Existence, Incentive Compatibility
and Efficiency of the Rational Expectations Equilibrium

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Abstract

The rational expectations equilibrium (REE), as introduced in Radner (1979) in a general equilibrium setting à la Arrow-Debreu-McKenzie, often fails to have normative properties such as universal existence, incentive compatibility and efficiency. We resolve those problems by providing a new model which makes the standard REE a desirable solution concept. In particular, we consider an asymmetric information economy with a continuum of agents whose private signals are independent conditioned on the macro states of nature; for such an economy, the REE universally exists, is incentive compatible and efficient. Also, we introduce the notion of REE with aggregate signals which extends the standard REE concept by allowing agents to use the information generated by a macroeconomic statistic, namely agents’ aggregate signals. It is shown that this new REE concept also possesses the desirable properties as the standard REE does, but with much more general conditions on agents’ utility functions.

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1 Introduction

In seminal papers, Radner [17] and Allen [1] extended the finite agent Arrow-Debreu-McKenzie economy to allow for asymmetric information. In this asymmetric information economy, each agent is characterized by a random utility function, random initial endowment, private information with a prior. The equilibrium notion that Radner put forward is called rational expectations equilibrium (REE), which is an extension of the deterministic Walrasian equilibrium of the Arrow-Debreu-McKenzie model. According to the REE, each individual maximizes interim expected utility conditioned on her own private information as well as the information generated by the equilibrium price.

By now it is well known that in a finite agent economy with asymmetric information, a rational expectations equilibrium, may not exist\(^1\) (see [13]), may not be incentive compatible, may not be Pareto optimal and may not be implementable as a perfect Bayesian equilibrium of an extensive form game (see [7], p. 31 and also Example 9.1.1, p.43). Thus, if the intent of the REE notion is to capture contracts among agents under asymmetric information, then such contacts not only do they not exist universally in well behaved economies (i.e., economies with concave, continuous, monotone utility functions and strictly positive initial endowments), but even if they exist, they fail to have any normative properties, such as incentive compatibility, Pareto optimality and Bayesian rationality.

The main difficulty that one encounters with the REE in a finite-agent economy is the fact that individuals are supposed to maximize their interim expected utility conditioned not only on their own private information, but also on the information generated by the equilibrium price (recall that the equilibrium price is endogenously determined); at the same time, the individuals can influence the equilibrium price to their own benefit by manipulating their private information. However, this would not have been a problem if each agent’s private information is negligible. This poses the following question. Is it possible to model the REE in such a way that each agent’s effect on the equilibrium price is negligible and therefore the REE concept overcomes the difficulties encountered above?

We introduce a new model where the REE concept becomes free of the problems mentioned above. In this model, agents’ perceived private signals conditioned on an exogenously given macro state of nature are independent, and thus, by the exact law of large numbers (see Lemma 1 below), the influence of each agent on the equilibrium price

\(^1\)It only exists in a generic sense as Radner [17] and Allen [1] have shown.
is negligible. In such a framework, the equilibrium price only reveals some information about the macro states. In particular, our Theorem 1 shows the existence of a REE, which is incentive compatible and ex-post Pareto optimal (as mentioned earlier, these results are false in a finite agent economy). Note that the REE price in this theorem depends on the exogenously given macro states of nature. Since an individual agent in such an economy cannot influence the macro states, the effect of each agent on the equilibrium price is exactly negligible, which assures incentive compatibility. The above theorem is based on the assumption that the utility of each agent depends only on her private information and is strictly concave. In contrast, Proposition 1 drops both conditions when we consider the special case that there is only one macro state (i.e, the private signals are unconditionally independent). In this case, the resulting REE price is constant.

Macroeconomic statistics can often provide a good description of an economy. In view of this, we introduce a macroeconomic statistic – aggregate signals into the REE model. In particular, each agent maximizes her expected utility conditioned on her private information and on the information generated by the aggregate signals. It should be noted that it is not necessary to include here the information generated by the equilibrium price that depends only on the macro states, because the aggregate signals are fully revealing of all the macro states.\(^2\) With this new feature on the aggregate signals, incentive compatibility for an REE is still preserved because each individual agent cannot change the aggregate signals by misreporting her private signal. The advantage for allowing the agents to use this additional information is that strict concavity and the dependence of the utility functions on the private information are no longer needed. What we need is just the mild assumption that agents’ private signals are independent conditioned on the macro states of nature.

Specifically, a new REE concept has been introduced in Theorem 2, where each agent conditions on the information generated by the aggregate signals instead of the equilibrium price. The reason that the information generated by the equilibrium price is no longer needed in the decision of an individual agent at the interim stage is the following. The aggregate signals are fully revealing of all the macro states while the information generated by the equilibrium price only depends on the macro states. Therefore the latter is redundant in the presence of aggregate signals. Theorem 2 shows the existence of an incentive compatible, interim efficient REE without strict concavity and the dependence of the utility functions on the private information. This extension of Radner’s

\(^2\)This means that different macro states correspond to different aggregate signals; see Subsection 3.1.
REE model is different from previous work in the literature since none of these models take into account the information carried in the aggregate signals.

This paper is organized as follows. In Section 2, we present the economic model, and the notions of REE, Pareto optimality and incentive compatibility. The assumptions and the main results are stated in Section 3. The proof of the main results is given in Section 4. In Section 5, we discuss related literature. Some concluding remarks are provided in Section 6.

2 The Economic model

In this section, we define the notion of a private information economy, followed by the definitions of rational expectations equilibrium, rational expectations equilibrium with aggregate signals, Pareto optimality and incentive compatibility.

2.1 Private information economy

We take an atomless probability space\(^3\) \((I, \mathcal{I}, \lambda)\) as the space of agents. Each agent receives a **private signal** of type \(q \in T^0 = \{q_1, q_2, \ldots, q_L\}\). Let \(T^0\) denote the power set of \(T^0\). A **signal profile** \(t\) is a function from \(I\) to \(T^0\). For \(i \in I\), \(t_i\) (also denoted by \(t_i\)) is the private signal of agent \(i\) while \(t_{-i}\) is the restriction of \(t\) to the set \(I \setminus \{i\}\). Let \(T\) be the set of all signal profiles. To model uncertainty, we associate \(T\) with a \(\sigma\)-algebra \(\mathcal{T}\) and a probability measure \(P\) on the measurable space \((T, \mathcal{T})\). For simplicity, we shall assume that \((T, \mathcal{T})\) has a product structure so that \(T\) is the product of \(T_{-i}\) and \(T^0\), while \(\mathcal{T}\) is the product \(\sigma\)-algebra of \(T^0\) and a \(\sigma\)-algebra \(\mathcal{T}_{-i}\) on \(T_{-i}\). For \(t \in T\) and \(t'_i \in T^0\), we shall adopt the usual notation \((t_{-i}, t'_i)\) to denote the signal profile whose value is \(t'_i\) for \(i\) and \(t_j\) for \(j \neq i\).

The **private signal process** is a function from \(I \times T\) to \(T^0\) such that \(f(i, t) = t_i\) for any \((i, t) \in I \times T\). For each \(i \in I\), let \(\hat{t}_i\) be the projection mapping from \(T\) to \(T^0\) with \(\hat{t}_i(t) = t_i\). The **private signal distribution** \(\tau_i\) of agent \(i\) over \(T^0\) is defined as \(\tau_i(\{q\}) = P(\hat{t}_i = q)\) for \(q \in T^0\). \(P^{T_{-i}}(\cdot|q)\) is the conditional probability measure on the measurable space \((T_{-i}, \mathcal{T}_{-i})\) when the private signal of agent \(i\) is \(q \in T^0\). If \(\tau_i(\{q\}) > 0\), then it is clear that for \(D \in \mathcal{T}_{-i}\), \(P^{T_{-i}}(D|q) = P(D \times \{q\})/\tau_i(\{q\})\).

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\(^3\)We use the convention that all probability spaces are countably additive.

\(^4\)In the literature, one usually assumes that different agents have different sets of private signals and requires that agents receive each of them with positive probability. For notational simplicity, we choose to work with a common set \(T^0\) of private signals, but allow zero probability for some of the redundant signals. There is no loss of generality in this latter approach.
We also would like to include another source of uncertainty in our model - the macro level uncertainty. Let $S = \{s_1, s_2, \ldots, s_K\}$ be the set of all possible macro states of nature and $\mathcal{S}$ be the power set of $S$. The $S$-valued random variable $\tilde{s}$ on $T$ models the macro level uncertainty. For each macro state $s \in \mathcal{S}$, denote the event $(\tilde{s} = s) = \{t \in T : \tilde{s}(t) = s\}$ that $s$ occurs by $C_s$. The probability that $s$ occurs is $\pi_s = P(C_s)$. Assume that $\pi_s > 0$.

Let $P_s$ be the conditional probability measure on $(T, T)$ when the random variable $\tilde{s}$ takes value $s$. Thus, for each $B \in T$, $P_s(B) = P(C_s \cap B)/\pi_s$. It is obvious that $P = \sum_{s \in \mathcal{S}} \pi_s P_s$. Note that the conditional probability measure $P_s$ is often denoted as $P(\cdot|s)$ in the literature.

The common consumption set for all the agents is the positive orthant $\mathbb{R}_+^m$. Let $u$ be a function from $I \times \mathbb{R}_+^m \times T$ to $\mathbb{R}_+$ such that for any given $i \in I$, $u(i, x, t)$ is the utility of agent $i$ at consumption bundle $x \in \mathbb{R}_+^m$ and signal profile $t \in T$. For any given $(i, t) \in I \times T$, we assume that $u(i, x, t)$ (also denoted by $u(i, t)(x)$ or $u_i(x, t)$) is continuous, monotonic in $x \in \mathbb{R}_+^m$. For each $x \in \mathbb{R}_+^m$, $u_x(\cdot, \cdot)$ is an integrable function on $I \times T$.

In our model, the initial endowment of an agent depends on her private signal. The initial endowment profile $e$ is a function from $I \times T^0$ to $\mathbb{R}_+^m$ such that for $(i, q) \in I \times T^0$, $e(i, q)$ is the initial endowment of agent $i$ when her private signal is $q$. We assume that for each $q \in T^0$, $e(\cdot, q)$ is $\lambda$-integrable over $I$, and $\int_I e(i, q)d\lambda$ is in the strictly positive cone $\mathbb{R}_+^{m+}$.

Formally, the private information economy is denoted by

$$\mathcal{E} = \{I \times T, u, e, f, (\tilde{i}_i, i \in I), \tilde{s}\}.$$ 

### 2.2 Rational expectations equilibrium, incentive compatibility and efficiency

In this subsection, we define the notion of a rational expectations equilibrium. Since the report of agents’ private signals plays a key role for determining the equilibrium price, the issue of incentive compatibility also arises.

A price is a normalized nonnegative vector $p$ in $\Delta_m$, where $\Delta_m$ is the unit simplex of $\mathbb{R}_+^m$. A price process $\tilde{p}$ is a function from $T$ to $\Delta_m$. For each $t \in T$, $\tilde{p}(t)$ is the price when the signal profile is $t$. For notational simplicity, the letter $p$ will be used both for a

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5In the sequel, we shall often use subscripts to denote some variable of a function that is viewed as a parameter in a particular context.

6The utility function $u(i, \cdot, t)$ is monotonic if for any $x, y \in \mathbb{R}_+^m$ with $x \leq y$ and $x \neq y$, $u(i, x, t) < u(i, y, t)$.

7The measure structure on the product space $I \times T$ will be specified in Section 4.

8A vector $x$ is in $\mathbb{R}_+^{m+}$ if and only if all of its components are positive.
price and a price process. The terms “price” and “price process” are used synonymously in this paper.

As we allow the agents to use their private information and the information revealed by the equilibrium price, we shall define an allocation $x$ to be a measurable mapping from $I \times \Delta_m \times T^0$ to $\mathbb{R}_+^m$. For each $(i, p, q) \in I \times \Delta_m \times T^0$, $x(i, p, q)$ is the consumption bundle of agent $i$ when the price is $p$ and her private signal is $q$.

Since an agent’s initial endowment is contingent on her private signal $q$, we denote the budget set for agent $i$ by $B_i(p, q)$ when the price is $p$ and her private signal is $q$. Hence, $B_i(p, q) = \{z \in \mathbb{R}_+^m : pz \leq pe(i, q)\}$.

Given a consumption bundle $z \in \mathbb{R}_+^m$, a private signal $q \in T^0$ and a price $p$, the interim (conditional) expected utility of agent $i$ is defined as follows:

$$U_i(z|p, q) = E\{u(i, z, t)|\tilde{p} = p, \tilde{t}_i = q\}. $$

In the rational expectations equilibrium, an agent updates her belief on the distribution of signal profiles based on her own private signal and observation of the equilibrium price. She computes her expected utility with the updated belief and aims to maximize the interim expected utility subject to her budget constraint. The formal definition of the rational expectations equilibrium is given below.

**Definition 1 (Rational Expectations Equilibrium (REE))** A rational expectations equilibrium for the private information economy $\mathcal{E} = \{I \times T, u, e, f, (\tilde{t}_i, i \in I), \tilde{s}\}$ is a pair of an allocation $x^*$ and a price process $p^*$ such that:

1. $x^*$ is feasible, i.e., $\int_I x^*(i, p^*(t), t_i)d\lambda = \int_I e(i, t_i)d\lambda$ for $P$-almost all $t \in T$;

2. for $\lambda$-almost all $i \in I$, and for $P$-almost all $t \in T$, $x^*(i, p^*(t), t_i)$ is a maximizer of the following problem:

$$\max_{z \in B_i(p^*(t), t_i)} U_i(z|p^*(t), t_i).$$

The following is a notion of incentive compatibility in the setting of REE. It says that an agent cannot increase her interim expected utility by mis-reporting her private signal.

**Definition 2 (Incentive Compatibility)** A REE $(x^*, p^*)$ is said to be incentive compatible if $\lambda$-almost all $i \in I$,

$$U_i(x^*(i, p^*(t), t_i)|p^*(t), t_i) \geq U_i(x^*(i, p^*(t_{-i}, t'_i), t_i)|p^*(t_{-i}, t'_i), t_i),$$

holds for all $t \in T$ and $t'_i \in T^0$. 
The following definitions of ex-post efficiency and interim efficiency are self-explanatory.

**Definition 3 (Ex-post Efficiency)** A REE \((x^*, p^*)\) is said to be ex-post efficient if for \(P\)-almost all \(t \in T\), there does not exist an integrable function \(y_t\) from \(I\) to \(\mathbb{R}_+^m\) such that

1. \(\int_I y_t(i) d\lambda = \int_I e(i, t_i) d\lambda\);
2. \(u(i, y_t(i), t_i) > u(i, x^*(i, p^*(t), t), t)\) for \(\lambda\)-almost all \(i \in I\).

**Definition 4 (Interim Efficiency)** A REE \((x^*, p^*)\) is said to be interim efficient if for \(P\)-almost all \(t \in T\), there does not exist an integrable function \(y_t\) from \(I\) to \(\mathbb{R}_+^m\) such that

1. \(\int_I y_t(i) d\lambda = \int_I e(i, t_i) d\lambda\);
2. \(U_i(y_t(i)|p^*, t_i) > U_i(x^*|p^*, t_i)\) for \(\lambda\)-almost all \(i \in I\).

2.3 Rational expectations equilibrium with aggregate signals, incentive compatibility and efficiency

In the standard REE model, agents use information transmitted by commodity prices to better estimate their utility. We shall now take a further step by allowing agents to use one more macroeconomic statistic, namely, the aggregate signals. We call such an equilibrium concept the **rational expectations equilibrium with aggregate signals**.

We follow the notation as in Subsection 2.2.

For any \(t \in T\), the empirical signal distribution is \(\lambda f_t^{-1}\), where \(f_t = f(\cdot, t)\). For \(q \in T^0\), \(\lambda f_t^{-1}(\{q\})\) is the fraction of agents whose private signal is \(q\) when the signal profile is \(t \in T\). Let \(\Delta(T^0)\) be the set of all probability distributions on \(T^0\). It is clear that \(\lambda f_t^{-1} \in \Delta(T^0)\). The empirical signal distribution process \(\tilde{\mu}\) is a function from \(T\) to \(\Delta(T^0)\) such that \(\tilde{\mu}(t) = \lambda f_t^{-1}\), which is also called the aggregate signals.

As noted in the introduction, the information generated by the equilibrium price becomes redundant in the presence of aggregate signals that reveal all the macro states. Thus, an agent can make contingent trades based the private and aggregate signals. An **allocation** \(x\) is a measurable mapping from \(I \times T^0 \times \Delta(T^0)\) to \(\mathbb{R}_+^m\). For \((i, q, \mu) \in I \times T^0 \times \Delta(T^0)\), \(x(i, q, \mu)\) is the consumption bundle of agent \(i\) when her private signal is \(q\) and the empirical signal distribution is \(\mu\).

Given a consumption bundle \(z \in \mathbb{R}_+^m\), the **interim (conditional) expected utility** of agent \(i\) is defined as follows.

\[
U_i(z|q, \mu) = E\{u(i, z, t)|\tilde{t}_i = q, \tilde{\mu} = \mu\}
\] (1)

\[^9\text{The measurability is stated with respect to the usual product \(\sigma\)-algebra of } I, T^0, \text{ and } B_{\Delta(T^0)}.\]
where \( q \in T^0 \) is the private signal of agent \( i \) and \( \mu \in \Delta(T^0) \) is the empirical signal distribution.

In the REE with aggregate signals, agent \( i \) updates her belief on the distribution of signal profiles upon observing her own private signal \( q \) and the empirical signal distribution \( \mu \). Each agent aims to maximize her conditional expected utility subject to her budget constraint. We state the formal definition of REE with aggregate signals below.

**Definition 5 (Rational Expectations Equilibrium (REE) with Aggregate Signals)** Let \( \mathcal{E} = \{ I \times T, u, e, f, (\tilde{t}_i, i \in I), \tilde{s} \} \) be a private information economy. A REE with aggregate signals for \( \mathcal{E} \) is a pair of an allocation \( x^* \) and a price \( p^* \) such that:

1. \( x^* \) is feasible, i.e., \( \int_I x^*(i, t_i, \lambda f_t^{-1})d\lambda = \int_I e(i, t_i)d\lambda \) for \( P \)-almost all \( t \in T \), and
2. \( x_i^* \) is a maximizer of the following problem:

\[
\max_{z} U_i(z | t_i, \lambda f_t^{-1}) \\
\text{subject to } z \in B_i(p^*(t), t_i)
\]

for \( \lambda \)-almost all \( i \in I \) and \( P \)-almost all \( t \in T \).

Condition (1) is standard. Condition (2) indicates that each agent, upon observing the aggregate signals and her own private signal, maximizes her interim expected utility subject to the budget constraint.

One may ask naturally what the relationship between the REE and REE with aggregate signals is. When the REE price in either Definition 1 or Definition 5 fully reveals the macro states,\(^{10}\) one can obtain a corresponding REE allocation from the other with the same price. This means that the two notions are equivalent. Otherwise, the two notions are different. For details, see Subsection 4.4.

Since the aggregate signals depend on the “pooled” information, there is also the issue of incentive compatibility. The formal definition of incentive compatibility is given below.

**Definition 6 (Incentive Compatibility)** A REE with aggregate signals \((x^*, p^*)\) is said to be incentive compatible if \( \lambda \)-almost all \( i \in I \),

\[
U_i(x_i^*(t_i, \lambda f_t^{-1}) | t_i, \lambda f_t^{-1}) \geq U_i(x_i^*(t_i, \lambda f_{t_i}^{-1}) | t_i, \lambda f_{t_i}^{-1}),
\]

holds for all \( t \in T \) and \( t_i \in T^0 \).

\(^{10}\)This means that the price depends only on the macro states, and different macro states correspond to different prices for those states.
The following definition of interim efficiency is analogous to Definition 4.

**Definition 7 (Interim Efficiency)** A REE with aggregate signals \((x^*, p^*)\) is said to be interim efficient if for \(P\)-almost all \(t \in T\), there does not exist an integrable function \(y_t\) from \(I\) to \(\mathbb{R}^m_+\) such that

1. \(\int_I y_t(i) d\lambda = \int_I e(i, t_i) d\lambda;\)
2. \(U_i(y_t(i)|t_i, \lambda f^{-1}_t) > U_i(x^*|t_i, \lambda f^{-1}_t)\) for \(\lambda\)-almost all \(i \in I\).

### 3 The main results

#### 3.1 Assumptions

In order to prove our main results, we shall need several assumptions on the private information economy. The first two assumptions are standard and self-explanatory. The latter two assumptions involve (conditional) independence of the signal process \(f\).

**A1:** For any fixed \(i \in I\) and \(t \in T\), the utility function \(u(i, \cdot, t)\) is strictly concave.

**A2:** The utility function depends only on agent’s private signal. That is, there is a function \(v\) from \(I \times \mathbb{R}^m_+ \times T^0\) to \(\mathbb{R}_+\) such that for each \((i, x, t) \in I \times \mathbb{R}^m_+ \times T\), \(u(i, x, t) = v(i, x, t_i)\).

**A3:** The signal process \(f\) is essentially pairwise independent conditioned on the macro state of nature \(\tilde{s}\). That is, for \(\lambda\)-almost all \(i \in I\), \(\tilde{t}_i\) and \(\tilde{t}_j\) are independent conditioned on \(\tilde{s}\) for \(\lambda\)-almost \(j \in I\).\(^{11}\)

**A4:** The signal process \(f\) is essentially pairwise independent in the sense that for \(\lambda\)-almost all \(i \in I\), \(\tilde{t}_i\) and \(\tilde{t}_j\) are independent for \(\lambda\)-almost all \(j \in I\).\(^{12}\)

When assumptions **A3** or **A4** are imposed, an immediate technical difficulty arises, which is the so-called measurability problem of independent processes. In our context, a signal process that is essentially independent, conditioned on the macro states of nature is never jointly measurable in the usual sense except for trivial cases.\(^ {13}\) Hence, we need

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\(^{11}\)In this paper, we need to work with \(f\) that is independent conditioned on the macro states \(s \in S\). As shown in Theorem 1 of [11], such kind of assumption on conditional independence holds under very general conditions.

\(^{12}\)For a detailed discussion on pairwise (conditional) independence, see [19].

\(^{13}\)See Proposition 2.1 in [19], Proposition 4 in [11], and also [9] for detailed discussion of the measurability problem.
to work with a joint agent-probability space \((I \times T, \mathcal{I} \otimes T, \lambda \otimes P)\) that extends the usual measure-theoretic product \((I \times T, \mathcal{I} \otimes T, \lambda \otimes P)\) of the agent space \((I, \mathcal{I}, \lambda)\) and the probability space \((T, \mathcal{T}, P)\), and retains the Fubini property.\(^{14}\) Its formal definition is given in Definition 8 of Section 4.

Fix \(s \in S\). For each \(A \in \mathcal{I} \otimes T\), let \(\lambda \otimes P_s(A) = \lambda \otimes P((I \times C_s) \cap A) / \pi_s\). Then, \((I \times T, \mathcal{I} \otimes T, \lambda \otimes P_s)\) also extends the usual measure-theoretic product \((I \times T, \mathcal{I} \otimes T, \lambda \otimes P_s)\), and retains the Fubini property. Thus, one can view \(\lambda \otimes P_s\) as the conditional probability measure on \(I \times T\), given \(\tilde{s} = s\).

We shall assume that \(f\) is a measurable process from \((I \times T, \mathcal{I} \otimes T)\) to \(T^0\). When the macro state is \(s\), the signal distribution of agent \(i\) conditioned on the macro state is \(P_{si} f_i^{-1}\), i.e., the probability for agent \(i\) to have \(q \in T^0\) as her signal is \(P_{si}(f_i^{-1}\{q\})\), where \(f_i = f(i, \cdot)\). Let \(\mu_s\) be the agents’ \textbf{average signal distribution} conditioned on the macro state \(s\), i.e.,

\[
\mu_s(\{q\}) = \int_I P_s(f_i^{-1}(\{q\}))d\lambda = \int_I \int_T 1_{\{q\}}(f(i, t))dP_s d\lambda, \tag{2}
\]

where \(1_{\{q\}}\) is the indicator function of the singleton set \(\{q\}\). Throughout the rest of this paper, the following \textbf{non-triviality assumption} on the process \(f\) will be imposed:

\[
\forall s, s' \in S, s \neq s' \Rightarrow \mu_s \neq \mu_{s'}.. \tag{3}
\]

This says that different macro states of nature correspond to different average conditional distributions of agents’ signals.

### 3.2 Main Theorems

Suppose that the agents’ utility functions only depends on their noisy private signals. If the private signals are pairwise independent conditioned on the macro state of nature, then there is a REE whose price depends only on the macro state of nature and each individual’s private signal has negligible influence on such a REE price. Hence such a REE is incentive compatible. This is the content of the following theorem.

As we consider an economy with a continuum of agents, it is expected that the idiosyncratic uncertainty will be canceled out and the equilibrium price only carry information about the macro states.

\(^{14}\)\(\mathcal{I} \otimes T\) is a \(\sigma\)-algebra that contains the usual \(\sigma\)-algebra \(\mathcal{I} \otimes T\), and the restriction of the countably additive probability measure \(\lambda \otimes P\) to \(\mathcal{I} \otimes T\) is \(\lambda \otimes P\).
Theorem 1 Under assumptions A1, A2 and A3, there exists an incentive compatible and interim efficient REE in which the equilibrium price \( p \) depends on the macro state of nature. Moreover, such a REE is also ex-post efficient.

When the economy has only idiosyncratic level information, i.e., the macro state variable \( \tilde{s} \) is constant (or the agents’ private signals are unconditionally independent), then the conclusion of Theorem 1 holds for general utility functions (without requiring strict concavity or dependence on agents’ own private signals in the utilities).

Proposition 1 Under assumption A4, there exists an incentive compatible and interim efficient REE in which the equilibrium price is constant.

Under the general assumption A3, Theorem 2 below shows the existence of an incentive compatible, interim efficient REE without strict concavity and the dependence of the utility functions on the private information as in Theorem 1. An additional advantage of this theorem over Theorem 1 is that agents need not condition their expected utilities on the endogenous equilibrium price. The relationship between the two REE notions in Theorems 1 and 2 will be discussed in Subsection 4.4.

Theorem 2 Under assumption A3, there exists an incentive compatible, interim efficient REE with aggregate signals in which the equilibrium price \( p \) depends only on the macro state of nature.

4 Proof of the main results

In order to work with (conditionally) independent processes constructed from signal profiles, we need to work with an extension of the usual measure-theoretic product that retains the Fubini property. Below is a formal definition of the Fubini extension in Definition 2.2 of [19].

Definition 8 A probability space \((I \times \Omega, \mathcal{W}, Q)\) extending the usual product space \((I \times \Omega, \mathcal{I} \otimes \mathcal{F}, \lambda \otimes P)\) is said to be a Fubini extension of \((I \times \Omega, \mathcal{I} \otimes \mathcal{F}, \lambda \otimes P)\) if for any real-valued \(Q\)-integrable function \(f\) on \((I \times \Omega, \mathcal{W}),\)

\(1)\) the two functions \(f_i\) and \(f_\omega\) are integrable respectively on \((\Omega, \mathcal{F}, P)\) for \(\lambda\)-almost all \(i \in I\), and on \((I, \mathcal{I}, \lambda)\) for \(P\)-almost all \(\omega \in \Omega\);
applications of the exact law of large numbers, see, for example, [3], [4], and [22].

for the classical Lebesgue unit interval. For a most general existence result, see [16]. For some recent
see Section 5 of [19]. It is shown in [21] that one can take the relevant agent space to be an extension
of the classical Lebesgue unit interval. For a most general existence result, see [16]. For some recent
applications of the exact law of large numbers, see, for example, [3], [4], and [22].

\[ \int_I f d\mathcal{P} = \int_I (\int_\Omega f d\mathcal{P}) d\lambda = \int_\Omega (\int_I f d\mathcal{P}) d\lambda. \]

To reflect the fact that the probability space \((I \times \Omega, \mathcal{W}, Q)\) has \((I, \mathcal{I}, \lambda)\) and \((\Omega, \mathcal{F}, P)\)

The following is an exact law of large numbers for a continuum of independent
random variables, which is stated here as a lemma for the convenience of the reader.\(^{16}\)

It will be used in the proof of Theorem 1.

Lemma 1 If a \(\mathcal{I} \otimes \mathcal{T}\)-measurable process \(G\) from \(I \times T\) to a complete separable metric
space \(X\) is essentially pairwise independent\(^{17}\) conditioned on \(\mathcal{I}\) in the sense that for \(\lambda\)-
almost all \(i \in I\), the random variables \(G_i\) and \(G_j\) from \((T, \mathcal{T}, P_s)\) to \(X\) are independent
for \(\lambda\)-almost all \(j \in I\), then for each \(s \in S\), the cross-sectional distribution \(\lambda G_i^{-1}\) of
the sample function \(G_i(\cdot) = G(t, \cdot)\) is the same as the distribution \((\lambda \otimes P_s)G^{-1}\) of the process
\(G\) viewed as a random variable on \((I \times T, \mathcal{I} \otimes \mathcal{T}, \lambda \otimes P_s)\) for \(P_s\)-almost all \(t \in T\). In
addition, for each \(s \in S\), when \(X\) is the Euclidean space \(\mathbb{R}^m\) and \(G\) is \((\lambda \otimes P_s)\)-integrable,
\[ \int_I G d\lambda = \int_{I \times T} G d(\lambda \otimes P_s) \] holds for \(P_s\)-almost all \(t \in T\).

4.1 Proof of Theorem 1

We outline the proof first. In step 1, we construct a large deterministic economy for each
macro state of nature \(s \in S\). Applying the standard Walrasian equilibrium existence
results for a large deterministic economy, we can obtain a Walrasian equilibrium \((y_s, p_s)\)
for such an economy. In step 2, we construct a price \(p\) and an allocation \(x\) from the
collection of allocation-price pairs \(\{(y_s, p_s) : s \in S\}\), and show that \((x, p)\) is a REE in
which the equilibrium price depends only on the macro state of nature.

Step 1: Define a function \(\Gamma\) from \(I \times T\) to \(I \times T^0\) by letting \(\Gamma(i, t) = (i, t_i)\) for \((i, t) \in I \times T\).
\(\Gamma\) is a measurable mapping in the sense that \(\Gamma^{-1}(D) \in \mathcal{I} \otimes \mathcal{T}\) for all \(D \in \mathcal{I} \otimes T^0\).

---

\(^{15}\)The classical Fubini Theorem is only stated for the usual product measure spaces. It does not apply
to integrable functions on \((I \times \Omega, \mathcal{W}, Q)\) since these functions may not be \(\mathcal{I} \otimes \mathcal{F}\)-measurable. However,
the conclusions of that theorem do hold for processes on the enriched product space \((I \times \Omega, \mathcal{W}, Q)\) that
extends the usual product.

\(^{16}\)See Corollaries 2.9 and 2.10 in [19]. We state the result using a complete separable metric
space \(X\) for the sake of generality. In particular, a finite space or an Euclidean space is a complete separable
metric space.

\(^{17}\)For the existence of non-trivial independent processes that are measurable in a Fubini extension,
see Section 5 of [19]. It is shown in [21] that one can take the relevant agent space to be an extension
of the classical Lebesgue unit interval. For a most general existence result, see [16]. For some recent
applications of the exact law of large numbers, see, for example, [3], [4], and [22].
Since assumption A2 holds, there is a function $v$ from $I \times \mathbb{R}_+^m \times T^0$ to $\mathbb{R}_+$ such that for each $(i, x, t) \in I \times \mathbb{R}_+^m \times T$, $u(i, x, t) = v(i, x, t_i)$. Thus we can work with the utility function $v$.

For each $s \in S$, let $\nu_s$ be the measure on $I \times T^0$ defined as $\nu_s(D) = (\lambda \otimes P_s) \left( \Gamma^{-1}(D) \right)$ for $D \in \mathcal{I} \otimes T^0$.

We define a large deterministic economy $\bar{E}_s = \{(I \times T^0, \mathcal{I} \otimes T^0, \nu_s, v, e)\}$, where the utility function for agent $(i, q) \in I \times T^0$ is $v(i, q)(\cdot) = v(i, \cdot, q)$ and the initial endowment for agent $(i, q)$ is $e(i, q)$. By the standard Walrasian equilibrium existence results (see, for example, [2]), there is a Walrasian equilibrium allocation $y_s$ and a strictly positive equilibrium price $p_s \in \Delta_m$ for the economy $\bar{E}_s$ such that:

1. $y_s$ is feasible, i.e., $\int_{I \times T^0} y_s(i, q) d\nu_s = \int_{I \times T^0} e(i, q) d\nu_s$, and
2. for $\nu_s$-almost all agent $(i, q) \in I \times T^0$, $y_s(i, q)$ is a maximal element in her budget set $B_{(i,q)}(p_s) = \{y \in \mathbb{R}_+^m : p_s y \leq p_s e(i, q)\}$.\(^{18}\)

**Step 2:** We define a mapping $x$ from $I \times \Delta_m \times T^0$ to $\mathbb{R}_+^m$ by letting

$$x(i, p, q) = \begin{cases} y_s(i, q) & \text{if } p = p_s \text{ for some } s \in S \\ e(i, q) & \text{otherwise.} \end{cases}$$

For any $p \in \{p_s : s \in S\}$, if there is only one macro state of nature $s \in S$ such that $p_s = p$, the choice of $s$ is unique. A problem will occur if there exist at least two different macro states of nature $s, s' \in S$ such that $p_s = p_{s'} = p$. In this case, the choice of $s$ is not unique. However, when $p_s = p_{s'}$, agent $(i, q)$ has the same budget set in these two different macro states of nature. By the strict concavity of the utility function, we know that the maximal element in the budget set is unique. Hence, $y_s(i, q) = y_{s'}(i, q)$, and it follows immediately that $x$ is well-defined.

Define the following sets

$$\forall s \in S, L_s = \{t \in T : \lambda f_t^{-1} = \mu_s\}; \quad L_0 = T - \cup_{s \in S} L_s.$$ 

The non-triviality assumption in equation (3) implies that for any $s, s' \in S$ with $s \neq s'$, $L_s \cap L_{s'} = \emptyset$. The measurability of the sets $L_s$, $s \in S$ and $L_0$ follows from the

\(^{18}\)By modifying the values of $y_s$ on a null set (if necessary), we can assume that for every $(i, q) \in I \times T^0$, $y_s(i, q)$ is a maximal element in her budget set.
dependence of the random variables and information economy compatibility in Definition 2 is satisfied.

Since any agent has for each macro state we can also write the price as

For any \( s \), \( s' \in S \), \( P_s(C_{s'}) = P_s(L_{s'}) \) is one when \( s = s' \) and zero when \( s \neq s' \). It is clear that \( P(C_s \Delta L_s) = 0 \). Hence \( p(t) \) depends essentially on the macro state of nature. For this reason, we can also write the price as \( p(\bar{s}) \).

Since the society’s signal distribution cannot be influenced by a single agent, we have for each \( i \in I \), \( \lambda f_{(t-i,t_i)}^{-1} = \lambda f_{(t-i,t'_i)}^{-1} \) for any \( t \in T \) and \( t'_i \in T^0 \). This means that for any \( i \in I \), \( t \in T \), \( t'_i \in T^0 \), and \( s \in S \),

\[
t \in L_s \Leftrightarrow \lambda f_{t_i}^{-1} = \mu_s \Leftrightarrow \lambda f_{(t-i,t'_i)}^{-1} = \mu_s \Leftrightarrow (t-i, t'_i) \in L_s.
\]

Since \( L_0 \) is \( T \setminus \cup_{s \in S} L_s \), we also know that \( t \in L_0 \Leftrightarrow (t-i, t'_i) \in L_0 \). Hence, we have \( p(t) = p(t-i, t'_i) \) for any \( t \in T \) and \( t'_i \in T^0 \). Therefore, the condition of incentive compatibility in Definition 2 is satisfied.

We will complete the proof by showing that \( (x, p) \) is a REE for the private information economy \( \mathcal{E} \). By the definition of \( \nu_s \), we have

\[
\int_{I \times T^0} g_s(i, q)d\nu_s = \int_{I \times T} g_s(i, t_i)d\lambda \otimes P_s
\]

and

\[
\int_{I \times T^0} e(i, q)d\nu_s = \int_{I \times T} e(i, t_i)d\lambda \otimes P_s.
\]

Given a macro state \( s \in S \), assumption A3 implies the essential pairwise independence of the random variables \( g_s(i, t_i) \) \( i \in I \) and the random variables \( e(i, t_i) \) \( i \in I \) respectively. By the exact law of large numbers in Lemma 1, there exists \( T_s \in T \) with \( P_s(T_s) = 1 \) such that for every \( t \in T_s \),

\[
\int_I g_s(i, t_i)d\lambda = \int_{I \times T} g_s(i, t_i)d\lambda \otimes P_s
\]
and
\[ \int_I e(i, t) d\lambda = \int_{I \times T} e(i, t) d\lambda \otimes P_s. \]  
(8)

By Equations (5) - (8), we have for any \( t \in T_s, \)
\[ \int_I y_s(i, t) d\lambda = \int_{I \times T_0} y_s(i, q) d\nu_s, \]  
(9)
and
\[ \int_I e(i, t) d\lambda = \int_{I \times T_0} e(i, q) d\nu_s. \]  
(10)

Since \( y_s \) is a feasible allocation for the economy \( \overline{E}_s \), combined with Equations (9) and (10), we have, for any \( t \in T_s, \)
\[ \int_I y_s(i, t) d\lambda = \int_I e(i, t) d\lambda. \]  
(11)

Let \( \Omega_0 = \bigcup_{s \in S} L_s \cap T_s. \) Then \( P(\Omega_0) = 1. \) For any \( t \in \Omega_0, \) there is a unique \( s \in S \) such that \( t \in L_s \cap T_s, \) and thus \( u_i(p(t), t) = y_s(i, t) \) and furthermore
\[ \int_I x(i, p(t), t) d\lambda = \int_I y_s(i, t) d\lambda = \int_I e(i, t) d\lambda. \]  
Consequently, \( x \) is feasible.

It is left to show that for each agent \( i \in I, x_i \) maximizes the conditional expected utility \( U_i \) subject to her budget constraint. Note that for any \( z \in \mathbb{R}^m_+ \) and \( t \in T, \)
\[ U_i(z | p(t), t) = E\{v(i, z, t) | \tilde{p} = p(t), \tilde{t}_i = t_i\} = v(i, z, t). \]  
(12)

As noted above, for any \( t \in \Omega_0, \) there is a unique \( s \in S \) such that \( t \in L_s \cap T_s. \) The budget set for agent \( i \) is \( B_i(p(t), t_i) = B_i(p_s, t_i) = \{z \in \mathbb{R}^m_+ : p_s z \leq p_s e(i, t_i)\}. \) This is exactly the same as the budget set \( B_{i(t_i)}(p_s) \) of agent \( (i, t_i) \) in the deterministic economy \( \overline{E}_s. \) Since \( u(i, \cdot, t) = v(i, \cdot, t_i) = U_i(\cdot | p(t), t_i), \) the following two problems are equivalent:

Problem (I) \[ \max_{\mathbf{z} \in B_{i(t_i)}(p_s)} v(i, \mathbf{z}, t_i) \]
subject to \( \mathbf{z} \in B_{i(t_i)}(p_s) \)

and

Problem (II) \[ \max_{\mathbf{z} \in B_i(p(t), t_i)} U_i(\mathbf{z} | p(t), t_i) \]
subject to \( \mathbf{z} \in B_i(p(t), t_i) \)

Since \( y_s(i, t_i) \) is a maximizer for Problem (I), it must be a maximizer for Problem (II) as well. By definition, \( x(i, p(t), t_i) = y_s(i, t_i). \) Hence, \( x(i, p(t), t_i) \) is a maximizer for
Problem (II). That is to say that $x(i, p(t), t_i)$ maximizes $U_i(\cdot | p(t), t_i)$ subject to the budget set $B_i(p(t), t_i)$.

Hence, $(x, p)$ constitutes a REE for the private information economy $\mathcal{E}$.

Now, we shall prove that $(x, p)$ is interim efficient. Suppose the two conditions in the definition of REE hold for any $t \in \bar{T}$, where $\bar{T} \in T$ and $P(\bar{T}) = 1$. Fix $t \in \bar{T}$, we can construct a large deterministic economy $\mathcal{E}_t = \{(I, \mathcal{I}, \lambda), \bar{u}_t, \bar{e}_t\}$, where $\bar{u}_t(i, \cdot) = U_i(\cdot | p(t), t_i)$ and $\bar{e}_t(i) = e(i, t_i)$ for each agent $i \in I$. Let $\bar{p}_t = p(t)$ and $\bar{x}_t(i) = x(i, p(t), t_i)$ for each $i \in I$. Then, it is easy to see that $(\bar{x}_t, \bar{p}_t)$ is a Walrasian equilibrium for $\mathcal{E}_t$. It follows that $\bar{x}_t$ is efficient in the sense that there is no allocation $\bar{y}$ for the economy $\mathcal{E}_t$ such that

$$
\int_I \bar{y}(i)d\lambda = \int_I \bar{e}_t(i)d\lambda, \quad \bar{u}_t(i, \bar{y}(i)) > \bar{u}_t(i, \bar{x}_t(i)) \text{ for } \lambda\text{-almost all } i \in I.
$$

The above two conditions are equivalent to

$$
\int_I \bar{y}(i)d\lambda = \int_I e(i, t_i)d\lambda, \quad U_i(\bar{y}| p(t), t_i) > U_i(x(i, p(t), t_i)| p(t), t_i) \text{ for } \lambda\text{-almost all } i \in I.
$$

Hence, $(x, p)$ is interim efficient.

The ex-post efficiency follows from the observation in Equation (12) that $U_i(z| p(t), t_i) = v(i, z, t_i)$. Q.E.D.

### 4.2 Proof of Proposition 1

Though we work with general utility functions in this proposition without the special restrictions in assumptions A1 and A2, we can convert the problem to the setting of Theorem 1 and then follow the proof of that theorem.

Define the conditional expected utility $v(i, z, q) = E\{u(i, z, t)| \bar{t}_i = q\}$ for any $i \in I$, $z \in \mathbb{R}^n_+$ and $q \in T^0$. Let $u'(i, z, t) = v(i, z, t_i)$ for any $i \in I$, $z \in \mathbb{R}^n_+$ and $t \in T$. By working with $u'$, we follow exactly the same proof as in Theorem 1 to obtain the allocation $x$. The corresponding conditional expected utility $E\{u'(i, z, t)| \bar{p} = p, \bar{t}_i = q\}$ will be denoted by $U_i'(z| p, q)$.

Note that the strict concavity condition as in assumption A1 is only used in the proof of Theorem 1 to show that the allocation $x(i, p, q)$ is well-defined when two different macro states correspond to the same equilibrium price. In the setting of this proposition, since $S$ is a singleton set, $x(i, p, q)$ is always well-defined without the condition of strict concavity.
We then follow the same proof as in Theorem 1 to obtain the price system \( p \), the incentive compatibility and feasibility of the allocation \( x \). Since \( S \) is a singleton set, \( p \) is constant. By the definition of the conditional expected utility \( U_i' \) and the fact that \( \hat{p} \) is constant,

\[
U_i'(z|p, q) = E\{u'(i, z, t) | \tilde{t}_i = q\} = E\{v(i, z, t_i) | \tilde{t}_i = q\} = v(i, z, q).
\]

Again by the fact that \( \hat{p} \) is constant, we have

\[
U_i(z|p, q) = E\{u(i, z, t) | \tilde{t}_i = q\} = v(i, z, q).
\]

Hence, \( U_i' = U_i \). As in the proof of Theorem 1, \( x_i(t, p(t), t_i) \) maximizes \( U_i(\cdot | p(t), t_i) \) subject to the budget set \( B_i(p(t), t_i) \). Hence \( (x, p) \) constitutes a REE for the private information economy \( E \).

The proof for interim efficiency is similar to that of Theorem 1, we will not repeat it. Q.E.D.

### 4.3 Proof of Theorem 2

This proof is similar to that of Theorem 1, we only highlight the differences.

**Step 1:** We introduce a large deterministic economy \( \bar{E}_s = \{(I \times T^0, I \otimes T^0, \nu_s), V_s, e\} \), where the utility function for agent \((i, q) \in I \times T^0\) is

\[
V_s(i, \cdot, q) = E\{u(i, \cdot, t) | \tilde{s} = s, \tilde{t}_i = q\}
\]

and the initial endowment for agent \((i, q)\) is \( e(i, q) \). Let \((y_s, p_s)\) be an equilibrium for the economy \( \bar{E}_s \) (with \( p_s \) strictly positive).

**Step 2:** Define a mapping \( x \) from \( I \times T^0 \times \Delta(T^0) \) to \( \mathbb{R}^n_+ \) by letting

\[
x(i, q, \mu) = \begin{cases} y_s(i, q) & \text{if } \mu = \mu_s \text{ for some } s \in S \\ e(i, q) & \text{otherwise.} \end{cases}
\]

By the non-triviality assumption in equation (3) in Subsection 3.1, there is at most one \( s \) such that \( \mu = \mu_s \). Hence, \( x \) is well-defined.

For \( t \in T \), we define \( p(t) \) in the same way as in the proof of Theorem 1. Thus, the price system depends only on the macro state of nature.

As shown in the proof of Theorem 1, we have for each \( i \in I \), \( \lambda f^{-1}_{(t_i, t_i')} = \lambda f^{-1}_{(t-t_i, t_i')} \) for any \( t \in T \) and \( t_i' \in T^0 \). Therefore,

\[
U_i(x_i^*(t, \lambda f^{-1}_t) | t, \lambda f^{-1}_t) = U_i(x_i^*(t_i, \lambda f^{-1}_{(t-t_i, t_i')} | t_i, \lambda f^{-1}_{(t-t_i, t_i')})),
\]

holds for all \( t \in T \) and \( t_i' \in T^0 \), i.e., the condition of incentive compatibility in Definition 6 is satisfied.
The feasibility of $x$ follows from the same proof as in Theorem 1.

Note that for each $s \in S$, $\tilde{s}(t) = s$ and $\tilde{\mu}(t) = \mu_s$ hold for $P$-almost all $t \in L_s$. Thus, the $\sigma$-algebras generated by $\tilde{s}$ and $\tilde{\mu}$ are essentially the same. Hence

$$U_i(\cdot|q, \mu_s) = E\{u(i, \cdot, t)|\tilde{t}_i = q, \tilde{\mu} = \mu_s\} = E\{u(i, \cdot, t)|\tilde{s} = s, \tilde{t}_i = q\} = V_s(i, \cdot, q).$$

A similar argument as in the proof of Theorem 1 shows that $x(i, t, \lambda f_t^{-1})$ maximizes $U_i(\cdot|t, \lambda f_t^{-1})$ subject to the budget set $B_i(p(t), t_i)$ for $P$-almost all $t \in T$. Hence, $(x, p)$ constitutes a REE with aggregate signals for the private information economy $\mathcal{E}$.

The proof for interim efficiency is similar to that of Theorem 1, we will not repeat it. Q.E.D.

4.4 The relationship between the REE with aggregate signals and the REE

The purpose of this subsection is to demonstrate that when the equilibrium price fully reveals the macro states, the REE with aggregate signals and the standard REE are equivalent. Without the full revelation of the macro states, the two notions need not be equivalent. We illustrate this equivalence by converting a REE with aggregate signals to a standard REE. The other direction is similar.

Let $(x^*, p^*)$ be a REE with aggregate signals. Assume that the REE price $p^*$ fully reveals the macro states. That is, the price depends only on the macro states, and different macro states correspond to different prices for those states. Namely, $p^*(t)$ is a constant $p^*_s$ on $C_s$, and $p^*_s \neq p^*_{s'}$ when $s \neq s'$. We can define an allocation $\bar{x}^*$ for the REE notion in Theorem 1 by letting

$$\bar{x}^*(i, p, q) = \begin{cases} x^*(i, q, \mu_s) & \text{if } p = p^*_s \text{ for some } s \in S \\ e(i, q) & \text{otherwise.} \end{cases}$$

By the feasibility of $x^*$ as in Definition 5, we have $\int_I x^*(i, t, \lambda f_t^{-1})d\lambda = \int_I e(i, t)\lambda d\lambda$ for $P$-almost all $t \in T$. We also know that for $P$-almost all $t \in T$, there exists a unique $s \in S$ such that $p^*(t) = p^*_s$ and $\lambda f_t^{-1} = \mu_s$. Thus, we obtain that $\int_I \bar{x}^*(i, p^*(t), t_i)d\lambda = \int_I e(i, t)\lambda d\lambda$ for $P$-almost all $t \in T$.

Next, since both the REE price $p^*$ and the aggregate signals $\lambda f_t^{-1}$ fully reveal the macro states, $\tilde{s}$, $p^*$ and $\lambda f_t^{-1}$ generate the same partition modulo the null events. Hence, $U_i(\cdot|t, \lambda f_t^{-1}) = U_i(\cdot|p^*(t), t_i)$. Therefore, it follows from condition (2) of Definition 5 that for $\lambda$-almost all $i \in I$, and for $P$-almost all $t \in T$, $\bar{x}^*(i, p^*(t), t_i)$ is a maximizer of the following problem:

$$\max_{z} U_i(z|p^*(t), t_i),$$

subject to $z \in B_i(p^*(t), t_i)$

Hence, $(\bar{x}^*, p^*)$ is a REE as in Definition 1.
5 Discussion

In view of the non-existence example of Kreps (1977), the earlier contributions on the REE were focused on the generic existence (Radner (1979), Allen (1981)). By now, it is well-known that with a finite number of agents, the REE not only does not exist but also fails to be Pareto efficient and incentive compatible; see [7], p. 31 and also Example 9.1.1, p.43.

In contrast to the above, the current paper demonstrates that the REE exists universally, and also is Pareto efficient and incentive compatible. Thus, the paper resolves the difficulties that REE faces as a solution concept. Our results enable us to conclude that in the presence of a continuum of agents with non-trivial but negligible private information, the REE becomes an appealing concept as it does have desirable properties, contrary to the finite agent case.

There are several other papers dealing with the REE with a continuum of agents that we discuss below. We first discuss the relationship of our Theorem 1 and Proposition 1 with related results in the literature. Our Theorem 2 proves some results about a new concept introduced in this paper, which have no analogs in the literature.

The early contributions by Einy, Moreno and Shitovitz [5] and [6] consider an asymmetric information economy with a continuum of agents and a finite number of states of nature. The private information of each agent is a partition of the finite state space. Since a finite state space has only finitely many different partitions, we can find a partition on the measure space of agents so that all the agents in each partitioning set of agents share the same private information. Therefore, there is little heterogeneity on the private information side of their model. In contrast, our model allows agents to have non-trivial idiosyncratic private information. Hence, their work is not directly related to ours.

Another interesting paper by Heifetz and Minelli [12] models the idea of informational smallness without aggregate uncertainty for an asymmetric information economy with a continuum of independent replica economies. The definition of an allocation in their paper is based on the notion of Pettis integral (Definition 6, p. 213). Instead of the usual state-wise feasibility condition, the authors used the condition that the value of the aggregate consumption is equal to the value of the aggregate endowment for every price function (see p. 212 and p. 213). Note that this does not imply market clearing, which requires the equality of aggregate consumption and aggregate endowment for almost all the state of nature. It seems to us that the reason they used this approach is to avoid
the so-called measurability problem associated with a continuum of independent random variables.\footnote{See, for example, \cite{19}.} Our Proposition 1 has a similar flavor of a constant REE price as in their Theorem 2 on a continuum of independent replica economies. A main difference here is that we work with a general independence condition, and a Fubini extension, where the state-wise feasibility condition is fulfilled (see part 1 of Definition 1). Our Theorem 1 allows for a more general set-up where aggregate uncertainty is included in addition to idiosyncratic private information.

Gul-Postlewaite (1992), and Mclean-Postlewaite (2002, 2003) model the idea of informational smallness (i.e., roughly speaking, approximate perfect competition) in countable replica economies. Their main objective is to show the consistency of ex post efficiency and incentive compatibility.\footnote{The corresponding limiting results for a continuum of agents are considered in \cite{20}.} They do not consider the REE notion in those papers. Reny-Perry (2006) consider the REE in the setting of a double auction with many buyers and sellers, which is not in the general equilibrium framework with asymmetric information as in Radner \cite{17} and Allen \cite{1}.

In Theorem 2, a new REE concept has been introduced, where each agent conditions her expected utility on the information generated by the aggregate signals instead of the equilibrium price. The reason that the information generated by the equilibrium price is no longer needed in the decision of an individual agent at the interim stage is the following. The aggregate signals are fully revealing of all the macro states while the information generated by the equilibrium price only depends on the macro states. Therefore the latter is redundant in the presence of aggregate signals. Theorem 2 shows the existence of an incentive compatible, interim efficient REE without strict concavity and the dependence of the utility functions on the private information. This extension of Radner-Allen REE model is different from previous work in the literature since none of these models take into account the information carried in the aggregate signals.

6 Concluding remarks

We model the REE notion in an asymmetric information economy where each agent’s private signal has negligible influence on the equilibrium price. This way, we are able to overcome the problems of the universal existence, ex post efficiency and incentive compatibility of the REE.

We also introduce a new notion of REE with aggregate signals, i.e., agents maxi-
mize their expected utility conditioned on their own private information, and the information generated by the aggregate signals. For such a REE notion we proved that it exists, it is interim efficient and incentive compatible.

To the best of our knowledge, this is the first paper which models the REE notion with a continuum of agents where both aggregate uncertainty and idiosyncratic private information are included. In such a general model, the difficulties that one encounters in the finite agent setting are no longer present.

Finally, it should be noted that the exact limiting results in this paper on an atomless economy with asymmetric information have asymptotic analogs for large but finite asymmetric information economies. For the general methodology of obtaining asymptotic results, see Section 6 of [20].

References


