International Capital Flows and World Production Efficiency

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Abstract

We develop a tractable two-country overlapping-generations model with domestic financial frictions and show that cross-country differences in financial development explain three recent empirical patterns of international capital flows. In our model, domestic financial frictions create two distinct distortions on the interest rates and production efficiency in the less financially developed country. International capital flows help ameliorate the two distortions.

Financial capital flows and foreign direct investment not only facilitate cross-country resource reallocation, but also trigger within-country resource reallocation among individuals with different productivity. From the efficient perspective, world output in the steady state is strictly higher under full capital mobility than under international financial autarky. However, if either financial capital flows or foreign direct investment is restricted, the world output may be lower.

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1 Introduction

This paper analyzes how the recent empirical patterns of international capital flows may affect world production efficiency. According to the standard neoclassical macroeconomic theory, capital should flow “downhill” from the rich country where the marginal return on capital is low to the poor country where the marginal return on capital is high. As a result, world output should be higher than under international financial autarky (IFA, hereafter). Meanwhile, there would be no difference between gross and net capital flows because capital flows would be unidirectional.

The recent empirical patterns of international capital flows are in stark contrast to these predictions (Lane and Milesi-Ferretti, 2001, 2007a,b). First, capital in the net term flows “uphill” from poor to rich countries (Prasad, Rajan, and Subramanian, 2006, 2007). Second, financial capital flows from poor to rich countries, while foreign direct investment (FDI, hereafter) flows in the opposite direction (Ju and Wei, 2007). Third, despite of its negative net positions of international investment since 1986, the U.S. has been receiving a positive net investment income until 2005 (Gourinchas and Rey, 2007; Hausmann and Sturzenegger, 2007; Higgins, Klitgaard, and Tille, 2007). According to conventional neoclassical models, the recent pattern of “uphill” net capital flows reduce the world output. However, in order to evaluate its efficiency effects, we need a model with the theoretical predictions in line with the current patterns of capital flows.

Recent research offers two main explanations to these empirical facts. Devereux and Sutherland (2009) and Tille and van Wincoop (2008, 2010) focus on the risk-sharing that investors can achieve by diversifying investment globally. International portfolio investment is determined by the cross-correlation patterns of aggregate shocks hitting individual economies. These models do not distinguish between FDI and portfolio investment.

The second strand of literature emphasizes the implications of domestic financial market imperfections on the patterns of international capital flows (Antras and Caballero, 2009; Antras, Desai, and Foley, 2009; Aoki, Benigno, and Kiyotaki, 2009a,b; Caballero, Farhi, and Gourinchas, 2008; Mendoza, Quadrini, and Rios-Rull, 2009; Smith and Valderrama, 2008). Matsuyama (2004) shows that in the presence of domestic credit market imperfections, financial market globalization may lead to a steady-state equilibrium in which fundamentally identical countries end up with different levels of output per capita, a result he calls “symmetry breaking”. In the steady state, capital flows “uphill” from the poor to the rich country. Given the concave production function at the country level, the world output in the steady state is lower than under international financial autarky. Ju and Wei (2007, 2008) analyze how cross-country differences in various aspects jointly generate the two-way flows of financial capital and FDI. The distinction between FDI and portfolio investment plays a key role in their models.

Following the second strand of the literature, we develop a two-country overlapping-
generations model with domestic financial frictions and prove analytically that cross-
country differences in financial development explain these three empirical facts. Furthermore,
despite of “uphill” net capital flows, world output in the steady state is higher than
under IFA, in contrast to the prediction of conventional neoclassical models.

In our model, individuals differ in productivity and the credit markets channel re-
sources from those with low to those with high productivity. If the credit market were
perfect, production would be efficiently conducted in the sense that the rates of return
on loans and equity capital would be equal to the social rate of return. However, due
to domestic financial frictions, the more productive individuals are subject to borrowing
constraints. Under IFA, the constraint on the aggregate credit demand keeps the rate of
return on loan, i.e., the loan rate, inefficiently lower while the rate of return to equity
capital, i.e., the equity rate, higher than social rate of return. Thus, financial frictions
distort the two interest rates. Meanwhile, due to the constraint on the aggregate credit
demand, the investment of the more (less) productive individuals is lower (higher) than
the socially efficiently level. Thus, financial frictions also distort production efficiency.

Our model economy consists of two countries, N (North) and S (South), which are
fundamentally identical except that country N is more financially developed than country
S. Suppose that the two countries are in the steady state under IFA before capital mobility
is allowed. Thus, the loan rate is higher, the equity rate is lower, and aggregate output
is higher in country N than in country S initially. Under full capital mobility, the initial
interest rate differentials drive financial capital flows from from country S to country N
and FDI flows in the opposite direction. since country N has the larger credit market, net
capital flows are “uphill” from country S to country N. By receiving a higher return on
its foreign assets than what it pays for its foreign liabilities, country N obtains a positive
net investment income despite of its negative net international investment position. This
way, our model generates the theoretical predictions in line with the three empirical facts.

International capital flows affect world output through affecting the size and the com-
position of aggregate investment in the two countries. Take financial capital flows as
an example. By cross-country resource reallocation, financial capital flows reduce
(raise) the size of aggregate investment in country S (N), which widens the cross-country
aggregate output gap. Given the concave aggregate production function with respect to
aggregate investment in each country, financial capital flows have a negative net impact
on the steady-state world output via affecting the size of aggregate investment in the two
countries, according to the Jensen’s inequality. For simplicity, we call it the net invest-
ment size effect of financial capital flows. Meanwhile, financial capital flows raise (reduce)
the loan rate in country S (N), which reduces (raises) inefficient production in country S
(N). Thus, financial capital flows also trigger within-country resource reallocation
among individuals with different productivity. Since the initial distortion on pro-
duction efficiency is more severe in country S than in country N, the efficiency gains in
country S dominate the efficiency losses in country N so that financial capital flows have a positive net impact on the steady-state world output via affecting the composition of aggregate investment in the two countries. For simplicity, we call it the net investment composition effect of financial capital flows. The same mechanism applies to FDI flows.

The net investment size effect essentially depends on the net capital flows, while the net investment composition effect depends on gross capital flows. Under full capital mobility, two-way capital flows imply that gross flows are much larger than net flows. Thus, the net composition effect strictly dominates the net size effect so that world output is higher than under IFA. However, if either financial capital flows or FDI flows are restricted, net capital flows coincide with gross capital flows. Thus, the net size effect may dominate the net composition effect so that world output may be lower than under IFA.

Our model is closely related to von Hagen and Zhang (2010) and Matsuyama (2004). Both papers assume that only some individuals can produce while others just lend all the savings inelastically to them. As a result, the loan rate adjusts to clear the credit market and the total savings are entirely invested by the productive individuals. This way, financial frictions only distort interest rates but not production efficiency under IFA. International capital flows only affect the size of aggregate investment in the two countries, which widens the cross-country output gap and world output in the steady state is strictly lower than under IFA. Our model differs from theirs in the presence of within-country resource reallocation between individuals with different productivity. The efficiency gains resulting from capital flows create the space for world output gains.

As widely documented in the empirical literature (Barlevy, 2003; Hsieh and Klenow, 2009; Jeong and Townsend, 2007; Levine, 1997; Midrigan and Xu, 2009), financial frictions distort production efficiency in the sense that some resources are inefficiently allocated into the less productive projects. If such distortions were not considered, the efficiency analysis of international capital flows would be incomplete and misleading.

The rest of the paper is organized as follows. Section 2 describes the model under IFA and discusses how domestic financial frictions distort interest rates and production efficiency. Section 3 shows how cross-country differences in financial development drive international capital flows and discusses the impacts on world production efficiency. Section 4 concludes with the main results. The appendix collects proofs and related issues.

2 The Model under International Financial Autarky

2.1 The Model Setting

The world economy consists of two countries, N (North) and S (South). There are a final good, which is internationally tradable and serves as numeraire, and two types of intermediate goods, A and B, which are not traded internationally. The prices of
intermediate goods in country \( i \in \{N, S\} \) and period \( t \) are denoted by \( v_t^i, A \) and \( v_t^i, B \). In the following, variables in country \( i \) are denoted with the superscript \( i \). The final good can be consumed or transformed into intermediate goods. At the beginning of each period, final goods \( Y_t^i \) are produced with intermediate goods, \( M_{t}^{i, A} \) and \( M_{t}^{i, B} \), and labor \( L_t^i \) in a Cobb-Douglas fashion. Labor and intermediate goods are priced at their respective marginal products in terms of final goods. To summarize,

\[
Y_t^i = \left( \frac{M_{t}^{i, A}}{\alpha - \alpha_B} \right)^{\frac{\alpha - \alpha_B}{\alpha B}} \left( \frac{M_{t}^{i, B}}{\alpha_B} \right)^{\frac{\alpha B}{1 - \alpha}}, \quad \text{where } \alpha \in (0, 1), \ \alpha_B \in (0, \alpha),
\]

\[
\omega_t^i L_t^i = (1 - \alpha) Y_t^i \quad v_t^{i, A} M_{t}^{i, A} = (\alpha - \alpha_B) Y_t^i, \quad v_t^{i, B} M_{t}^{i, B} = \alpha_B Y_t^i.
\]

\( \omega_t^i \) denotes the wage rate. \((1 - \alpha), (\alpha - \alpha_B) \) and \( \alpha_B \) measure the respective shares of labor, intermediate good A and B in aggregate production. There is no uncertainty in the economy. In this section, we assume that international capital flows are not allowed.

In both countries, the population consists of two generations, the old and the young, which live for two periods each. There is no population growth and the population size of each generation in each country is normalized to one. Agents when young are endowed with a unit of labor which they supply inelastically to the production of final goods \( L_t^i = 1 \) at the wage rate \( \omega_t^i \) in period \( t \). Each generation consists of two types of agents of mass \( \eta \) and \( 1 - \eta \), respectively, which we call entrepreneurs and households. Each household (entrepreneur) when young is endowed with a linear technology to produce intermediate good \( A \) (\( B \)) one-to-one from final goods. Individual \( j \) born in country \( i \) and period \( t \) have the preference over consumption in two periods of life,

\[
U_t^{i,j} = (1 - \beta) \ln \frac{c_{y,t}^{i,j}}{(1 - \beta)} + \beta \ln \frac{c_{o,t+1}^{i,j}}{\beta}
\]

where \( c_{y,t}^{i,j} \) and \( c_{o,t+1}^{i,j} \) denote his consumption when young and when old; \( j \in \{e, h\} \) denotes entrepreneur and household, respectively; \( \beta \in (0, 1) \) measures the relative importance of utility from consumption when old with respect to the lifetime welfare. By setting \( \beta = 0.5 \), we assume that individuals put equal weights on utility from consumption when young and when old; by setting a higher \( \beta \), we assume that individuals care more about utility from consumption when old; in the extreme case of \( \beta = 1 \), individuals only care about consumption when old, which is the case in von Hagen and Zhang (2010).

Consider any particular household born in country \( i \) and period \( t \). In period \( t \), the household receives the labor income \( \omega_t^i \), consumes \( c_{y,t}^{i,h} \), save \( i_t^{i,h} \) and save \( i_t^{i,h} \) in the form of the investment in his own production project \( v_t^{i,A} \) and the loans to entrepreneurs \( d_t^i \) at the gross loan rate of \( R_t^i \). In period \( t + 1 \), the household receives the project revenue \( v_{t+1}^{i,A} \) and the gross deposit return \( R_t^i d_t^i \). The no-arbitrage condition is

\[
R_t^i = v_{t+1}^{i,A}.
\]
The household consumes his total wealth $c^{i,e}_{o,t+1} = v^{i,e}_{t+1}i^{i,h}_t + R^i_t i^{i,h}_t = R^i_t (\omega^i_t - c^{i,h}_{y,t})$ before exiting from the economy. His lifetime budget constraint is $c^{i,h}_{y,t} + c^{i,h}_{o,t+1} = \omega^i_t$. Given the logarithm utility function (3), his optimal consumption choices in two periods are,

$$c^{i,h}_{y,t} = (1 - \beta)\omega^i_t$$ and $c^{i,h}_{o,t+1} = R^i_t \beta \omega^i_t$. (5)

Substituting them into the utility function, the household lifetime utility becomes log-linear in the labor income and the loan rate,

$$U^i_{t,h} = \ln \omega^i_t + \beta \ln R^i_t.$$

Consider any particular entrepreneur born in country $i$ and period $t$. In period $t$, the entrepreneur receives the labor income $\omega^i_t$, consumes $c^{i,e}_{y,t}$, and finances his investment $i^{i,e}_t$ using own funds $n^i_t = \omega^i_t - c^{i,e}_{y,t}$ together with debts $z^i_t = i^{i,e}_t - n^i_t = i^{i,e}_t - (\omega^i_t - c^{i,e}_{y,t})$. (6)

In period $t + 1$, he receives the project revenue $v^{i,B}_{t+1}i^{i,e}_t$. After repaying the debts, he consumes the rest, $c^{i,e}_{o,t+1} = v^{i,B}_{t+1}i^{i,e}_t - R^i_t z^i_t$, before exiting from the economy.

Assumption 1. $\eta \in (0, \frac{\alpha B}{\alpha})$.

Assumption 1 guarantees that total savings of entrepreneurs are less than the size of socially desired aggregate investment in sector B. In equilibrium, entrepreneurs finance their project investment using external funds. Due to credit market imperfections, entrepreneurs can borrow only up to a fraction of the future project revenue,

$$R^i_{t,z^i_t} \leq \theta^i i^{i,e}_ti^{i,B}_{t+1}. (7)$$

Following Matsuyama (2004, 2007), we use $\theta^i \in [0, 1]$ to measure the level of financial development in country $i$. $\theta^i$ captures a wide range of institutional factors\(^1\) and is higher in countries with more sophisticated financial and legal systems, better creditor protection, and more liquid asset market. The two countries are fundamentally identical except that country N is more financially developed than country S, $0 \leq \theta^S < \theta^N \leq 1$.

The equity rate is defined as the rate of return on the entrepreneur’s own funds,

$$\Gamma^i_t \equiv \frac{v^{i,B}_{t+1}i^{i,e}_t - R^i_t z^i_t}{n^i_t} = v^{i,B}_{t+1} + (v^{i,B}_{t+1} - R^i_t)(\lambda^i_t - 1) \geq R^i_t,$$ (8)

where $\lambda^i_t \equiv \frac{i^{i,e}_t}{n^i_t}$ denotes the investment-equity ratio. For a unit of equity capital invested, the entrepreneur gets $v^{i,B}_{t+1}$ as the marginal return. In addition, he can borrow $(\lambda^i_t - 1)$

\(^1\)The pledgeability, $\theta^i$, can be argued in various forms of agency costs (Hart and Moore, 1994; Holmstrom and Tirole, 1997; Townsend, 1979). The strictness of the borrowing constraint may also depend on idiosyncratic features of entrepreneurs and their projects, e.g., the credit records, the availability of collateral assets, the project rating, etc. Since we focus here on the aggregate implications of financial development, we assume that the entrepreneurial projects invested in country $i$ are homogeneous and subject to the same $\theta^i$ for simplicity.
unites of debt which provides him the extra return \((v_{i,t+1}^{i,B} - R_t^i)\). The term \((v_{i,t+1}^{i,B} - R_t^i)(\lambda_t^i - 1)\) captures the leverage effect, depending positively on the debt-equity ratio, \((\lambda_t^i - 1)\) and the spread, \((v_{i,t+1}^{i,B} - R_t^i)\). In equilibrium, the equity rate should be no less than the loan rate; otherwise, the entrepreneur would rather lend than borrow. The inequality (8) is equivalent to \(R_t^i \leq v_{i,t+1}^{i,B}\) and we call it the participation constraint for the entrepreneur.

If \(R_t^i < v_{i,t+1}^{i,B}\), the entrepreneur borrows to the limit, i.e., he finances the investment \(i_t^{i,e}\) using \(\theta v_{i,t+1}^{i,B} + \eta_{i,t+1}^{i,e}\) units of loan and \(n_t^{i,e}\) units of equity capital in period \(t\). After repaying the debt in period \(t + 1\), the entrepreneur gets \((1 - \theta) v_{i,t+1}^{i,B} i_t^{i,e}\) in the net term. If \(R_t^i = v_{i,t+1}^{i,B}\), the entrepreneur does not borrow to the limit. According to equation (8), the equity rate is equal to the loan rate, \(\Gamma_t^i = R_t^i\). To summarize,

\[
\Gamma_t^i = \begin{cases} 
\frac{(1-\theta)v_{i,t+1}^{i,B} i_t^{i,e}}{n_t^{i,e}} & \text{if } R_t^i < v_{i,t+1}^{i,B}, \\
 v_{i,t+1}^{i,B} & \text{if } R_t^i = v_{i,t+1}^{i,B}.
\end{cases}
\]

(9)

The entrepreneur’s lifetime budget constraint is \(c_t^{i,e} + \frac{c_{o,t+1}^{i,e}}{R_t^i} = \omega_t^i\). Given the logarithm utility function (3), his optimal consumption choices in two periods are,

\[
c_t^{i,e} = (1 - \beta) \omega_t^i \quad \text{and} \quad c_{o,t+1}^{i,e} = \Gamma_t^i \beta \omega_t^i.
\]

(10)

Substituting them into the utility function, the entrepreneur’s lifetime utility becomes log-linear in the labor income and the equity rate, \(U_t^i = \ln \omega_t^i + \beta \ln \Gamma_t^i\).

Aggregate outputs of intermediate good A and B in period \(t + 1\) are

\[
M_{t+1}^{i,A} = (1 - \eta) i_t^{i,h} \quad \text{and} \quad M_{t+1}^{i,B} = \eta_t^{i,e}.
\]

(11)

The credit market and the final good market clear in equilibrium

\[
(1 - \eta) d_t^i = \eta z_t^i, \quad \Rightarrow \quad (1 - \eta) i_t^{i,h} + \eta i_t^{i,e} = \beta \omega_t^i,
\]

(12)

\[
C_t^i + I_t^i = Y_t^i,
\]

(13)

where \(C_t^i = \eta (c_{y,t}^{i,e} + c_{o,t}^{i,e}) + (1 - \eta) (c_t^{i,h} + c_{o,t}^{i,h})\) and \(I_t^i = \eta_t^{i,e} + (1 - \eta) i_t^{i,h}\) denote aggregate consumption and aggregate investment in country \(i\) and period \(t\).

**Definition 1.** Given the level of financial development \(\theta^i\), the market equilibrium in country \(i \in \{N, S\}\) under IFA is a set of allocations of households, \(\{\eta^{i,e}, c_{o,t}^{i,e}, c_{o,t}^{i,h}\}\), entrepreneurs, \(\{\eta_t^{i,e}, z_t^{i,e}, c_{y,t}^{i,e}, c_{o,t}^{i,e}\}\), and aggregate variables, \(\{Y_t^i, M_{t+1}^{i,A}, M_{t+1}^{i,B}, \omega_t^i, i_t^{i,A}, i_t^{i,B}, R_t^i, \Gamma_t^i\}\), satisfying equations (1)-(2), (4)-(7), (9)-(12),

Under IFA, young individuals invest their labor income in the production of intermediate goods in period \(t\), \(I_t^i = \beta \omega_t^i = \beta (1 - \alpha) Y_t^i\). In period \(t + 1\), aggregate revenue of
intermediate goods is \( v_{t+1}^i M_{t+1}^i + v_{t+1}^B M_{t+1}^B = \alpha Y_{t+1}^i = \rho \omega_{t+1} \), where \( \rho \equiv \frac{\alpha}{1-\alpha} \). We define

\[
\Psi_t^i \equiv \frac{v_{t+1}^i M_{t+1}^i + v_{t+1}^B M_{t+1}^B}{I_t^i} = \frac{\rho \omega_{t+1}^i}{\beta \omega_t^i}.
\]

It is constant in the steady state at \( \Psi_i = \frac{\rho}{\beta} \), independent of \( \theta_t \).

Let \( \chi_{t+1}^i \equiv \frac{v_{t+1}^A}{v_{t+1}^B} \) denote the relative price of intermediate goods, which reflects the distortions of financial frictions on interest rates and production efficiency as below.

### 2.2 Existence, Uniqueness, and Stability of The Steady State

We prove the existence, uniqueness and stability of the steady-state equilibrium by analyzing the phase diagram of wages. For simplicity, we drop the country superscripts.

**Proposition 1.** Let \( \bar{\theta} \equiv 1 - \frac{\omega_t}{\alpha_B} \eta \). For \( \theta_t \in [\bar{\theta}, 1] \), the borrowing constraints are not strictly binding under IFA and the economic allocation is independent of \( \theta_t \). Aggregate savings \( \beta \omega_t \) is allocated efficiently in the two sectors, according to their factor shares in the aggregate production function, \( \frac{M_{t+1}^A}{\alpha - \alpha_B} = \frac{M_{t+1}^B}{\alpha_B} = \frac{\beta \omega_t}{\alpha} \). The relative price is constant at unity, \( \chi_{t+1} \equiv \frac{v_{t+1}^A}{v_{t+1}^B} = 1 \) and aggregate production is efficient. The private rates of return are equal to the social rate of return, \( R_t = \Gamma_t = \Psi_t \). Individuals in the same generation have the same welfare, \( U_{t}^i = U_{t}^e = \ln \omega_t + \beta \ln \Psi_t \).

The model dynamics are characterized by the dynamic equation of wages,

\[
\omega_{t+1} = \left( \frac{\beta \omega_t}{\rho} \right)^\alpha.
\]

Given \( \alpha \in (0,1) \), there exists a unique and stable non-zero steady state with the wage at \( \omega = \left( \frac{\beta}{\rho} \right)^\rho \) and the rate of investment return at \( R_t = \Gamma_t = \Psi_t = \frac{\rho}{\beta} \).

Suppose that entrepreneurs are credit constrained and they borrow to the limit. Given aggregate savings \( \beta \omega_t \), aggregate output of intermediate goods A and B are \( M_{t+1}^A = (1 - \eta) \phi_t \beta \omega_t \) and \( M_{t+1}^B = \eta \lambda_t \beta \omega_t \), respectively, where \( \phi_t = \frac{v_{t+1}^B}{\beta \omega_t} \in (0,1) \) denote the fraction of household savings invested in his own production project. The model dynamics are characterized by the equation system of six variables in each country, \( \omega_t, R_t, v_t^A, v_t^B, \lambda_t, \phi_t \),

\[
\omega_{t+1} = \left( \frac{(1 - \eta) \phi_t \beta \omega_t}{\rho - \beta B} \right)^{\alpha - \alpha_B} \left( \frac{\eta \lambda_t \beta \omega_t}{\rho B} \right)^{\alpha_B}
\]

\[
(1 - \phi_t)(1 - \eta) \beta \omega_t = (\lambda_t - 1) \eta \beta \omega_t.
\]

\[
R_t = v_t^A.
\]

\[
\lambda_t \equiv \frac{1}{1 - \frac{\theta_{t+1}}{\rho_t}},
\]

\[
v_{t+1}^A (1 - \eta) \phi_t \beta \omega_t = (\rho - \beta B) \omega_{t+1}, \quad v_{t+1}^B \eta \lambda_t \beta \omega_t = \rho_B \omega_{t+1}.
\]
where \( \rho_B \equiv \frac{\alpha_B}{1-\alpha} \). Equation (16) shows the wage dynamics, equation (17) is the credit market clearing condition, equation (18) is the household no-arbitrage condition, equation (19) defines the entrepreneurs’ investment-equity ratio, and the two equations in (20) determine the prices of intermediate goods. \( v^A_{t+1} \) and \( v^B_{t+1} \) also represent the rates of return on the household and entrepreneurial projects, respectively. The solutions to some endogenous variables are,

\[
\chi_{t+1} = 1 - \frac{\bar{\theta} - \theta}{1 - \eta} \tag{21}
\]

\[
v^B_{t+1} = \Psi_t \left[ \frac{1}{\chi} \left( \frac{\alpha - \alpha_B}{\alpha} \right) + \frac{\alpha_B}{\alpha} \right] \tag{22}
\]

\[
\lambda_t = \frac{1}{\eta} \left( \frac{\alpha - \alpha_B}{\alpha_B} \right) \frac{1}{\chi} + 1 \tag{23}
\]

\[
\phi_t = \frac{\alpha (1 - \eta)}{\alpha(1 - \eta)\left(1 - \frac{\alpha B}{\alpha B} (1 - \chi)\right)} \tag{24}
\]

\[
R_t = v^A_{t+1} = \Psi_t \left[ 1 - \frac{\alpha B}{\alpha_B} (1 - \chi) \right] \tag{25}
\]

\[
\Gamma_t = \Psi_t \left[ 1 + \frac{\alpha B}{\alpha} \frac{1 - \eta}{\eta} (1 - \chi) \right] \tag{26}
\]

\[
\omega_{t+1} = \left( \frac{\Lambda \beta}{\rho \omega_t} \right)^{\alpha} \text{ where } \Lambda = \frac{\chi^{\alpha_B}}{1 - \frac{\alpha B}{\alpha} (1 - \chi)} \tag{27}
\]

As summarized in Proposition 1, for \( \theta \in [\bar{\theta}, 1] \), the borrowing constraints are not binding and the relative price is constant at unity. The socially efficient allocation is obtained simply by plugging \( \chi^i = 1 \) into equations (21)-(27). We take this allocation as the benchmark and analyze the case where the borrowing constraints are binding as follows.

For \( \theta \in [0, \bar{\theta}] \), according to equation (21), the relative price is constant, smaller than unity, and increasing in \( \theta \). Intuitively, the binding borrowing constraints affect aggregate investment in the two sectors. Equation (23) shows that the investment-equity ratio is lower than the socially efficient level and the underinvestment in sector B keeps the price of intermediate good B higher than the social rate of return, as shown in equation (22). Meanwhile, equation (24) shows that sector A is overinvested so that the price of intermediate good A is lower than the social rate of return, as shown in equation (25). Thus, the distortion on cross-sectoral resource allocation keeps the relative price lower than unity. The rise in \( \theta \) facilitates resource reallocation from sector A to sector B, which improves production efficiency and the relative price rises, accordingly. This way, the relative price reflects the revenue splitting rule in the credit market.

In addition, the relative price also reflects the revenue splitting rule in the credit market. At the aggregate level, the total savings of households in period \( t \), \((1 - \eta)\beta \omega_t\), are rewarded at the loan rate, \( R_t \), in period \( t + 1 \), while those of entrepreneurs, \( \eta \beta \omega_t \), are rewarded at the equity rate, \( \Gamma_t \). Thus, aggregate revenue of intermediate goods
\( \rho \omega_{t+1} \) is distributed between households and entrepreneurs,

\[
R_t (1 - \eta) \beta \omega_t + \Gamma_t \eta \beta \omega_t = \rho \omega_{t+1} \quad \Rightarrow \quad R_t (1 - \eta) + \Gamma_t \eta = \frac{\rho \omega_{t+1}}{\beta \omega_t} = \Psi_t. \tag{28}
\]

For \( \theta \in [0, \bar{\theta}) \), the constraint on aggregate credit demand keeps the loan rate (the equity rate) lower (higher) than the social rate of return, as shown in equations (25) and (26). Intuitively, savers (households) obtain less surplus than borrowers (entrepreneurs) from the credit relationship, due to the general-equilibrium effect. Thus, in the less financially developed country, entrepreneurs have a stronger position in the credit relation at the aggregate level so that the equity rate is higher and the loan rate is lower. For \( \theta \geq \bar{\theta} \), the borrowing constraints are not binding. Thus, households and entrepreneurs obtain the same rate of return from the credit relationship, \( R_t = \Gamma_t = \Psi_t \).

Equation (27) shows the dynamic equation of wages, where the domestic composite productivity factor \( \Lambda \) measures how efficiently domestic savings are converted into aggregate output in the next period. \( \Lambda \) is a constant, depending positively on the relative price and eventually on the level of financial development.

\[
\frac{\partial \ln \Lambda}{\partial \theta} = \begin{cases} 
\frac{\alpha - \alpha_B (1 - \chi)}{(1 - \eta) \chi [\frac{\alpha - \alpha_B}{\alpha_B} + \chi]} > 0, & \text{for } \theta \in (0, \bar{\theta}) \\
0 & \text{for } \theta = \bar{\theta}.
\end{cases}
\]

For \( \theta \in [\bar{\theta}, 1] \), the domestic composite productive factor is implicitly equal to unity, according to equation (15). For \( \theta \in [0, \bar{\theta}) \), the distortion on cross-sectoral resource allocation keeps aggregate output lower than its efficient level, \( \Lambda < 1 \). The rise in \( \theta \) improves cross-sectoral resource allocation and raises \( \Lambda \), implying that domestic savings are more efficiently transformed into output.

Let a variable with subscript \( IFA \) denotes its steady-state value under IFA.

Proposition 2. For \( \theta \in [0, \bar{\theta}) \), the borrowing constraints are strictly binding under IFA. Sector B is under-invested while sector A is over-invested. The cross-sectoral distortion keeps the relative price and aggregate output lower than their socially efficient levels. The loan rate is lower while the equity rate is higher than the social rate of return. In the same generation, entrepreneurs are better off than households, \( U^h_e < U^e_i \), due to the rate-of-return differential, \( R_t < \Gamma_i \).

There exists a unique and stable non-zero steady state with the wage at \( \omega_{IFA} = \left( \frac{\Lambda_B}{\rho} \right)^{\rho} \). The steady-state wage under IFA can also be reformulated as

\[
\omega_{IFA} = \left( \frac{1}{R_{IFA}} \right)^{\rho} \left( \frac{\beta \alpha}{\rho \alpha_B} R_{IFA} - \frac{\alpha - \alpha_B}{\alpha_B} \right)^{\rho \alpha_B}.
\]

The model of von Hagen and Zhang (2010) is a special case here with \( \alpha_B = \alpha \), i.e., intermediate goods are produced only by entrepreneurs. In that case, \( \Lambda = 1 \) implies that financial frictions do not affect production efficiency since all savings are invested by
entrepreneurs and the loan rate adjusts to clear the credit market. Thus, financial frictions in von Hagen and Zhang (2010) only distort the interest rates, while financial frictions in the current model affect not only the interest rates but also production efficiency.

For illustration purpose, we choose the parameter values as follows, \( \eta = 0.1 \) implies that entrepreneurs account for 10% of population, \( \alpha = 0.36 \) implies that the labor income accounts for 64% of aggregate output, \( \alpha_B = 0.18 \) implies that intermediate goods A and B enter equally into aggregate production function, \( \beta = 0.4 \) implies that individuals consume 60% of their income when young and save 40% for future. The threshold value is \( \bar{\theta} = 0.8 \). Figure 1 shows the steady-state patterns of endogenous variables with \( \theta \in [0, \bar{\theta}] \) on the horizontal axis. The vertical axis denotes the levels of endogenous variables.

![Figure 1: Steady-State Pattern under IFA](image)

As \( \theta^i \) rises from 0 to \( \bar{\theta} \), entrepreneurs can borrow more from households. The rise in the effective aggregate credit demand pushes up the loan rate, which induces households to raise their lending and to reduce their own project investment. Thus, the investment-equity ratio \( \lambda \) rises while the household portfolio share \( \phi \) declines. Aggregate output of intermediate good A (B) declines (rises) and the price of intermediate good A (B) rises (declines), accordingly. The improvement in the cross-sectoral resource allocation enhances aggregate production efficiency, which raises aggregate output. The social rate of return is constant at \( \Psi = \rho \beta \). The equity rate declines in \( \theta \), because the declines in the price of intermediate good B and the spread of leveraged investment dominate the rise in the debt-equity ratio, according to equation (8).
Households benefit from a rise in $\theta$ via the positive labor income effect and the positive loan rate effect. For $\theta$ is close to zero, entrepreneurs benefit from a marginal rise in $\theta$ as the positive labor income effect dominates the negative equity rate effect; otherwise, entrepreneurs lose from a marginal rise in $\theta$. The social welfare, defined as the weighted average of individual welfare, $U^s \equiv (1-\eta)U^h + \eta U^e = \ln \omega + (1-\eta) \ln R + \eta \ln \Gamma$, rises in $\theta$, as the labor income effect dominates the interest rate effects at the aggregate level.

3 International Capital Flows

We consider three scenarios of capital mobility, free mobility of financial capital under which individuals are allowed to lend abroad but entrepreneurs are not allowed to make direct investment abroad, free mobility of FDI under which entrepreneurs are allowed to make direct investment abroad but individuals are not allowed to lend abroad, and full capital mobility under which individuals are allowed to lend abroad and entrepreneurs are allowed to make direct investments abroad.

Without loss of generality, we assume $0 \leq \theta_S < \theta_N \leq \bar{\theta}$, which guarantees that the borrowing constraints are strictly binding in the steady state in both countries under the three scenarios of capital mobility. We also assume that both countries are initially in the steady state under IFA before capital mobility is allowed from period $t = 0$ on.\(^3\)

Let $\Upsilon_i^t$ and $\Omega_i^t$ denote the aggregate outflows of financial capital and FDI from country $i$ in period $t$, respectively, with negative values indicating capital inflows. Financial capital outflows reduce the domestic credit supply, $(1-\eta)(1-\phi_i^t)\beta \omega_i^t - \Upsilon_i^t$, while FDI outflows reduce the aggregate equity capital for domestic investment, $\eta \beta \omega_i^t - \Omega_i^t$. Thus, FDI flows raise (reduce) the aggregate credit demand in the host (parent) country.\(^4\) With these changes, the analysis in section 2 carries through for the cases of capital mobility, due to the linearity of preferences, productive projects, and borrowing constraints.

Let a variable with subscript $FCF$, $FDI$, and $FCM$ denotes its steady-state value under the scenarios of free mobility of financial capital, free mobility of FDI, and full capital mobility, respectively.

\(^2\)Entrepreneurs can either bring their funds and projects abroad for investment or make equity investment in the foreign entrepreneurial project. The two alternatives are analytically equivalent in our model. Without the necessary skills, households cannot make direct or equity investment abroad.

\(^3\)Given the model structure of overlapping generations, capital mobility from period $t = 0$ on does not affect the behaviors of individuals born before period $t = 0$, even if announced in advance.

\(^4\)In the case of debt default, the project liquidation value depends on the efficiency of the legal institution, the law enforcement, and the asset market in the host country. Thus, we assume that entrepreneurs making FDI borrow only from the host country and are subject to the borrowing constraints there. Alternatively, we can assume that entrepreneurs borrow only in their parent country no matter where they invest, because the financial institutions in their parent country may have better information on their credit record, social network, and business activities. The realistic case should be a hybrid of the two. Our results hold under the alternative assumption.
3.1 Free Mobility of Financial Capital

Financial capital flows equalize the loan rate across the border and the credit markets clear in each country as well as at the world level,

\[ R^N_t = R^S_t = R^*_t, \quad (1 - \phi^*_t)(1 - \eta)\beta \omega^*_t = (\lambda^*_t - 1)\eta \beta \omega^*_t + \Upsilon^*_t, \quad \text{and} \quad \Upsilon^N_t + \Upsilon^S_t = 0. \]

Except for these conditions, the equations governing the model dynamics are same as under IFA. The solutions to some endogenous variables are,

\[ \Upsilon^*_t = (1 - \eta)\beta \omega^*_t \left[ 1 - \frac{1}{1 + \frac{\alpha_B}{\alpha}(\chi^*_t - \chi^*_IFA)} \right] = (1 - \eta)\beta \omega^*_t \left[ 1 - \frac{\omega^*_t + R^*_IA}{\omega^*_t} \right] \quad (29) \]

\[ \lambda^*_t = \frac{1}{\eta} \left( \frac{1}{\chi^*_IFA} + 1 + \frac{\alpha_B}{\alpha}(\chi^*_t - \chi^*_IFA) \right) \quad (30) \]

\[ \phi^*_t = \frac{\alpha}{\alpha - \alpha_B} \left[ 1 - \frac{\alpha_B}{\alpha}(\chi^*_t - \chi^*_IFA) + \frac{\alpha_B}{\alpha\eta}(\chi^*_t - \chi^*_IFA) \right] \quad (31) \]

\[ R^*_t = \psi^*_t \left[ 1 - \frac{\alpha_B}{\alpha}(1 - \chi^*_IFA) + \frac{\alpha_B}{\alpha\eta}(\chi^*_t - \chi^*_IFA) \right] \quad (32) \]

\[ \Gamma^*_t = \psi^*_t \left[ 1 + \frac{\alpha_B}{\alpha}(1 - \chi^*_IFA) - \frac{\alpha_B}{\alpha}(1 - \chi^*_IFA) \right] \quad (33) \]

\[ \omega^*_t = \left( \frac{\Lambda^*_t \beta \omega^*_t}{\rho} \right)^\alpha \quad (34) \]

where \[ \Lambda^*_t = \frac{(\chi^*_t)^{\frac{\alpha_B}{\alpha}}}{\frac{\alpha_B}{\alpha}(\chi^*_t - \theta^*_t)} = \left( \frac{\alpha_B}{\alpha} \right)^{\frac{\alpha}{\alpha_B}} \frac{(\chi^*_t)^{\frac{\alpha_B}{\alpha}}}{\frac{\alpha_B}{\alpha}(\chi^*_t - \chi^*_IFA)} \] \( \quad (35) \]

Comparing with equations (23)-(27), the solutions to the entrepreneurial investment-equity ratio, the household portfolio share, the loan rate, and the domestic composite productivity factor here include an extra component \( \chi^*_t - \chi^*_IFA \). Thus, the change in the relative price is key to understand the model dynamics.

Under IFA, the steady-state loan rate is lower in country S than in country N. In period \( t = 0 \), households in country S lend abroad for a higher interest rate by reducing their own project investment and domestic deposits. In equilibrium, the loan rate rises in country S and entrepreneurs reduce their project investment. In country N, financial capital inflows reduce the loan rate and entrepreneurs raise their project investment. Meanwhile, due to the decline in the loan rate, households in country N raise their own project investment by reducing their domestic deposits. Since financial capital flows directly affect households, the decline (rise) in aggregate output of intermediate good A dominates that of intermediate good B so that the relative price rises (declines) in country S (N), as confirmed by equation (29) and the patterns of financial capital flows, \( \Upsilon^*_t > 0 > \Upsilon^N_t \). According to equations (30) and (31), the changes in the relative price reduce (raise) the
entrepreneurial investment-equity ratio and the household portfolio share in country S (N), which also confirms our intuitions. The relative price reflects production efficiency. This way, financial capital flows improve (worsen) production efficiency in country S (N) by affecting the composition of aggregate investment.

Financial capital flows affect aggregate output directly by reducing (raising) the size and indirectly by improving (worsening) the composition of aggregate investment in country S (N). Equation (34) shows the dynamic equation of wages, where the domestic composite productivity factor Λ_i is negatively related to the relative price. Thus, the changes in the relative price imply that the size effect dominates the component effect and that, in the net term, aggregate output in country S (N) and period t = 1 is lower (higher) than the steady-state level under IFA.

More formally, given the predetermined labor income in period t, ω^S_i and ω^N_i, the loan rate equalization and the international credit market clearing condition,

\[ \Upsilon^S_t + \Upsilon^N_t = 0, \Rightarrow \frac{1 - \eta}{\eta} (\omega^S_t + \omega^N_t) = \frac{\omega^S_t \left( \frac{\alpha - \alpha_B}{\alpha B} + \theta^S \right)}{\chi^{S}} - \frac{\omega^N_t \left( \frac{\alpha - \alpha_B}{\alpha B} + \theta^N \right)}{\chi^{N}}, \]

\[ R^S_t = R^N_t = R^*_i, \Rightarrow (\chi^{S} - \theta^S) (\chi^{N} + \theta^N) \frac{1}{\omega^S_t} = (\chi^{S} - \theta^S) (\chi^{N} + \theta^N) \frac{1}{\omega^N_t}, \]

uniquely determine χ^S_{i+1} and χ^N_{i+1} in each period.

**Proposition 3.** Given the world loan rate, R^*_i, the phase diagram of wages in each country is monotonically increasing and concave for ω^i_t ∈ (0, ω^*_i), where ω^*_i = \frac{\alpha_B}{\rho \alpha} (1 - \theta^i) (R^*_i)^{\frac{\rho}{1 - \rho}}; for ω^i_t > ω^*_i, the phase diagram is flat at \omega^i_{i+1} = (R^*_i)^{-\rho} < ω^i_t. It has a positive intercept on the vertical axis at \omega^i_{i+1} = (R^*_i)^{-\rho (\theta^i)^{\rho_B}} and crosses the 45 degree line once and only once from the left. Thus, there exists a unique and stable steady state.

**Proof.** See appendix A.

**Proposition 4.** The steady-state wage in country i and financial capital flows are

\[ \omega^i_{FCF} = \left( \frac{1}{R^*_i} \right)^{\rho} \left\{ \frac{\beta \alpha}{\rho \alpha_B} R^*_{FCF} [\eta + (1 - \eta) \frac{R^S_{IFA}}{R^N_{IFA} R^*_{FCF}} - \frac{\alpha - \alpha_B}{\alpha B} \right\}^{\rho_B}, \]

\[ \Upsilon^i_{FCF} = (1 - \eta) \beta \omega^i_{FCF} \left( 1 - \frac{R^S_{IFA}}{R^*_{FCF}} \right). \]

Given R^S_{IFA} < R^N_{IFA}, equation (37) and the world credit market clearing condition \Upsilon^S_{FCF} + \Upsilon^N_{FCF} = 0 imply that R^*_{FCF} ∈ (R^S_{IFA}, R^N_{IFA}). Accordingly, financial capital flows from country S to country N in the steady state, \Upsilon^S_{FCF} > 0 > \Upsilon^N_{FCF}.

**Proposition 5.** The steady-state relative price in country S (N) is higher (lower) than that under IFA. Under free mobility of financial capital, the steady-state relative price in country S is still lower than in country N. To summarize, \chi^S_{IFA} < \chi^S_{FCF} < \chi^N_{FCF} < \chi^N_{IFA}.

**Proof.** See appendix A.
Proposition 6. The steady-state aggregate output in country S (N) is lower (higher) than that under IFA. Under free mobility of financial capital, the steady-state aggregate output in country S is lower than in country N. To summarize, \( Y_{SFCF}^N < Y_{IFA}^S < Y_{IFA}^N < Y_{FCF}^N \).

Proof. See appendix A.

Proposition 7. The equity rate in the steady state is same as under IFA, \( \Gamma_{FCF}^i = \Gamma_{IFA}^i \).

According to equations (33) and (26), the equity rate has the same solution under free mobility of financial capital and under IFA. Intuitively, financial capital flows affect the equity rate through three channels during the transitional process. Take country S as an example. First, financial outflows raise the domestic loan rate, which tends to reduce the equity rate. Second, the decline in the aggregate investment in sector B in period \( t \) raises the price of intermediate good B in period \( t + 1 \), which tends to raise the equity rate. Third, financial capital outflows reduce aggregate domestic investment in period \( t \), which reduces aggregate output and labor income in period \( t + 1 \). Thus, aggregate domestic savings in period \( t + 1 \) is also lower and so is aggregate domestic investment. This dynamic “capital accumulation” effect further enhances the second effect, which tends to raise the equity rate. In period \( t = 0 \), the first effect dominates the second effect and the equity rate falls. In the long run, the second and the third effects fully neutralize the first effect and the equity rate returns to its initial steady-state level as under IFA.

Consider the welfare at the individual level and at the country level. As mentioned in subsection 2.1, the individual welfare of households and entrepreneurs essentially depends on the labor income when young and the interest rates. Take country S as an example. In period \( t = 0 \), entrepreneurs lose from the decline in the equity rate, given the predetermined labor income, \( \omega_S^0 \). In the long run, since the steady-state equity rate is same as but the steady-state labor income is lower than under IFA, entrepreneurs in the later generations also lose from financial capital outflows. In period \( t = 0 \), households benefit from the higher loan rate, given the predetermined labor income, \( \omega_H^0 \). In the long run, since the steady-state loan rate is higher but the steady-state labor income is lower than under IFA, households in the later generations may or may not benefit from financial capital outflows, depending on which effect dominates. Thus, financial capital flows may have opposite welfare impacts on households and entrepreneurs in the same generation. Meanwhile, financial capital flows may have opposite welfare impacts on households in the early and later generations. In country N, entrepreneurs benefit from financial capital flows via the positive equity rate effect and the positive labor income effect, while households in the early generation are worse off due to the negative loan rate effect and those in the later generation may be better or worse off, depending on the relative magnitude of the positive labor income effect and the negative loan rate effect.

Figure 2 shows the impulse responses of the model economy with respect to free mobility of financial capital from period \( t = 0 \) on, given that the world economy is
initially in the steady state under IFA with $\theta^N = \overline{\theta} = 0.8$ and $\theta^S = \frac{\overline{\theta}}{2} = 0.4$ before period $t = 0$. The horizontal axes denote the time periods, the vertical axes of the three panels in the first row denote the dynamic paths of relevant variables in levels, and the vertical axes of the three panels in the second row denote the percentage differences of relevant variables versus their respective steady-state levels under IFA.

![Graph](image)

Figure 2: Transitional Dynamics from IFA to FCF

The patterns of capital flows, aggregate output and interest rates in both countries confirm our analytical results. Here, we focus on the world output implications. As mentioned above, financial capital flows affect aggregate output in each country through affecting the size and the composition of aggregate investment. First, financial capital flows reduce (raise) the size of aggregate investment in country S (N) through cross-country resource reallocation, which widens the initial cross-country output gap. Since the aggregate production function is concave in aggregate investment in each country, the “uphill” financial capital flows tend to have a negative effect on world output, due to the Jensen’s inequality. For simplicity, we call it the net investment size effect. Second, financial capital flows improve (worsen) production efficiency in country S (N) through affecting the composition of aggregate investment. Since production inefficiency is initially more severe in country S than in country N, the efficiency gains in country S dominate the efficiency losses in country N so that financial capital flows tend to have a positive effect on world output. For simplicity, we call it the net investment composition effect. Given our parameter choice of $\theta^S = 0.4$ and $\theta^N = 0.8$, the net size effect dominates the
net composition effect and world output is lower than under IFA.

Figure 3 compare the steady-state patterns under free mobility of financial capital versus under IFA, given $\theta^N = \bar{\theta}$ and $\theta^S \in [0, \bar{\theta}]$. The horizontal axes denote $\theta^S \in [0, \bar{\theta}]$, the vertical axes of the three panels in the first row denote the levels of relevant variables, and the vertical axes of the three panels in the second row denote the percentage difference of endogenous variables versus their respective steady-state levels under IFA.

Figure 3: Steady-State Pattern under FCF versus IFA

For $\theta^S$ close to $\theta^N$, the net investment size effect due to cross-country resource reallocation dominates the net investment composition effect due to the within-country resource reallocation among individuals with different productivity. Thus, the steady-state world output is lower than under IFA. For $\theta^S$ much smaller than $\theta^N$, production inefficiency is significantly more severe in country S under IFA. Thus, the net investment composition effect may dominate the net investment size effect so that the steady-state world output is higher than under IFA.

Figure 4 shows the threshold values of $\theta^N$ as a function $\theta^S$ where the steady-state world output under free mobility of financial capital coincide with that under IFA. The horizontal and vertical axes measure $\theta^S, \theta^N \in [0, \bar{\theta}]$. For $(\theta^S, \theta^N)$ in region A (B), the net investment size effect dominates (is dominated by) the net investment composition effect and the steady-state world output is lower (higher) than that under IFA.
3.2 Free Mobility of FDI

The analysis for free mobility of FDI follows that for free mobility of financial capital. Here, we summarize the main results and leave the detailed analysis in appendix B.

The initial cross-country equity-rate differential under IFA induces entrepreneurs in country N to make direct investment in country S in period \( t = 0 \) for a higher equity return and, in equilibrium, the equity rate declines (rises) in country S (N). FDI inflows raise the size and improve the composition of aggregate domestic investment in country S, which raises aggregate output in country S. The improvement in production efficiency is reflected as the rise in the relative price. The opposite is true for country N.

FDI flows affect the loan rate through three channels during the transitional process. Take country S as an example. First, FDI inflows raise aggregate credit demand, which tends to raise the loan rate. Second, due to the rise in the aggregate investment in sector B in period \( t \), the price of intermediate good A in period \( t + 1 \) is higher, which tends to raise the loan rate. Third, FDI inflows raise aggregate output and labor income in period \( t + 1 \). Thus, aggregate domestic savings in period \( t + 1 \) is also higher and so is aggregate domestic investment. This dynamic “capital accumulation” effect tends to reduce the loan rate. In period \( t = 0 \), the first and the second effects raise the loan rate. In the long run, the third effect fully neutralizes the first and the second effects and the loan rate returns to its initial steady-state level as under IFA.

If both \( \theta^S \) and \( \theta^N \) are close to \( \bar{\theta} \), FDI flows reduce the steady-state world output as the net investment size effect dominates; otherwise, FDI flows raise the steady-state world output as the net investment composition effect dominates.
3.3 Full Capital Mobility

Financial capital flows equalize the loan rate across the border and the credit markets clear in each country as well as at the world level. FDI flows equalize the equity rate across the border and the world equity capital market clears. FDI flows directly affect aggregate output of intermediate good B in each country. To summarize,

\[ \Upsilon_t^N + \Upsilon_t^S = \Omega_t^N + \Omega_t^S = 0, \quad R_t^N = R_t^S = R^*_t, \quad \Gamma_t^N = \Gamma_t^S = \Gamma^*_t, \]

\[ (1 - \phi_i^t)(1 - \eta)\beta^i \omega^i_t = (\lambda^i_t - 1)(\eta \beta^i \omega^i_t - \Omega^i_t) + \Upsilon^i_t, \]

\[ M_{t+1}^B = \lambda^i_t(\eta \beta^i \omega^i_{t+1} - \Omega^i_t). \]

Except for these conditions, the equations governing the model dynamics are same as under IFA. The solutions to some endogenous variables are,

\[ \Omega_t^i = \eta \beta^i \omega^i_t \left[ 1 - \frac{\omega^i_{t+1}}{\omega^i_t} \frac{\Gamma^i_{IFA}}{\Gamma_t} \right], \quad (38) \]

\[ \Upsilon_t^i = (1 - \eta)\beta^i \omega^i_t \left[ 1 - \frac{\omega^i_{t+1}}{\omega^i_t} \frac{R^i_{IFA}}{R^*_t} \right], \quad (39) \]

\[ \Omega_t^i + \Upsilon_t^i = \beta \omega^i_t \left\{ 1 - \frac{\omega^i_{t+1}}{\omega^i_t} \frac{\Gamma^i_{IFA}}{\Gamma_t} + (1 - \eta) \frac{R^i_{IFA}}{R^*_t} \right\}, \quad (40) \]

\[ \chi^i_{t+1} = (1 - \theta^i) \frac{R^*_t}{\Gamma_t} \Gamma^*_t + \theta^i = (1 - \theta^i) \left( \frac{R^*_t}{\Gamma_t} - \frac{R^i_{IFA}}{\Gamma^i_{IFA}} \right) + \chi^i_{IFA}, \quad (41) \]

\[ \omega^i_{t+1} = \left[ (1 - \theta^i) \frac{R^*_t}{\Gamma_t} + \theta^i \right]^{\rho_B} \left( \frac{1}{\Gamma_t} \right)^{\rho_B}, \quad (42) \]

Under IFA, the steady-state loan rate is lower while the steady-state equity rate is higher in country S than in country N. In period \( t = 0 \), households in country S lend abroad for a higher interest rate, \( \Upsilon^S_t > 0 > \Upsilon^N_t \), while entrepreneurs in country N make direct investment abroad for a higher equity return, \( \Omega^S_t < 0 < \Omega^N_t \). Financial capital outflows and FDI inflows raise the loan rate in country S, which reduces the inefficient household investment in sector A. The declines in the price of intermediate good B and the spread imply that the equity rate falls in country S. The opposite is true for country N.

The changes in the interest rates imply that the ratio of the loan rate versus the equity rate rises (decline) in country S (N) and so does the relative price, according to equation (41). This way, financial capital and FDI flows improve (worsen) production efficiency in country S (N) via the composition effect. Furthermore, since country N has a larger credit market, net capital flows are from country S to country N, which reduces (raises) the size of aggregate investment in country S (N). Since the size effect dominates the composition effect, aggregate output in country N rises; while aggregate output in country S may rise or decline, depending on which effect dominates.

**Proposition 8.** Given the world loan rate \( R^*_t \) and the world equity rate, \( \Gamma^*_t \), the phase diagram of wages \( \omega^i_{t+1} = \left( \frac{1 - \theta^i}{\Gamma_t} \right)^{\rho_B} \left( \frac{1}{\Gamma_t} \right)^{\rho_B} \) is flat. It crosses the 45 degree line once and only once. Thus, there exists a unique and stable steady state in the model economy.
Proof. See appendix A.

**Proposition 9.** The steady-state wage in country $i$ and capital flows from country $i$ are

$$
\omega^i_{FCM} = \left( \frac{1}{R^i_{FCM}} \right)^{\rho - \rho_B} \left( 1 - \theta^i \right) \left( \frac{\theta^i}{R^i_{FCM}} \right) \rho_B = \left( \frac{1}{R^i_{FCM}} \right)^{\rho} (\chi^i_{FCM})^{\alpha_B}, \tag{43}
$$

$$
\Upsilon^i_{FCM} = (1 - \eta) \beta \omega^i_{FCM} \left( 1 - \frac{R^i_{IFA}}{R^i_{FCM}} \right) \tag{44}
$$

$$
\Omega^i_{FCM} = \eta \beta \omega^i_{FCM} \left( 1 - \frac{\Gamma^i_{IFA}}{\Gamma^i_{FCM}} \right), \tag{45}
$$

$$
\Omega^i_{FCM} + \Upsilon^i_{FCM} = \beta \omega^i_{FCM} \left[ 1 - \eta \frac{\Gamma^i_{IFA}}{\Gamma^i_{FCM}} - (1 - \eta) \frac{R^i_{IFA}}{R^i_{FCM}} \right]. \tag{46}
$$

Given $R^S_{IFA} < R^N_{IFA}$ and $\Gamma^S_{IFA} > \Gamma^N_{IFA}$, the world interest rates are $R^*_{FCM} \in (R^S_{IFA}, R^N_{IFA})$ and $\Gamma^*_{FCM} \in (\Gamma^N_{IFA}, \Gamma^S_{IFA})$. In particular,

$$
(1 - \eta) R^*_{FCM} + \eta \Gamma^*_{FCM} = \frac{\rho}{\beta}. \tag{47}
$$

Financial capital flows from country $S$ to country $N$ $\Upsilon^S_{FCM} > 0 > \Upsilon^N_{FCM}$ and FDI flows in the opposite direction $\Omega^S_{FCM} < 0 < \Omega^N_{FCM}$. The net capital flows are from country $S$ to country $N$, $\Upsilon^S_{FCM} + \Omega^S_{FCM} > 0 > \Upsilon^N_{FCM} + \Omega^N_{FCM}$.

Proof. See appendix A. \qed

**Proposition 10.** The steady-state relative price in country $S$ ($N$) is higher (lower) than under IFA. Under full capital mobility, the steady-state relative price in country $S$ is still lower than in country $N$. To summarize, $\chi^S_{IFA} < \chi^S_{FCM} < \chi^N_{FCM} < \chi^N_{IFA}$.

Proof. See appendix A. \qed

**Proposition 11.** In the steady state, the gross factor payment in each country sum up to zero, $\Upsilon^i_{FCM} R^i_{FCM} + \Omega^i_{FCM} \Gamma^i_{FCM} = 0$. Thus, although country $N$ has net capital inflows, $\Upsilon^F_{FCM} + \Omega^F_{FCM} < 0$, it receives a positive net factor payment, $\Upsilon^F_{FCM} (R^F_{FCM} - 1) + \Omega^F_{FCM} (\Gamma^F_{FCM} - 1) = 0 - (\Upsilon^F_{FCM} + \Omega^F_{FCM}) > 0$. Intuitively, country $N$ receives a higher rate of return on its foreign assets than what it pays for its foreign liabilities.

Proof. See appendix A. \qed

Figure 5 shows the impulse responses of the model economy with respect to full capital mobility from period $t = 0$ on, given that the world economy is initially in the steady state under IFA with $\theta^N = \bar{\theta} = 0.8$ and $\theta^S = \hat{\theta}/2 = 0.4$ before period $t = 0$. The axis scalings are same as those of figure 2.

The patterns of capital flows, aggregate output and interest rates in both countries confirm our analytical results. Here, we focus on the world output implications. International capital flows affect aggregate output in each country through affecting the size and
the composition of aggregate investment. First, net capital flows raise (reduce) the size of aggregate investment in country S (N) through cross-country resource reallocation. Given our parameter choice of $\theta^S = 0.4$ and $\theta^N = 0.8$, net capital flows widens the initial cross-country output gap and the net investment size effect is negative for world output. Second, both financial capital and FDI flows improve (worsen) production efficiency in country S (N) through affecting the composition of aggregate investment and the net investment composition effect is positive for world output. Note that the net investment size effect depends on net capital flows, while the net composition effect depends on gross capital flows. Under full capital mobility, two-way capital flows imply that net flows are much smaller than gross flows. Thus, the net composition effect dominates the net size effect and world output is higher than under IFA.

Figure 6 compare the steady-state patterns under full capital mobility versus under IFA, given $\theta^N = \tilde{\theta}$ and $\theta^S \in [0, \tilde{\theta}]$. The axis scalings are same as in figure 3. As mentioned above, two-way capital flows imply small net flows and large gross flows. Thus, the net investment composition effect dominates the net investment size effect and world output is strictly higher than under IFA.
Figure 6: Steady-State Pattern under FCM versus IFA

4 Conclusion

In our model, domestic financial frictions create two distinct distortions on the interest rates and production efficiency in the less financially developed country. International capital flows help ameliorate the two distortions. Our model generates the theoretical predictions in line with the recent empirical evidences.

Intuitively, financial capital flows and FDI not only facilitate cross-country resource reallocation, but also trigger within-country resource reallocation among individuals with different productivity. We show that net capital flows matter for the size while gross capital flows matter for the composition of aggregate investment in each country. In contrast to the predictions of conventional neoclassical models, our model shows that the steady-state world output is strictly higher under full capital mobility than under IFA, despite “uphill” net capital flows. However, if international mobility of either financial capital or FDI is restricted, the steady-state world output may be lower. This is a standard result of the second-best theory. In the presence of domestic financial frictions, capital flows to the place where the private rate of return is higher rather than to the place where the marginal product of capital is higher. In this sense, whether international factor mobility can improve allocation or production efficiency depends significantly on various sources of market imperfections or incompleteness.
References


A Proofs of Propositions

Proof of Propositions 3

Proof. Let $P^i \equiv \frac{\partial \alpha}{\partial \rho A}$. Combining equations (32), (34), (35), the phase diagram of wages can be reformulated as,

$$
(\chi^i_{t+1} - \theta^i) = R^i_t \frac{\omega^i_t}{\omega^i_{t+1}} P^i,
$$

$$
\chi^i_t = R^i_t \frac{\omega^i_t}{\omega^i_{t+1}} P^i + \theta^i,
$$

$$(1 - \alpha) \ln \omega^i_{t+1} = -\alpha \ln R^i_t + \alpha B \ln \left( \frac{\omega^i_t}{\omega^i_{t+1}} P^i R^i_t + \theta^i \right).$$

The property of the first derivative of $\omega^i_{t+1}$ w.r.t. $\omega^i_t$,

$$
\frac{\partial \ln \omega^i_{t+1}}{\partial \ln \omega^i_t} = \frac{\alpha B}{1 - \alpha} \frac{\omega^i_t R^i_t P^i}{\chi^i_t} (1 - \frac{\partial \ln \omega^i_{t+1}}{\partial \ln \omega^i_t})
$$

$$
\frac{\partial \ln \omega^i_{t+1}}{\partial \ln \omega^i_t} = 1 + \frac{1 - \alpha}{{\alpha B}} \frac{\omega^i_t \chi^i_{t+1}}{\omega^i_t R^i_t P^i} \in (0, 1)
$$

$$
\frac{\partial \ln \omega^i_{t+1}}{\partial \ln \omega^i_t} = \frac{1 - \alpha}{{\alpha B}} + \frac{1 - \alpha}{{\alpha B}} \frac{\theta^i}{\omega^i_t R^i_t P^i}
$$

$$
1 = \frac{\partial \omega^i_{t+1}}{\partial \omega^i_t} \left[ \frac{\omega^i_t}{\omega^i_{t+1}} \frac{1 - \alpha_A}{\alpha_B} + 1 - \alpha \frac{\theta^i}{R^i_t P^i} \right]
$$

$$
\frac{\partial \omega^i_{t+1}}{\partial \omega^i_t} = 1 - \frac{\partial \ln \omega^i_{t+1}}{\partial \ln \omega^i_t} \left( \frac{\partial \omega^i_{t+1}}{\partial \omega^i_t} \right)^2 < 0
$$

Since $\frac{\partial \ln \omega^i_{t+1}}{\partial \ln \omega^i_t} \in (0, 1)$, we get $\frac{\partial \omega^i_{t+1}}{\partial \omega^i_t} > 0$ and $\frac{\partial \omega^i_{t+1}}{\partial \omega^i_t} < 0$.

For $\omega^i_t = 0$, the phase diagram has a positive intercept on the vertical axis at $\omega^i_{t+1} = (R^i_t)^{-p} \theta^i$. Define a threshold value $\tilde{\omega}^i_t = \frac{(1 - \theta^i)}{\rho A} (R^i_t)^{-1 - \alpha}$. For $\omega^i_t \in (0, \tilde{\omega}^i_t)$, the phase diagram of wages is monotonically increasing and concave, as proved above. For $\omega^i_t > \tilde{\omega}^i_t$, aggregate saving and investment in sector B is so high that the relative price reaches unity. In that case, the borrowing constraints do not bind any more and the phase diagram is flat with $\omega^i_{t+1} = \tilde{\omega}^i_{t+1} = (R^i_t)^{-p}$. Given $R^i_t < \tilde{\omega}^i_t$, we get $\tilde{\omega}^i_{t+1} < \tilde{\omega}^i_{FCF}$.

Thus, the phase diagram of wages crosses the 45 degree line once and only once from the left, and the intersection is in its concave part. Thus, the model economy has a unique and stable steady state under free mobility of financial capital.

Proof of Propositions 5

Proof. The steady-state loan rates in country i under IFA and under free mobility of
financial capital are

\[ R_{IF\text{A}}^i = \frac{\rho}{\beta} \left[ 1 - \frac{\alpha_B}{\alpha} (1 - \chi_{IF\text{A}}^i) \right] \]
\[ R_{FC\text{F}}^i = \frac{\rho}{\beta} \left[ 1 - \frac{\alpha_B}{\alpha} (1 - \chi_{IF\text{A}}^i) + \frac{\alpha_B}{\eta\alpha} (\chi_{FC\text{F}}^i - \chi_{IF\text{A}}^i) \right]. \]

Since the steady-state loan rate is lower in country S than in country N under IFA, the cross-country loan rate equalization implies that the steady-state loan rate in country S (N) is higher (lower) than under IFA. Accordingly, we get \( \chi_{IF\text{A}}^N > \chi_{IF\text{A}}^F \) and \( \chi_{FC\text{F}}^S > \chi_{IF\text{A}}^F \).

Under free mobility of financial capital, the steady-state loan rate in country \( i \) can be reformulated as \( R_{FC\text{F}}^i = \frac{\rho}{\beta} \frac{\alpha_B}{\eta\alpha} (\chi_{FC\text{F}}^i - \theta^i) \). The cross-country loan rate equalization implies \( \chi_{FC\text{F}}^S - \theta^S = \chi_{FC\text{F}}^N - \theta^N \). Given \( \theta^N > \theta^S \), we get \( \chi_{IF\text{A}}^N > \chi_{FC\text{F}}^N \). \hfill \Box

**Proof of Propositions 6**

*Proof.* According to the wage dynamic equation (34), the steady-state wage \( \omega^i = \left( \frac{\Lambda^i\beta}{\rho} \right)^{\rho} \), where the domestic composite productivity factor \( \Lambda^i = \frac{(\chi_{t+1}^i)^{\alpha_B}}{\eta\alpha (\chi_{t+1}^i - \theta^i)} \), depends negatively on \( \chi_{t+1}^i \). In the steady state, \( \chi_{FC\text{F}}^N < \chi_{IF\text{A}}^F \) and \( \chi_{FC\text{F}}^S > \chi_{IF\text{A}}^F \) imply that \( \omega_{FC\text{F}}^N > \omega_{IF\text{A}}^F \) and \( \omega_{FC\text{F}}^S < \omega_{IF\text{A}}^F \). Thus, financial capital flows widen the initial cross-country output gap, \( \omega_{FC\text{F}}^S < \omega_{IF\text{A}}^S < \omega_{IF\text{A}}^N < \omega_{FC\text{F}}^N \). \hfill \Box

**Proof of Propositions 8**

*Proof.* The aggregate production function implies that \( \omega_{t+1}^i = (v_{t+1}^i)^{-(\rho - \rho_B)} (v_{t+1}^i)^{-\rho_B} \). Given \( v_{t+1}^i = R^i_{FC\text{F}} \) and \( \chi_{t+1}^i = \frac{R^i_{FC\text{F}}}{v_{t+1}^i} \), the factor prices equation can be written as \( \omega_{t+1}^i = (R^i_{FC\text{F}})^{-\rho} (\chi_{t+1}^i)^{\rho_B} \). Using equation (41) to substitute away \( \chi_{t+1}^i \), the phase diagram of wages \( \omega_{t+1}^i = \left[ (1 - \theta^i) \frac{R^i_{FC\text{F}}}{R^i_{FC\text{F}}} + \theta^i \right]^{\rho_B} \left( \frac{1}{R^i_{FC\text{F}}} \right)^{\rho} \) is a function of the world interest rates. \hfill \Box

**Proof of Propositions 9**

*Proof.* In the steady state, \( \frac{\omega_{t+1}^i}{\omega_{t}^i} = 1 \). Substitute it into equations (38)-(42), we get the steady-state patterns of wages and capital flows.

The world credit market clearing condition, \( \Upsilon_{FC\text{F}}^S + \Upsilon_{FC\text{F}}^N \) and the steady-state financial capital flows described by equation (44) imply that \( (1 - \frac{R_{IF\text{A}}^S}{R_{FC\text{F}}^S})(1 - \frac{R_{IF\text{A}}^N}{R_{FC\text{F}}^N}) < 0 \).

Given \( R_{IF\text{A}}^S > R_{IF\text{A}}^N \), the world loan rate must be \( R_{FC\text{F}}^S \in (R_{IF\text{A}}^S, R_{IF\text{A}}^N) \). Similarly, we can prove \( \Gamma_{FC\text{F}}^S \in (\Gamma_{IF\text{A}}^N, \Gamma_{IF\text{A}}^F) \). The interest rate patterns imply that \( \Upsilon_{FC\text{F}}^S > 0 > \Upsilon_{FC\text{F}}^N \) and \( \Omega_{FC\text{F}}^S < 0 < \Omega_{FC\text{F}}^N \).

In both countries, households receive the same rate of return, \( R^i_{FC\text{F}} \), on their savings \( (1 - \eta)\beta(\omega_{t}^S + \omega_{t}^N) \) and entrepreneurs also receive the same rate of return, \( \Gamma^i_{FC\text{F}} \), on their savings, \( \eta\beta(\omega_{t}^S + \omega_{t}^N) \). Aggregate revenue of intermediate goods \( \rho(\omega_{t+1}^H + \omega_{t+1}^F) \) is then distributed among households and entrepreneurs,

\[ (1 - \eta)R^i_{FC\text{F}} + \eta\Gamma^i_{FC\text{F}} = \frac{\rho(\omega_{t+1}^H + \omega_{t+1}^F)}{\beta(\omega_{t+1}^H + \omega_{t+1}^F)} \implies (1 - \eta)R^i_{FC\text{F}} + \eta\Gamma^i_{FC\text{F}} = \frac{\rho}{\beta}. \]
Under IFA, equation (28) implies that

\[(1 - \eta)R^i_{IFA} + \eta \Gamma^i_{IFA} = \rho / \beta.\]  

(49)

Let \(Z \equiv \eta \Gamma^i_{IFA} + (1 - \eta)R^i_{IFA} \). Using equations (48) and (49), we get \(Z = \eta \Gamma^i_{IFA} + (1 - \eta)\frac{\Gamma^i_{IFA}}{R^i_{FCM}}\) as a function of \(\Gamma^i_{IFA}\), given \(\Gamma^*_i\). Since \(\frac{\partial Z}{\partial \Gamma^i_{IFA}} = \eta \frac{\Gamma^i_{IFA}}{\eta \Gamma^i_{IFA} - (1 - \eta)R^i_{IFA}}\) < 0 and \(\Gamma^i_{IFA} > \Gamma^*_i\), it is obvious that \(Z(\Gamma^i_{S}) < Z(\Gamma^*_i) = 1\), or equivalently, \(\eta \Gamma^i_{IFA} + (1 - \eta)\frac{R^i_{IFA}}{R^i_{FCM}} < 1\). Thus, according to equation (46), net capital flows from country S to country N in the steady state.

\[\square\]

**Proof of Propositions 10**

**Proof.** Under IFA, the gross equity premium \(\frac{\Gamma^i}{R^i} = \frac{\frac{1}{\chi^i_{IFA}} - \theta^i}{\theta^i + \frac{\alpha B}{\alpha B}}\) declines in \(\theta^i\). Thus, the gross equity premium is higher in country S than in country N under IFA. Under full capital mobility, the cross-country interest rate equalization also implies that the gross equity premium is equalized across the border, i.e., the gross equity premium declines (rises) in country S (N). According to equation (41), the relative price rises (declines) in country S (N), \(\chi^S_{FCM} > \chi^S_{IFA}\) and \(\chi^N_{FCM} < \chi^N_{IFA}\). Under full capital mobility, the cross-border equalization of gross equity premium implies that \(\chi^S_{t+1} < \chi^N_{t+1}\). In the steady state, \(\chi^S_{FCM} < \chi^N_{FCM}\).

\[\square\]

**Proof of Propositions 11**

**Proof.** The steady-state gross international factor payment of country \(i\) is

\[R^*_i = \beta \omega^i_{FCM}(1 - \eta) + \Gamma^*_i \eta - (1 - \eta)R^i_{IFA} - \eta \Gamma^i_{IFA}].\]

Plugging equations (48) and (49), the gross foreign factor payment is constant at zero,

\[R^*_i = \beta \omega^i_{FCM} + \Gamma^*_i \Omega^i_{FCM} = 0.\]

\[\square\]

**Proof of Propositions 12**

**Proof.** Let \(P^i \equiv \frac{\rho_{AB}}{(1 - \eta)\alpha B}(\theta^i + \frac{\alpha - \alpha B}{\alpha B})\). Combining equations (57) and (58), we get

\[
\begin{align*}
\chi^i_{t+1} - \theta^i &= \frac{\omega^i_{t+1} P^i}{\omega^i_t} \frac{1 - \theta^i}{\Gamma^*_t}, \\
\chi^i_{t+1} &= \frac{\omega^i_{t+1} P^i}{\omega^i_t} \frac{1 - \theta^i}{\Gamma^*_t} + \theta^i, \\
\ln \omega^i_{t+1} &= \alpha \ln \omega^i_t - \alpha \ln P^i + \alpha_B \ln \left(\frac{\omega^i_{t+1} (1 - \theta^i) P^i}{\omega^i_t \Gamma^*_t} + \theta^i\right).
\end{align*}
\]
Let \( K^i_t = \frac{\alpha_B}{\chi^i_{t+1}} \frac{(1-\theta^i)\rho^i}{\Gamma^i_t} \omega^i_{t+1} \) and \( S^i = \frac{\Gamma^i_t}{(1-\theta^i)\rho^i} \).

\[
K^i_t = \frac{\alpha_B}{1 + \theta^i S^i \frac{\omega^i_{t+1}}{\omega^i_{t+1}}} < \alpha_B < \alpha
\]

\[
\frac{\partial K^i_t}{\partial \omega^i_t} = -\frac{(K^i_t)^2 S^i \theta^i}{\alpha_B \omega^i_{t+1}} \left( 1 - \frac{\partial \ln \omega^i_{t+1}}{\partial \ln \omega^i_t} \right) < 0
\]

\[
= -\frac{(K^i_t)^2 S^i \theta^i}{\alpha_B \omega^i_{t+1}} \left( 1 - \frac{1}{1 - K^i_t} \right).
\]

The property of the first derivative of \( \omega^i_{t+1} \) w.r.t. \( \omega^i_t \),

\[
\frac{\partial \ln \omega^i_{t+1}}{\partial \ln \omega^i_t} = \alpha + \frac{\alpha_B}{\chi^i_{t+1}} \frac{(1-\theta^i)\rho^i}{\Gamma^i_t} \frac{\partial \ln \omega^i_{t+1}}{\partial \ln \omega^i_t} - 1
\]

\[
\frac{\partial \ln \omega^i_{t+1}}{\partial \ln \omega^i_t} = \frac{\alpha - K^i_t}{1 - K^i_t} \in (0, 1)
\]

\[
\frac{\partial \omega^i_{t+1}}{\partial \omega^i_t}(1 - K^i_t) = (\alpha - K^i_t) \frac{\omega^i_{t+1}}{\omega^i_t}
\]

\[
\frac{\partial \omega^i_{t+1}(1 - K^i_t)}{\partial \omega^i_t} = \left( \frac{\partial \omega^i_{t+1}}{\partial \omega^i_t} \frac{\omega^i_{t+1}}{\omega^i_t} \right) \left[ \frac{\partial K^i_t}{\partial \omega^i_t} + (\alpha - K^i_t) \frac{1}{\omega^i_t} \right]
\]

\[
= \frac{\partial \omega^i_{t+1}}{(\omega^i_t)^2} \left( \frac{\partial \ln \omega^i_{t+1}}{\partial \ln \omega^i_t} - 1 \right) G^i_t
\]

where \( G^i_t = \left[ -\left( \frac{K^i_t}{1 - K^i_t} \right)^2 \frac{S^i \theta^i(1 - \alpha)}{\alpha_B \omega^i_{t+1}} \frac{\omega^i_{t+1}}{\omega^i_t} + \frac{\partial \ln \omega^i_{t+1}}{\partial \ln \omega^i_t} \right] \)

\[
= \left( \frac{K^i_t}{1 - K^i_t} \right)^2 \left[ \frac{1}{K^i_t} - 1 \right] \left( \frac{\alpha}{K^i_t} - 1 - \frac{G^i_t}{\alpha_B} (1 - \alpha) \right)
\]

\[
= \left( \frac{K^i_t}{1 - K^i_t} \right)^2 \frac{1}{\alpha_B} \left[ \frac{\alpha - \alpha_B}{\alpha_B} [(1 + S^i \theta^i \frac{\omega^i_t}{\omega^i_{t+1}})^2 - \alpha_B] + (S^i \theta^i \frac{\omega^i_t}{\omega^i_{t+1}})^2 \right] > 0.
\]

Since \( \frac{\partial \ln \omega^i_{t+1}}{\partial \ln \omega^i_t} \in (0, 1) \), we get \( \frac{\partial \omega^i_{t+1}}{\partial \omega^i_t} > 0 \) and \( \frac{\partial \omega^i_{t+1}}{\partial \omega^i_t} < 0 \).

Define a threshold value \( \omega^i_t = \mathcal{P}^i(\Gamma^i_t)^{-\frac{1}{1-\alpha}} \). For \( \omega^i_t \in [0, \omega^i_t] \), aggregate saving and investment in sector B is so low that the relative price reaches unity. In that case, the borrowing constraints do not bind any more and the phase diagram is flat with \( \omega^i_{t+1} = \omega^i_{t+1} = (\Gamma^*_t)^{-\theta} \). For \( \omega^i_t > \omega^i_t \), the phase diagram is monotonically increasing and concave, as proved above. Given \( \Gamma^*_t > \frac{\ell}{\theta} \), we get \( \omega^i_{t+1} > \omega^i_t \).

Thus, the phase diagram of wages crosses the 45 degree line once and only once from the left, and the intersection is in its concave part. Thus, the model economy has a unique and stable steady state under free mobility of FDI.

\[\square\]

**Proof of Propositions 14**

*Proof.* The steady-state equity rates in country \( i \) under IFA and under free mobility of
FDI are
\[
\begin{align*}
\Gamma_{IFA}^i &= \frac{\rho}{\beta} \left[ 1 + \frac{\alpha_B}{\alpha} \frac{1 - \eta}{\eta} (1 - \chi_{IFA}^i) \right] \\
\Gamma_{FDI}^i &= \frac{\rho}{\beta} \left[ 1 + \frac{\alpha_B}{\alpha} \frac{1 - \eta}{\eta} (1 - \chi_{IFA}^i) \right] \frac{1}{1 + \frac{\alpha_B}{\alpha} (\chi_{IFI}^i - \chi_{IFA}^i)}. \end{align*}
\]

Since the steady-state equity rate under IFA is higher in country S than in country N, the cross-country equity rate equalization implies that the steady-state equity rate in country S (N) is lower (higher) than under IFA, i.e., \(\chi_{FDI}^S > \chi_{IFA}^S\) and \(\chi_{FDI}^N < \chi_{IFA}^N\).

Under free mobility of FDI, the steady-state equity rate in country \(i\) can be reformulated as \(\Gamma_{FDI}^i = \frac{\theta}{\beta (1 - \eta)} (\frac{\alpha_B}{\alpha} (1 - \theta^i))^{\frac{\chi_{IFA}^i \omega_i}{\chi_{IFA}^i - \theta^i}}\). The cross-country equity rate equalization implies \(\frac{\chi_{S} - \theta^i}{\chi_{S} - \theta^i} (1 - \theta^i) = \frac{\chi_{N} - \theta^i}{\chi_{N} - \theta^i} (1 - \theta^i)\). Define \(Z(\theta, \chi) = \frac{(1 - \theta^i) + (\alpha_B + \chi)}{\chi - \theta^i} > 0\) as a function of \(\theta\) and \(\chi\), where \(\theta \in [0, \theta^i, \theta^N]\) and \(\chi \in [0, 1]\). Then, \(Z(\theta^S, \chi) < Z(\theta^N, \chi)\). The equity-rate equalization implies that \(\chi_{FDI}^S < \chi_{FDI}^N\) in the steady state.

**Proof of Propositions 15**

**Proof.** According to the wage dynamic equation (58), the steady-state wage \(\omega^i = (\frac{\Lambda^i \beta}{\rho})^\theta\), where the domestic composite productivity factor \(\Lambda^i\) depends positively on \(\chi^i\). In the steady state, \(\chi_{FDI}^S > \chi_{IFA}^S\) and \(\chi_{FDI}^N < \chi_{IFA}^S\) imply that \(\omega_{FDI}^S > \omega_{IFA}^S\) and \(\omega_{FDI}^N < \omega_{IFA}^S\). Given \(\theta^N\), we can find \(\tilde{\theta}^S, \chi^S, \chi^N\) that solve the three equations implied by \(\omega^S = \omega^N\).

Alternatively, we can prove it using the equation
\[
\left( \frac{\chi^N}{\chi^S} \right) \alpha_B = \frac{2\eta}{1 - \eta} \left( \frac{\chi^S - \theta^S}{\chi^S - \theta^N} \right) \left( \frac{\chi^N - \theta^N}{\chi^N - \theta^S} \right). \tag{50}
\]

The solutions of \(\chi^S\) and \(\chi^N\) are given as follows,
\[
\left( \frac{\chi^S - \theta^S}{\chi^S - \theta^N} \right) = \left( \frac{\chi^N - \theta^N}{\chi^N - \theta^S} \right) \left( \frac{\chi^N}{\chi^S} \right) \alpha_B.
\]

Plug in the solutions of \(\chi^S\) and \(\chi^N\) into equation (50), we can get \(\tilde{\theta}^S\) as a function of \(\theta^N\). Plot the function in the \((\theta^S, \theta^N)\) space. Thus, for \(\theta^S \in (\tilde{\theta}^S, \theta^N)\), aggregate output is higher in country S than in country N, while the opposite is true for \(\theta^S \in [0, \tilde{\theta}^S]\). \(\square\)
B Free Mobility of FDI

FDI flows equalize the equity rate across the border and the credit markets clear in each country as well as at the world level. FDI flows directly affect aggregate output of intermediate good B. To summarize,

$$\Gamma_t^N = \Gamma_t^S = \Gamma_t^*, \quad (1 - \phi_t^i)(1 - \eta)\beta\omega_t^i = (\lambda_t^i - 1)(\eta\beta\omega_t^i - \Omega_t^i), \quad \Omega_t^N + \Omega_t^S = 0,$$

$$M_{t+1} = \lambda^i(\eta\beta\omega_t^i - \Omega_t^i).$$

Except for these conditions, the equations governing the model dynamics are same as under IFA. The solutions to some endogenous variables are,

$$\Omega_t^i = \frac{1}{\eta} \left(1 - \frac{\alpha - \alpha_B}{\alpha} \frac{1}{\chi_{t+1}^i} \right) + \left(\lambda_{t+1}^i - \chi_{IFA}^i\right) \frac{\eta\omega_t^i}{\eta\chi_{t+1}^i} \Gamma_{IFA}^i$$

$$\lambda_t^i = \frac{1}{\eta} \left(1 - \frac{\alpha - \alpha_B}{\alpha} \frac{1}{\chi_{t}^i} \right) + \left(\lambda_{t}^i - \chi_{IFA}^i\right) \frac{\eta\omega_t^i}{\eta\chi_{t+1}^i} \Gamma_{IFA}^i$$

$$\phi_t^i = \frac{1}{\alpha(1 - \eta)} \left(1 - \frac{\alpha - \alpha_B}{\alpha} \frac{1}{\chi_{IFA}^i} \right)$$

$$R_t^i = \Psi_t^i \left[1 - \frac{\alpha_B}{\alpha} \left(1 - \chi_{IFA}^i\right) \right]$$

$$\Gamma_t^i = \Psi_t^i \left[1 + \frac{\alpha_B}{\alpha} \frac{1 - \eta}{\eta} \left(1 - \chi_{IFA}^i\right) \right] \left[1 + \frac{\alpha_B}{\alpha} \frac{1}{\eta\chi_{t+1}^i} \frac{1}{\frac{\alpha_B}{\alpha} \left(1 - \chi_{IFA}^i\right)} \right]$$

$$\omega_{t+1}^i = \left(\frac{\Lambda_t^i \beta \omega_t^i}{\rho}\right)^\alpha, \quad \text{where} \quad \Lambda_t^i = \frac{(\chi_{t+1}^i)^\alpha}{\left[1 - \frac{\alpha - \alpha_B}{\alpha} (1 - \chi_{IFA}^i)\right]}.$$
FDI flows affect aggregate output directly by raising (reducing) the size and indirectly by improving (worsening) the composition of aggregate investment in country S (N). Thus, aggregate output rises (declines) in country S (N). Equation (58) shows the dynamic equation of wages, where the domestic composite productivity factor $\Lambda_t$ is positively related to the relative price, which confirms the patterns of aggregate output.

More formally, given the predetermined labor income in period $t$, $\omega^S_t$ and $\omega^N_t$, the equity rate equalization and the international equity capital market clearing condition,

$$\Omega^S_t + \Omega^N_t = 0, \quad \Rightarrow \quad \frac{\eta}{1 - \eta}(\omega^S_t + \omega^N_t) = \frac{\omega^S_t(\chi^S_{t+1} - \theta^S)}{\alpha - \alpha_B} + \frac{\omega^N_t(\chi^N_{t+1} - \theta^N)}{\alpha - \alpha_B + \theta^N}$$

$$\Gamma^S_t = \Gamma^N_t = \Gamma^*_t, \quad \Rightarrow \quad \frac{(\alpha - \alpha_B + \theta^S)}{(\alpha - \alpha_B + \theta^N)} \left( \frac{1 - \theta^S}{\chi^S_{t+1} - \theta^S} \right)^{\frac{1}{\alpha}} \left( \frac{\chi^S_{t+1}}{\chi^S_{t+1} - \theta^S} \right)^{\rho_B} \frac{\omega^N_t}{\omega^S_t} = \left( \frac{1 - \theta^N}{\chi^N_{t+1} - \theta^N} \right)^{\frac{1}{\alpha}}$$

uniquely determine $\chi^S_{t+1}$ and $\chi^N_{t+1}$ in each period.

**Proposition 12.** Given the world equity rate, $\Gamma^*_t$, the phase diagram of wages in each country is monotonically increasing and concave for $\omega^i_t > \omega^i_t$, where $\omega^i_t \equiv \frac{\rho_B}{\rho(1 - \eta)}(\theta^i + \frac{\alpha - \alpha_B}{\alpha_B})(\Gamma^*_t)^{-\frac{1}{\alpha}}$; for $\omega^i_t \in (0, \omega^i_t)$, the phase diagram is flat at $\omega^i_t = (\Gamma^*_t)^{-\rho} > \omega^i_t$. The phase diagram crosses the 45 degree line once and only once. Thus, there exists a unique and stable steady state.

**Proof.** See appendix A. \hfill $\blacklozenge$

**Proposition 13.** The steady-state wage in country $i$ and FDI flows from country $i$ are

$$\omega^i_{FDI} = \left( \frac{1}{R^i_{IFA}} \right)^{\rho} \left\{ \frac{\beta_{\alpha}}{\rho \alpha_B} R^i_{IFA} \left[ \frac{\Gamma^i_{IFA}}{\Gamma^i_{FDI}} \right] \right\} . \quad (59)$$

$$\Omega^i_{FDI} = \eta \beta \omega^i_{FDI} \left( 1 - \frac{\Gamma^i_{IFA}}{\Gamma^i_{FDI}} \right) . \quad (60)$$

Given $\Gamma^S_{IFA} > \Gamma^N_{IFA}$, equation (60) and the world equity capital market clearing condition $\Omega^S_{FDI} + \Omega^N_{FDI} = 0$ imply that $\Gamma^*_{FDI} \in (\Gamma^N_{IFA}, \Gamma^S_{IFA})$. Accordingly, FDI flows from country N to country S in the steady state, $\Omega^S_{FDI} < 0 < \Omega^N_{FDI}$.

**Proposition 14.** The steady-state relative price in country S (N) is higher (lower) than that under IFA. Under free mobility of FDI, the steady-state relative price in country S is still lower than in country N. To summarize, $\chi^S_{IFA} < \chi^S_{FDI} < \chi^N_{FDI} < \chi^N_{IFA}$.

**Proof.** See appendix A. \hfill $\blacklozenge$

**Proposition 15.** The steady-state aggregate output in country S (N) is higher (lower) than that under IFA. $Y^S_{FDI} > Y^S_{IFA}$ and $Y^N_{FDI} < Y^N_{IFA}$. Under free mobility of FDI, the steady-state aggregate output in country S can be higher or lower in country N, depending on parameter values.
Proof. See appendix A.

Equation (55) shows that FDI flows do not affect the household portfolio choice. Take country S as an example. FDI inflows increase aggregate credit demand, which raises the loan rate in period \( t = 0 \). Meanwhile, the rise in the aggregate investment in intermediate good B raises the price of intermediate good A in period \( t = 1 \). The two forces just offset each other so that the household portfolio choice is unaffected.

**Proposition 16.** The loan rate in the steady state is same as under IFA, \( R_{FCF}^S = R_{IFA}^S \).

See subsection 3.2 for the loan rate dynamics.

Consider the welfare at the individual level and at the country level. Take country S as an example. In period \( t = 0 \), entrepreneurs lose from the decline in the equity rate, given the predetermined labor income, \( \omega_0^S \). In the long run, since the steady-state equity rate is lower but the steady-state labor income is higher than under IFA, entrepreneurs in the later generations may or may not benefit from FDI inflows, depending on which effect dominates. In period \( t = 0 \), households benefit from the higher loan rate, given the predetermined labor income, \( \omega_0^H \). In the long run, since the steady-state loan rate is same as but the steady-state labor income is higher than under IFA, households in the later generations also benefit from FDI inflows. The opposite welfare implications apply to individuals born in country F.

![Figure 7: Transitional Dynamics from IFA to Free Mobility of FDI](image)

Figure 7 shows the impulse responses of the model economy with respect to free mobility of FDI from period \( t = 0 \) on, given that the world economy is initially in the
steady state under IFA with $\theta^N = \bar{\theta} = 0.8$ and $\theta^S = \bar{\theta}/2 = 0.4$ before period $t = 0$. The axis scalings are same as those of figure 2.

The patterns of capital flows, aggregate output and interest rates in both countries confirm our analytical results. Here, we focus on the world output implications. FDI flows affect aggregate output in each country through affecting the size and the composition of aggregate investment. First, FDI flows raise (reduce) the size of aggregate investment in country S (N) through cross-country resource reallocation. Given our parameter choice of $\theta^S = 0.4$ and $\theta^N = 0.8$, FDI flows widens the initial cross-country output gap and the net investment size effect is negative for world output. Second, FDI flows improve (worsen) production efficiency in country S (N) through affecting the composition of aggregate investment and the net investment composition effect is positive for world output. Given our parameter choice, the net composition effect dominates the net size effect and world output is higher than under IFA.

Figure 8: Steady-State Pattern under Free Mobility of FDI versus IFA

Figure 8 compare the steady-state patterns under free mobility of FDI versus under IFA, given $\theta^N = \bar{\theta}$ and $\theta^S \in [0, \bar{\theta}]$. The axis scalings are same as in figure 3. For $\theta^S$ close to $\theta^N$, the net investment size effect due to cross-country resource reallocation dominates the net investment composition effect due to within-country resource reallocation among individuals with different productivity. Thus, the steady-state world output is lower than under IFA. For $\theta^S$ much smaller than $\theta^N$, production inefficiency is significantly more severe in country S under IFA. Thus, the net investment composition effect may dominate
the net investment size effect so that the steady-state world output is higher than under IFA.

Figure 9: World Output Gains under Free Mobility of FDI

Figure 9 shows the threshold values of $\theta^N$ as a function $\theta^S$ where the steady-state world output under free mobility of FDI coincide with that under IFA. The horizontal and vertical axes measure $\theta^S, \theta^N \in [0, \bar{\theta}]$. For $(\theta^S, \theta^N)$ in region A (B), the net investment size effect dominates (is dominated by) the net investment composition effect so that the steady-state world output is lower (higher) than that under IFA.