholders. The equilibrium property that the controlling shareholder must be punished for bad outcomes but the noncontrolling shareholders can be spared gives rise to a cost of corporate control and provides a rationale for (partial) separation of ownership and control. Finally, we provide a partial characterization of the optimal share composition in our setting.

Appendix

Proof of Proposition 0. Plugging $P = p$ into the incentive constraint (2) gives us

$$W \leq \tilde{W} (\delta, \theta) \equiv \frac{B - c^H}{\theta} + (p - e^H) - \frac{(1 - \delta p) \left( \frac{c^H - c^L}{\theta} + c^H - c^L \right)}{\delta (p - q)}. \quad (8)$$

A necessary condition for the sustainability of high effort, i.e., $\tilde{W} (\delta, \theta) \geq (B - e^L) / \theta + (q - c^L)$,
is given by

$$\frac{B - e^L}{\theta} + (q - c^L) \leq \frac{B - e^H}{\theta} + (p - c^H) - \frac{(1 - \delta) \left( \frac{c^H - e^L}{\theta} + c^H - c^L \right)}{\delta (p - q)}.$$  \hspace{1cm} (9)

As long as (9) is satisfied, there exists

$$W \in \left[ \frac{B - e^L}{\theta} + (q - c^L), \frac{B - e^H}{\theta} + (p - c^H) - \frac{(1 - \delta) \left( \frac{c^H - e^L}{\theta} + c^H - c^L \right)}{\delta (p - q)} \right]$$

such that (8) is satisfied. One immediate result is that $B$ does not affect the sustainability of high effort. This is because the controlling shareholder receives $B$ regardless. This inequality can be rewritten as

$$\delta \geq \hat{\delta} (\theta) = \frac{e^H - e^L + \theta (c^H - c^L)}{q (e^H - e^L + \theta (c^H - c^L)) + \theta (p - q)^2}.$$  

Note that $\hat{\delta} (\theta)$ is decreasing in $\theta$, $\hat{\delta} (0) = 1/q > 1$ and

$$\hat{\delta} (1) = \frac{e^H - e^L + c^H - c^L}{q (e^H - e^L + c^H - c^L) + (p - q)^2}.$$  

If $\hat{\delta} (1) > 1$, then $\hat{\delta} (\theta) > 1$ for all $\theta$ and high effort is unsustainable for discount factors. Therefore, for the analysis to be nontrivial, it is necessary that $\hat{\delta} (1) \leq 1$, which is equivalent to (3). When (3) holds, there exists $\theta$ such that $\hat{\delta} (\theta) \leq 1$ if and only if

$$\theta \geq \theta_0 = \frac{(1 - q) (e^H - e^L)}{(p - q)^2 - (1 - q) (c^H - c^L)}.$$  

Figure 1 depicts the relationship between $\hat{\delta}$ and $\theta$.

For $\delta \geq \hat{\delta} (\theta)$, the maximum value of the controlling share can be obtained by plugging $W = \bar{W} (\delta, \theta)$ from (8) into (1). Since $W$ is enforced by a discounted price, setting $W = \bar{W} (\delta, \theta)$ means the firm gives a discount just large enough to support the incentive to engage in high-quality production. In other words, setting $W = \bar{W} (\delta, \theta)$ maximizes $U$ and $V$, which in turn maximizes $S$.  

**Proof of Proposition 1.** (i) If $\delta < \hat{\delta} (\theta)$, then $\bar{W} (\delta, \theta) < (B - e^L) / \theta + (q - c^H)$. Therefore, there does not exist $T \in [(B - e^L) / \theta + (q - c^H), \bar{W} (\delta, \theta)]$. In other words, when high-quality production is not sustainable in the absence of transfer of controlling shares, allowing transfer of these shares will not improve the performance of the firm because the possibility of ownership turnover cannot lower the controlling shareholder’s continuation payoff below $B - e^L + \theta (q - c^L)$. Therefore, if $\delta < \hat{\delta} (\theta)$, only low effort can be supported in equilibrium even when turnover is allowed. Consequently, $\bar{V} = \frac{B - e^L}{\theta} + \bar{U}$ and $\bar{U} = q - e^L$.  

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(ii) When \( \delta = \hat{\delta}(\theta) \), in order to support high effort, the continuation payoff to the controlling shareholder must be set at \( \theta (q - c^L) + B - c^L \) following a bad outcome, for incentive provision. When controlling shares turnover is allowed, it requires a transaction price of \( \theta (q - c^L) + B - c^L \) to sustain effort. When the incumbent has positive bargaining power, this is possible only if the surplus from the trade of controlling shares is zero, i.e. the newly arrived entrepreneur has to be punished as severely as the incumbent after he takesovers the firm. Thus, the firm’s profit cannot be increased by turnover. Consequently, \( \bar{U} \) and \( \bar{V} \) stay at \( \bar{U}^0(\theta) \) and \( \bar{V}^0(\theta) \) respectively.

(iii)-(iv) In order to construct the optimal relational contract, we look for PPE that maximizes the total shareholder value \( S = \theta V + (1 - \theta) U \).

An upper bound on the equilibrium value of \( U \) in any PPE is \( p - c^H \).

Equation (5), which determines the value of controlling share, \( V \), in those equilibria with ownership turnover and high-quality production, can be rewritten as

\[
V = \frac{1 - \delta}{1 - \delta p} \left[ \frac{B - e^H}{\theta} + (p - c^H) \right] + \frac{\delta (1 - p) T}{1 - \delta p} \tag{10}
\]

Thus, the equilibrium value of \( V \) is increasing in the transaction price of controlling shares \( T \), as long as \( T \) is small enough to sustain incentive for high-quality production. More precisely, the following incentive constraint must be satisfied to motivate high-quality production

\[
T \leq \bar{W}(\delta, \theta) + \frac{(1 - \delta p) \left( \frac{e^H - e^L}{\theta} + e^H - c^L \right)}{\delta (p - q)} \tag{11}
\]

Therefore, the maximum possible value of \( V \) in any PPE with ownership turnover and high-quality production is given by equation (10) with \( T = \bar{W}(\delta, \theta) \). Consequently, an upper bound on the equilibrium value of \( V \) in any PPE is \( \bar{V}^0(\theta) \).

Now, we show the upper bounds on \( U \) and \( V \) can be achieved if the discount factor is large enough. This then implies that when the discount factor is large enough, the optimal relational contract yields \( \bar{U} = p - c^H \) and \( \bar{V} = \bar{V}^0(\theta) \).

First, in order for the controlling share to achieve the value \( \bar{V}^0(\theta) \), we have to set \( T = \bar{W}(\delta, \theta) \). Next, recall \( \hat{W} \) is the value of controlling shares to the new controlling shareholder following an ownership turnover. Suppose high-quality production can be supported with \( \hat{W} = \hat{V}^0(\theta) \). Then no price cut is needed in the equilibrium punishment phase and from equation (6), the noncontrolling shares achieve the value \( \bar{U} = p - c^H \). Substituting \( \hat{W} = \hat{V}^0(\theta) \) and \( T = \bar{W}(\delta, \theta) \) into equation (4) and rearrange, we can express the value of controlling shares to the incumbent following an off-equilibrium negotiation breakdown, \( W \), as

\[
W = \frac{\bar{W}(\delta, \theta) - \beta \bar{V}^0(\theta)}{1 - \beta}
\]

This off-equilibrium value of controlling share can be supported by requiring the incumbent to
offer a price discount to customers for one period. The discount is given by
\[
\frac{V^0(\theta) - W}{1 - \delta} = \frac{\bar{V}^0(\theta) - \bar{W}(\delta, \theta)}{(1 - \delta)(1 - \beta)}
\]

The equilibrium described above is feasible if and only if
\[
W \geq \frac{B - e^L}{\theta} + (q - c^L)
\]

This translates into the following condition:
\[
\delta \geq \tilde{\delta}(\beta, \theta) = \frac{e^H - e^L + \theta (e^H - c^L)}{(1 - q)\beta + q(e^H - e^L + \theta (c^H - c^L)) + (1 - \beta)\theta(p - q)^2}.
\]

In sum, if \(\delta \in [\tilde{\delta}(\beta, \theta), 1)\), \(\bar{U} = p - c^H\) and \(\bar{V} = \bar{V}^0(\theta)\) in the optimal relational contract. The equilibrium takes the form described in the text, with \(T = \bar{W}(\delta, \theta)\). On the off-equilibrium path punishment phase in which the controlling shares are retained by the incumbent, a price cut \(\frac{V^0(\theta) - W(\delta, \theta)}{(1-\delta)(1-\beta)}\) is offered to customers for one period. This concludes the proof for part (iv) of the proposition.

Next, suppose \(\delta \in [\hat{\delta}(\theta), \tilde{\delta}(\beta, \theta))\). In the optimal relational contract, \(\bar{V} > \bar{W}\), and a price discount is offered in the on-the-equilibrium punishment phase. It is therefore immediate that \(\bar{U} < p - c^H\). We now proceed to construct the optimal relational contract.

Equation (4) can be written as
\[
\bar{W} = \frac{T - (1 - \beta)W}{\beta}
\]

Using (14) and (10), the price cut \((V - \bar{W}) / (1 - \delta)\) can be written as
\[
\frac{V - \bar{W}}{1 - \delta} = \frac{1}{1 - \delta p} \left[ \frac{B - e^H}{\theta} + (p - c^H) \right] - \frac{1}{1 - \delta} \left[ \frac{1}{\beta} - \frac{\delta (1 - p)}{1 - \delta p} \right] T + \frac{1 - \beta}{\beta (1 - \delta)} W
\]

Note that because \(\delta \in [\hat{\delta}(\theta), \tilde{\delta}(\beta, \theta))\), the expression for \((V - \bar{W}) / (1 - \delta)\) above is positive for all \(T\) and \(W\) such that \(T \in [0, \bar{W}(\delta, \theta)]\) (by (11)) and \(W \geq (B - e^L) / \theta + (q - c^L)\) (by (12)). The price cut is therefore increasing in \(W\) and decreasing in \(T\) (since \(1/\beta > 1 > \delta (1 - p)/1 - \delta p\)). Therefore, to get the minimum equilibrium price cut, we should set \(W = (B - e^L) / \theta + (q - c^L)\) and \(T = \bar{W}(\delta, \theta)\). Note that by setting \(T = \bar{W}(\delta, \theta)\), we also achieve the upper bound on the equilibrium value of controlling shares \(V = \bar{V}^0(\theta)\). Thus, \(\bar{V} = \bar{V}^0(\theta)\).

By setting \(W = (B - e^L) / \theta + (q - c^L)\) and \(T = \bar{W}(\delta, \theta)\), the value of noncontrolling shares in
the optimal relational contract is

\[
\bar{U} = (p - c^H) - \delta \left( 1 - p \left( \frac{e^H - e^L}{p - q} + c^L - c^H \right) \right) \left( \beta^{-1}(\delta^{-1} - q) - (1 - q) \right) - (p - q) (\beta^{-1} - 1) \right) \tag{15}
\]

In sum, if \( \delta \in [\bar{\delta}(\theta), \hat{\delta}(\beta, \theta)) \), \( \bar{U} \) is given by (15) and \( \bar{V} = \bar{V}^0(\theta) \) in the optimal relational contract. The equilibrium takes the form described in the text, with \( T = \bar{W}(\delta, \theta) \). On the off-equilibrium path in which the controlling shares are retained by the incumbent, a price cut of \( \frac{\bar{V}^0(\theta) - (B - e^L)/\theta + (q - e^L)}{(1 - \delta)(1 - \beta)} \) is offered to customers for one period.

Finally, because \( \delta \in (\bar{\delta}(\theta), \hat{\delta}(\beta, \theta)) \), it can readily verified that

\[
\bar{U} \in \left( (p - c^H) - (1 - p) \left( \frac{e^H - e^L}{p - q} + c^L - c^H \right), p - c^H \right) = (\bar{U}^0(\theta), p - c^H)
\]

**Proof of Claim 1.** Fix a \( \theta \geq \theta \). If \( \delta \leq \bar{\delta}(\theta) \), then \( \beta \) has no effect on \( \bar{W} \) and hence no effect on the acquisition premium and the post-acquisition accounting profit. For the rest of the proof, we consider the case \( \delta > \bar{\delta}(\theta) \).

Recall that \( \bar{\delta}(0, \theta) = \bar{\delta}(\theta) \) and that \( \bar{\delta}(\beta, \theta) \) is strictly increasing in \( \beta \), we can thus define its inverse: let \( \bar{\beta}(\delta, \theta) \) be solution to \( \delta = \bar{\delta}(\beta, \theta) \). When \( \beta \leq \bar{\beta}(\delta, \theta) \), we have \( \delta \geq \bar{\delta}(\beta, \theta) \). Therefore, according to the proof of Proposition 1, \( \bar{W} = \bar{V}^0(\theta) \) and \( T = \bar{W}(\delta, \theta) \). Both the acquisition premium, \( T - \bar{U} \), and post-acquisition profit, \( p - c^H \), are locally invariant to \( \beta \).

When \( \beta > \bar{\beta}(\delta, \theta) \), we have \( \delta < \bar{\delta}(\beta, \theta) \). Therefore, according to the proof of Proposition 1, \( T \) remains at \( \bar{W}(\delta, \theta) \) and

\[
\bar{W} = \frac{\bar{W}(\delta, \theta) - (1 - \beta) \left( \frac{B - e^L}{\theta} + (q - e^L) \right)}{\beta}
\]

Note that \( \bar{W} \) is equal to \( \bar{V}^0(\theta) \) when \( \beta = \bar{\beta}(\delta, \theta) \) and is strictly decreasing for \( \beta \in (\bar{\beta}(\delta, \theta), 1) \).

The claim then follows because the acquisition premium varies inversely with \( \bar{W} \) while the post-acquisition accounting profit varies positively with \( \bar{W} \). ■

**Proof of Claim 2.** Based on Proposition 1, if \( \delta \geq \bar{\delta}(\beta, \theta) \), then \( \bar{V} = \bar{V}^0(\theta) \), \( \bar{U} = \bar{U} = p - c^H \), and \( T = \bar{W}(\delta, \theta) \). The expressions for \( \Delta^H \) and \( \Delta^L \) then follow by direct substitution. It follows immediately that \( \Delta^H > \Delta^L \). Also, \( \Delta^H < 0 \) if \( \frac{B - e^H}{\theta} \) is sufficiently small.

Next, if \( \delta \in (\bar{\delta}(\theta), \hat{\delta}(\beta, \theta)) \), then \( \bar{V} \) and \( T \) remain at \( \bar{V}^0(\theta) \) and \( \bar{W}(\delta, \theta) \), respectively. The expressions for \( \bar{U} \) and \( \bar{U} \) are given by (7). According to the proof of Proposition 1,

\[
\bar{W} = \frac{\bar{W}(\delta, \theta) - (1 - \beta) \left( \frac{B - e^L}{\theta} + (q - e^L) \right)}{\beta}
\]
Direct substitution gives the expressions for $\Delta^H$ and $\Delta^L$. Note that the term in the brackets is negative if and only if

$$\delta > \delta_1(\theta) = \frac{c^H - c^L + \frac{e^H - e^L}{\theta}}{(p-q)^2 + q \left( c^H - c^L + \frac{e^H - e^L}{\theta} \right)}.$$ 

It can be readily verified that $\delta_1(\theta) < \tilde{\delta}(\beta, \theta)$ under (3). Thus, if $\delta \in \left( \delta_1(\theta), \tilde{\delta}(\beta, \theta) \right)$ and $\frac{B-e^H}{\theta}$ is sufficiently small, then $\Delta^H$ can be negative. Finally, note that

$$\Delta^H - \Delta^L = (\bar{V}^0(\theta) - \bar{U}) - (\bar{W}(\delta, \theta) - \bar{U}) = \bar{W} - \bar{W}(\delta, \theta).$$

Because $\bar{W} - \bar{W}(\delta, \theta) > 0$, we obtain that $\Delta^H > \Delta^L$ as in the former case. ■

Proof of Proposition 2. (i) First, it is immediately apparent that when $\delta = \tilde{\delta}(1)$, $\theta^*$ is unique and equal to one as any lower $\theta$ cannot support high-quality production. Similarly, for any $\delta \in \left( \tilde{\delta}(1), 1 \right)$, it is suboptimal to set $\theta$ below $\tilde{\theta}(\delta)$.

Next, when $\theta = \tilde{\theta}(\delta)$ or equivalently, $\delta = \tilde{\delta}(\theta)$, we have that $\bar{U} = U^0(\tilde{\theta}(\delta))$ and $\bar{V} = V^0(\tilde{\theta}(\delta))$, according to Proposition 1. Therefore, $\bar{U} - \bar{V} = -\frac{B-e^H}{\theta(\delta)} < 0$ and non-controlling shares is less valuable than controlling shares. Since the value of each kind of shares is strictly increasing in $\theta$ for $\theta \in \left[ \tilde{\theta}(\delta), \min \left\{ \tilde{\theta}(\delta, \beta), 1 \right\} \right]$ and $\delta \in \left( \tilde{\delta}(1), 1 \right)$, the optimal ownership structure $\theta^*$ strictly exceeds $\tilde{\theta}(\delta)$.

Moreover, if $\tilde{\theta}(\delta, \beta) < 1$ and $\theta > \tilde{\theta}(\delta, \beta)$, then according to Proposition 1, $\bar{U}(\theta) = p - e^H$ and $\bar{V}(\theta) = V^0(\theta)$. This gives

$$S(\theta) = B - e^H + (p - c^H) - \frac{1-p}{p-q} \left[ c^H - c^L + \theta \left( c^H - c^L \right) \right].$$

Therefore, $S$ is strictly decreasing in $\theta$ for $\theta \geq \tilde{\theta}(\delta, \beta)$ and it is suboptimal to set $\theta$ above $\tilde{\theta}(\delta, \beta)$.

We have thus established that $\theta^*(\delta) \in \left( \theta(\delta), \min \left\{ \tilde{\theta}(\delta, \beta), 1 \right\} \right]$.

To see why $\theta^*(\delta)$ is unique when when $\delta \in \left( \tilde{\delta}(1), 1 \right)$, recall by definitions, $\theta \in \left( \tilde{\theta}(\delta), \min \left\{ \tilde{\theta}(\delta, \beta), 1 \right\} \right]$ if and only if $\delta \in \left( \tilde{\delta}(\delta), \tilde{\delta}(\beta, \theta) \right]$. Using part (iii) of Proposition 1, the total shareholder value $S(\theta) = \theta \bar{V}(\theta) + (1 - \theta) \bar{U}(\theta)$ for this range of $\theta$ is given by

$$S(\theta) = \theta \left[ \frac{B - e^H}{\theta} + (p - c^H) - \frac{1-p}{p-q} \left( \frac{e^H - e^L}{\theta} + c^H - c^L \right) \right]$$

$$+ \left( 1 - \theta \right) \left[ (p - c^H) - \delta \frac{1-p}{1-\delta} \left\{ \frac{e^H - e^L}{\theta} + c^H - c^L \right\} \right] \left[ \beta^{-1}(\delta^{-1} - q) - (1-q) \right] - (p-q) \left( \beta^{-1} - 1 \right) \right].$$

Direct computation gives the second derivative:

$$S''(\theta) = -2\theta^{-3} \left( \delta \frac{1-p}{1-\delta} \left( \frac{e^H - e^L}{\theta} \right) \right) \left( \beta^{-1}(\delta^{-1} - q) - (1-q) \right) < 0$$
Since $S$ is strictly concave in $\theta$ in the interval $(\tilde{\theta}(\delta), \min \{\tilde{\theta}(\delta, \beta), 1\})$, $\theta^*$ is unique when $\delta \in (\tilde{\delta}(1), 1)$.

Finally, it is easy to verify that both $\tilde{\theta}(\delta) \to \tilde{\theta}$ and $\tilde{\theta}(\delta, \beta) \to \tilde{\theta}$ as $\delta \to 1$. Therefore, $\theta^*$ converges to $\tilde{\theta}$.

(ii) For the rest of the proof, we fix a pair $(\delta, \beta) \in (0, 1)^2$ and drop the dependence of $\tilde{\theta}$ and $\tilde{\theta}$ on $(\delta, \beta)$ to simplify expressions.

Suppose first that $\delta \in [\tilde{\delta}(1), 1)$. In other words, $e^H - e^L + c^H - c^L < \frac{\delta(p-q)^2}{1-\delta q}$. Following part (i), $\theta^* = \min \{\tilde{\theta}, 1\}$ if and only if the left derivative of $S$ evaluated at $\min \{\tilde{\theta}, 1\}$ is non-negative, i.e. $S'_- (\min \{\tilde{\theta}, 1\}) \geq 0$. Note that $S'_- (\theta)$ can be decomposed into $S'_- (\theta) = \theta V'_- (\theta) + (1 - \theta) \tilde{U}'_-(\theta) - (\tilde{U}(\theta) - \tilde{V}(\theta))$. Moreover,

$$
\tilde{U} (\min \{\tilde{\theta}, 1\}) - \tilde{V} (\min \{\tilde{\theta}, 1\}) \leq \frac{1-p}{p-q} \left( \frac{e^H - e^L}{\min \{\tilde{\theta}, 1\}} + c^H - c^L \right) - \frac{B - e^H}{\min \{\tilde{\theta}, 1\}} = \min \{\tilde{\theta}, 1\} V'_- (\min \{\tilde{\theta}, 1\}) + \frac{1-p}{p-q} (c^H - c^L).
$$

Therefore, if $c^H - c^L = 0$, we have

$$
S'_- (\min \{\tilde{\theta}, 1\}) \geq (1 - \min \{\tilde{\theta}, 1\}) \tilde{U}'_- (\min \{\tilde{\theta}, 1\}) > 0
$$

Finally, we characterize the set of $(e^H - e^L, c^H - c^L)$ pair such that $\theta^*$ is unique and in the interval $(\tilde{\theta}, \min \{\tilde{\theta}, 1\})$. An obvious necessary condition is that $\delta > \tilde{\delta}(1)$, which can be equivalently expressed as

$$
e^H - e^L + c^H - c^L < \frac{\delta(p-q)^2}{1-\delta q} \tag{16}
$$

Suppose first that $\tilde{\theta} < 1$, i.e. $e^H - e^L + c^H - c^L < \frac{\delta(1-\beta)(p-q)^2}{1-\delta q - \beta(1-q)}$. In this case, $\theta^*(\delta) \in (\tilde{\theta}, \tilde{\theta})$ if and only if $S'_-(\tilde{\theta}) < 0$. The latter condition is equivalent to

$$
e^H - e^L > \frac{(\delta(1-\beta)(p-q)^2)}{(1-\delta q - \beta(1-q))} - \frac{(e^H - e^L)^2}{(1-\delta q - \beta(1-q))} \tag{17}
$$

Next suppose $\tilde{\theta} \geq 1$. Then $\theta^*(\delta) \in (\tilde{\theta}, \tilde{\theta})$ if and only if $S'(1) < 0$. The latter condition is equivalent to

$$
e^H - e^L < \frac{\delta(1-\beta)(p-q)^2}{1-\delta q - \beta(1-q)} - \frac{(1-\delta q)(1-\beta)}{1-\delta q - \beta(1-q)} (e^H - e^L) \tag{18}
$$

Define the set $M(\delta, \beta)$ by

$$
M(\delta, \beta) = \{(e^H - e^L, c^H - c^L) \in \mathbb{R}^2_+ : (16), (17) \text{ and } (18) \text{ hold.}\}
$$
From the discussion above, \( \theta^* (\delta) \in (\bar{\theta}, \min \{ \bar{\theta}, 1 \}) \) if and only if \((e^H - e^L, e^H - c^L) \in M (\delta, \beta) \). Finally, it is straightforward to verify that \( M (\delta, \beta) \) is a non-empty convex set in \( \mathbb{R}^2_+ \).

(iii) Suppose \((e^H - e^L, e^H - c^L) \in M (\delta, \beta) \). By the strict concavity of \( S (\theta) \) in the interval \((\bar{\theta}, \min \{ \bar{\theta}, 1 \})\], \( \theta^* \) is characterized by the first order condition \( S' (\theta^*) = 0 \), which can be simplified into

\[
\theta^* = \frac{(e^H - e^L) (1 - \delta q - \beta \delta (1 - q))}{(1 - \beta) \left[ \delta (p - q)^2 - (e^H - c^L) (1 - \delta q) \right]}
\]

Therefore, for all \((e^H - e^L, e^H - c^L) \in M (\delta, \beta) \), \( \theta^* \) is increasing in both \( e^H - e^L \) and \( e^H - c^L \). The proof is completed by recognizing that \( \bar{\theta} \) is also increasing in both \( e^H - e^L \) and \( e^H - c^L \).

References


